Response to the editor Q. Yin

« The paper of Didier Paillard provides a useful python package to compute several insolation quantities. I thank the two reviewers (Marie-France Loutre and Michel Crucifix) as well as Bryan Lougheed for their constructive comments on the paper, and also thank the author for his thoughtful reply. Based on these comments and the author reply, revisions are needed before the manuscript can be accepted for publication. I invite the author to submit a revised manuscript after considering these comments as well as my comments below. »

I thank the editor Q. Yin for these encouraging remarks and for the comments listed below.

« 1. Both reviewers mention that there is a lack of references in the manuscript. I share their opinion. In addition to the papers suggested by the reviewers, I would also recommend the papers below:
- Berger A., Loutre M.F., Tricot C., 1993. Insolation and Earth's orbital periods. Journal of Geophysical Research, 98 (D6), 10,341-10,362, https://doi.org/10.1029/93JD00222 .
- Berger A., 1978. Long-term variations of caloric insolation resulting from the earth's orbital elements. Quaternary Research,9 (2), 139-167, https://doi.org/10.1016/0033-5894(78)90064-9. »

In this revised version, I have now included references these two references as well as a few others:

- Crucifix, M. (2023). palinsol : a R package to compute Incoming Solar Radiation (insolation) for palaeoclimate studies (v1.0 (CRAN)). Zenodo. https://doi.org/10.5281/zenodo.14893715
- Oliveira, E. D.: Daily INSOLation (DINSOL-v1.0): an intuitive tool for classrooms and specifying solar radiation boundary conditions, Geosci. Model Dev., 16, 2371–2390, https://doi.org/10.5194/gmd-16-2371-2023, 2023.
- Lougheed B. : Orbitalchime, https://github.com/bryanlougheed/orbitalchime, 2025.
- Berger, A. : Long-Term Variations of Caloric Insolation Resulting from the Earth's Orbital Elements. *Quaternary Research*, *9*, 139–167, 1978b.
- Berger A., Loutre M.F., Tricot C.,: Insolation and Earth's orbital periods. *Journal of Geophysical Research*, 98 (D6), 10,341-10,362, https://doi.org/10.1029/93JD00222, 1993.
- Berger A., Yin Q., Wu Z.: Length of astronomical seasons, total and average insolation over seasons. *Quaternary Science Reviews,* 334, 108620. https://doi.org/10.1016/j.quascirev.2024.108620, 2024.

as well as references to the **code** from Berger (1978) and Laskar (2004):

- Berger, A. : Long-term variations of daily insolation and Quaternary climatic change, Journal of the Atmospheric Sciences, 35, 2362–2367, https://zenodo.org/records/7198109, 1978a.
- Laskar, J., Robutel, P., Joutel, F., Gastineau, M., Correia, A. C. M., & Levrard, B. : A long-term numerical solution for the insolation quantities of the Earth. Astronomy & Astrophysics, 428, 261–285 – doi: 10.1051–0004–6361:20041335, http://vo.imcce.fr/insola/earth/online/earth/online/index.php, 2004.

« 2. Regarding the "length of season" in section 2.3, can the author give in his paper the equation he used and mention the difference between his calculation and what has been published in Berger et al (2024)?
Berger A., Yin Q., Wu Z., 2024. Length of astronomical seasons, total and average insolation over seasons. Quat. Science Reviews, 334, 108620. https://doi.org/10.1016/j.quascirev.2024.108620. »

This is a welcomed suggestion. This reference has now been added, as shown in the above list, and the paper is mentioned when discussing season durations:

[line 177]:

« The length of seasons may itself be a quantity of interest for paleoclimatic studies, as discussed in Berger *et al.* (2024) »

The calculation is rather standard and is based on eccentric anomaly, as shown on Kepler's equation (equation (3) in the manuscript). It is the same as in Berger et al. (2024).

Point-by-point reply to M-F Loutre (RC1)

« The author presents and explains the different formula for the computation of the insolation, and other related values, that formed the basis for the a new version of AnalySeries, called "PyAnalySeries", to be released soon. As the author indicates some of the formulas presented in this paper are classical, while others are new. This paper is an important piece of work, to make all these formula available at one place. Many paleoscientists are using several of them (maybe even without knowing) and it will continue to be the case in the future. Therefore I consider that this paper is important and is worth to be published. Although the topic is difficult and technical, the author tried to be as pedagogical as possible. In that sense, the abstract clearly reflects the content of the paper. »

I warmly thank M.F. Loutre for these encouraging remarks and for the very useful and detailed comments listed below.

« Here are some more specific comments.
1. References
a. As the author recognizes that some formulas are classical, it would be good to offer a reference for that, in particular for section 2.2 »

Indeed, and this was also noted by Marie-France Loutre (therefore the same reply below).
In this revised version, I have included references to:

- Crucifix, M. (2023). palinsol : a R package to compute Incoming Solar Radiation (insolation) for palaeoclimate studies (v1.0 (CRAN)). Zenodo. https://doi.org/10.5281/zenodo.14893715
- Oliveira, E. D.: Daily INSOLation (DINSOL-v1.0): an intuitive tool for classrooms and specifying solar radiation boundary conditions, Geosci. Model Dev., 16, 2371–2390, https://doi.org/10.5194/gmd-16-2371-2023, 2023.
- Lougheed B. : Orbitalchime, https://github.com/bryanlougheed/orbitalchime, 2025.
- Berger, A. : Long-Term Variations of Caloric Insolation Resulting from the Earth's Orbital Elements. *Quaternary Research*, *9*, 139–167, 1978b.
- Berger A., Loutre M.F., Tricot C.,: Insolation and Earth's orbital periods. *Journal of Geophysical Research*, 98 (D6), 10,341-10,362, https://doi.org/10.1029/93JD00222, 1993.
- Berger A., Yin Q., Wu Z.: Length of astronomical seasons, total and average insolation over seasons. *Quaternary Science Reviews*, 334, 108620. https://doi.org/10.1016/j.quascirev.2024.108620, 2024.

as well as references to the code from Berger (1978) and Laskar (2004):

- Berger, A. : Long-term variations of daily insolation and Quaternary climatic change, Journal of the Atmospheric Sciences, 35, 2362–2367, https://zenodo.org/records/7198109, 1978a.
- Laskar, J., Robutel, P., Joutel, F., Gastineau, M., Correia, A. C. M., & Levrard, B. : A long-term numerical solution for the insolation quantities of the Earth. Astronomy & Astrophysics, 428, 261–285 – doi: 10.1051–0004–6361:20041335, http://vo.imcce.fr/insola/earth/online/earth/online/index.php, 2004.

I also provided a table to summarize different available software packages in Appendix D.

Name	Reference	code	interface	link to software	Daily inso	Integrated inso	Caloric seasons	Min/Max	above threshold
	Berger, 1978	Fortran		https://zenodo.org/records/7198109	x				
	Laskar et al, 2004	Fortran	web interface	http://vo.imcce.fr/insola/earth/online/earth/online/index.php	x				
AnalySeries	Paillard et al, 1996	C++	GUI on old macOS	... outdated	x	x	(*)		
PyAnalySeries	Hevia-Cruz et al, 2025	python	GUI multiplatform	https://github.com/PaleoIPSL/PyAnalySeries	x	x	(*)		
palinsol	Crucifix, 2023	R code		https://doi.org/10.5281/zenodo.14893715	x	x	(*)		x
DInsol v1.0	Oliveira, 2023	Fortran	GUI Windows/Linux		x				
orbitalchime	Lougheed, 2025	python		https://github.com/bryanlougheed/orbitalchime	x	x	(*)		x
inso	*this paper*	python		https://github.com/dpaillard/Insolation	x	x	x	x	x

Table A1: A short, non-exhaustive list of available software for insolation computations. The (*) symbol in the 'caloric season' column is meant to highlight that the software might not be able to compute the insolation over caloric seasons for all possible orbital configurations (in particular at low-latitudes with multiple maxima) but was designed mostly for the ice age problem (high latitudes and Earth-like parameters) as discussed in the main text (see for instance Figure 10).

But an exhaustive review on the subject would be quite a difficult endeavour, and certainly would go far beyond this paper. In particular, it would likely require some testing or benchmarking of these different packages.

Concerning section 2.2 (daily insolation), I now explicitly refer to the reference Berger (1978) [line 119].

« b. I also suggest to add a reference related to the use of the elliptical integral. I can suggest this one (Berger et al, 2010, https://doi.org/10.1016/j.quascirev.2010 .05.007) although there might be others as well. »

Thank you for this reference. I now have added it in the revised paper, as well as the older one, Loutre et al, 2004, which also introduces the elliptical integrals. [line 240].

« 2. Line 159: T=1 year. A few words about what is one year (Gregorian year, sidereal year, tropical year, anomalistic year, ...), which one is chosen and why might be welcome. »

I have added this precision in the new manuscript ("anomalistic year"):

[lines 163-165]:

« To be very precise, *T* should be one anomalistic year, ie. the (averaged) time for the planet to travel from perihelion to perihelion. But for the Earth, the difference with our civil year (based on the tropical year) is very small »

This choice is in fact not quite crucial for our purpose and a full justification (see AC1: https://doi.org/10.5194/egusphere-2025-2885-AC1) would be out of the scope of the paper.

« 3. Line 202. I do not fully agree with the author's definition of 'caloric insolation'. The caloric Summer half year is defined such that any day of the Summer half year receives more insolation than any day of the Winter half year. In particular, it means that in the tropics caloric half years may not be continuous »

Marie-France Loutre is right, and this point was also mentionned by Michel Crucifix (therefore the same reply below).
I changed the corresponding sentence in the introduction into:

[lines 41-42] :

More precisely, he [Milankovitch] defined the "caloric summer" as being the half-year during which the insolation, for any specific day, is larger that the insolation during the other half, the 'caloric winter'.

It happens that the computation of minima and maxima of daily insolation, developed later in the paper, allows for a very generic way to compute 'caloric insolation' following the correct definition given by Milankovitch for all latitudes or orbital parameters, even when the caloric half years are not continuous. I have therefore decided to include new functions in the library for this purpose. The corresponding 'true' caloric insolation is called WcalS (for caloric summer) while the former one computed as an approximation is now called WcalSSol (for caloric summer solstice-centered). The corresponding paragraph has been entirely rewritten:

[lines 205-218]

« In fact, Milankovitch used another possibility, which is to integrate the daily forcing over a fixed given duration, exactly half a year. More precisely, it is possible to find a threshold value such that the insolation exceeds it during half a year (the "caloric summer") while remaining below it during the other half (the "caloric winter") and to compute the corresponding integrated insolation over these two periods (J.m^{-2}), or equivalently the corresponding averaged insolation W_{CalS} and W_{CalW} (in W.m^{-2}) since the duration is by definition a constant (half a year). It is critical to note that W_S and W_{CalS} are in fact entirely different quantities as illustrated below on figure 2.

Though this definition is quite simple, the precise computation of W_{CalS} is difficult in the generic case, in particular when there are several maxima or minima of insolation during the annual cycle. This will be discussed later on in details in paragraphs 4 and 5 below. Still W_{CalS} is most often used in the context of Quaternary climates and ice ages, for Earth-like situations at rather high latitudes. Then, the "caloric summer" is extremely close to the half-year time period centered at the summer solstice as will be shown later (see figure 10). For such situations, we can therefore compute a very good approximation of W_{CalS} as $W_{CalS} \approx W_{CalSSol} = \frac{1}{T/2}\int_{\lambda_{Summer-year/4}}^{\lambda_{Summer+year/4}} W_D dt$. Similarly, we can also define the winter counterpart as $W_{CalW} \approx W_{CalWSol} = \frac{1}{T/2}\int_{\lambda_{Winter-year/4}}^{\lambda_{Winter+year/4}} W_D dt$. »

The new computation is presented in paragraph 5:

[lines 458-467]:

« But instead of using a fixed threshold, we can also use the same strategy in order to compute the Milankovitch insolation W_{CalS} (the integral over a 'caloric season') for all possible orbital situations, even when the daily insolation has several maxima during the annual cycle, as this is the case in low latitudes today. As explained in paragraph 3.1, we first need to find the threshold value such that the insolation exceeds it during exactly half a year (the "caloric summer"). Since, for any threshold $W_{Threshold}$, we can obtain as above the value(s) λ_{T1} and the value(s) λ_{T2} for which $W_D(\lambda_T)$ crosses it upwards and downwards, we can also compute the associated mean longitudes L_{T1} and L_{T2}, and the corresponding duration(s) $L_{T2} - L_{T1}$, and finally sum up all these durations in case there are several such intervals. In other words, it is quite straightforward to define a function $f(W_{Threshold})$ that computes the time during which the insolation remains above the threshold. By solving $f(W_{Threshold}) = 1/2$, we get the desired value of $W_{Threshold}$ and the caloric summer mean insolation W_{CalS} can then be computed as the integral above this threshold, using the above procedure. »

Finally, the two quantities are now compared on the new Figure 10:

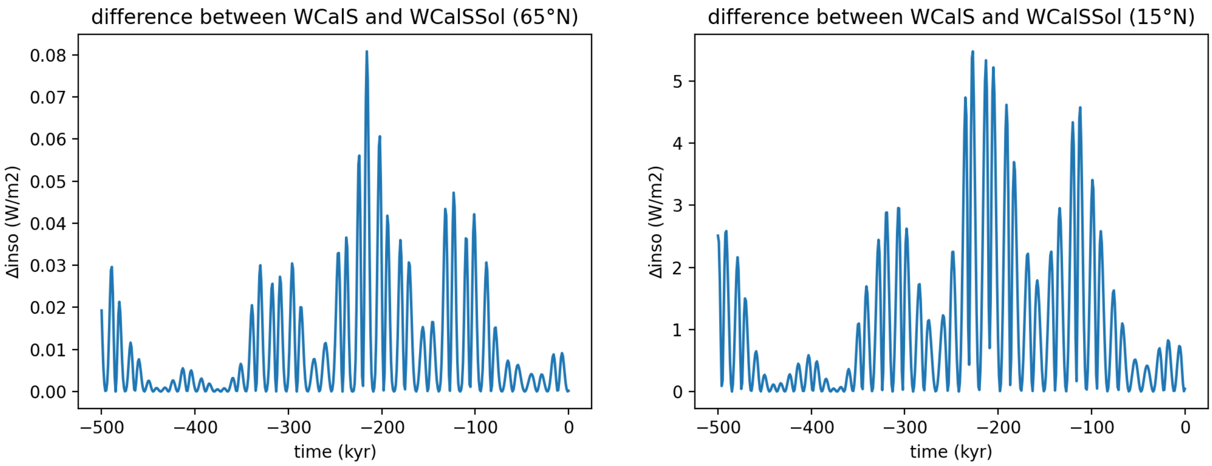

Figure 10: Comparison of W_{CalS} and $W_{CalSSol}$ at high northern latitudes (65°N) and at lower latitudes (15°N). The difference is negligible at 65°N (about 0.01%) but becomes significant at lower latitudes. The approximation $W_{CalSSol}$ in the context of Quaternary glaciations is therefore very good and much faster to compute, while it might fail in other contexts, like lower latitudes or other planetary settings.

« 4. Line 326. $\varphi_{Ext}(-\lambda,e,\varepsilon,-\varpi) = \varphi_{Ext}(\lambda,e,\varepsilon,\varpi)$. Isn't it also $\varphi_{Ext}(-\lambda,e,\varepsilon,-\varpi) = -\varphi_{Ext}(\lambda, e,\varepsilon,\varpi)$? »

Yes, thank you ! This is now corrected. [line 348]

« 5. Figure 4.
a. The titles of the bottom part read 'insolation %'. Percentage of what? Is it a percentage of the solar constant? This should be explained. »

Yes, it is indeed in percent of the solar constant, since these situations do not correspond to actual planets (at least, not in our solar system). This is now explained in the legend of Figure 4 :

[line 387].
« Note that the bottom panels correspond to fictitious planets: insolation is then expressed in percentage of the "solar" or "stellar" constant. »

« *b.* Would it be possible to explain in plain language what are 'turning points'? »

I guess that the difficulty was that the word 'turning point' was used in the figure legend before being defined properly in the main text. This is now corrected:

[line 355].
the points λ_M where $\varphi_{Ext}(\lambda_M)$ is extremal: these points λ_M ("turning points") give us the latitudes $\varphi_M = \varphi_{Ext}(\lambda_M)$ beyond which the number of extrema changes.

« 6. Line 450. Please provide here a full list of all the astronomical solutions that can be used in the library (or none of them). »

The list is actually given later on, [line 503-505] , so there is no need to repeat it. This list is now supressed.

« 7. Line 455. 'expressed in kyr AP'. Does that mean that time is expressed in thousands of year positive for the future and negative for the past? »

Yes, time is positive for the future (negative for the past). This is now specified:

[line 506].

« Time *t* is expressed in kyrAP (thousands of years after present), which means that *t* is positive in the future and negative in the past. »

« 8. Line 460. '...'Berger 1978' solution... is a trigonometric approximation of some older astronomical computations'. Berger (1978) is based on Bretagnon (1974) astronomical solution, which is a trigonometric solution of simplified equations (first order of the Lagrange equations) »

Indeed, this is more correct. In this revised version, I also added the Stockwell-Pilgrim solution and I reformulated this sentence accordingly:

[line 517].

« The 'Berger1978' or the 'StockwellPilgrim1904' solutions are given for historical purpose: they are trigonometric approximations: Berger (1978) is based on Bretagnon (1974) and Pilgrim (1904) is based on Stockwell (1873). »

« 9. Line 465. Laskar's solutions are given in years (or thousand years) before/after 2000A.D. while Berger (1978) is given in years (or thousand years) before/after 1950A.D. Does this affect the computation ? »

There is no correction applied to the time scale in my software. In fact, the « astro.py » library does not perform any computation at all beyond what is provided in the original publications (it is just a uniform interface). It is therefore the user's responsability to know what time scale is used as an input. This is now specified, and I have added a new function *info()* for the user to get easily this information, as well as some reference:

[line 507].

« The user should be aware that the starting time is different for these different solutions: for Laskar's solutions, *t* is in thousands of years after 2000 A.D; for Berger's solution, *t* is in thousands of years after 1950 A.D; for Stockwell's solution, *t* is in thousands of years after 1850 A.D.

This information, as well as the original references, can be easily obtained by calling the 'info()' function which is returning a string:

a.info() »

« 10. Lines 512 and 533. At line 512, refL is the true longitude of the reference point, while on line 533 refL is the mean longitude of the reference point. It is not so clear in the text. »

refL is always the **true** longitude, used as the zero for measuring « mean longitudes ». In other words, mean longitude (time) is computed from this point (usually = 0 for the march equinox). This is now explicitly mentioned in the description of the function:

[line 577].
« ... (or mean longitude) measured from a reference true longitude (refL) »

« 11. Line 535. Why is np.pi/2 used here? Isn't it the reference longitude? In that case why isn't it 0? »

The idea is to compute the integrated insolation over half a year centered at the june solstice. It is therefore convenient to use « june solstice » (true longitude = pi/2) as the reference longitude and then to compute mean longitudes (time) from this point, between -pi/2 and +pi/2 to get half a year.

« 12. Line 561. 'eps' should probably be 'obl'. »

Thank you. This is now corrected.

Point-by-point reply to M. Crucifix (RC2)

« The author proposes a new package written in Python for the computation of incoming solar radiation and integrals thereof, based on standard solutions for planetary motion and precession. The package is open source and licensed under the CeCILL free software licence agreement. Computation of insolation is fairly standard and already done in several other open source packages. Scientifically and computationally, a particularly valuable contribution of this article lies in section 4 where the author proposes numerical and analytical procedures for the computation of the minimum and maximum of the insolation for the year. One particularly interesting outcome is displayed in Figure 5, showing that in some realistic configurations equatorial insolation has only one and not two maxima. This could have some implications for the interpretation of the double-precession signal, which is relevant for cyclostratigraphic interpretations. The idea of computing insolation above a threshold by computing an elliptic integral between true solar longitudes that are first identified is elegant and welcome. »

I warmly thank M. Crucifix for his kind comments and for the very useful and detailed observations listed below.

 # Lacking references
Mention of existence of other, similar packages by other authors is only made once and very indirectly line 549: "while most of the routines available in 'astro.py' and 'insol.py', as described above, are available in several other software packages or languages"

Indeed, and this was also noted by Marie-France Loutre (therefore the same reply below).
In this revised version, I have included references to:

- Crucifix, M. (2023). palinsol : a R package to compute Incoming Solar Radiation (insolation) for palaeoclimate studies (v1.0 (CRAN)). Zenodo. https://doi.org/10.5281/zenodo.14893715
- Oliveira, E. D.: Daily INSOLation (DINSOL-v1.0): an intuitive tool for classrooms and specifying solar radiation boundary conditions, Geosci. Model Dev., 16, 2371–2390, https://doi.org/10.5194/gmd-16-2371-2023, 2023.
- Lougheed B. : Orbitalchime, https://github.com/bryanlougheed/orbitalchime, 2025.
- Berger, A. : Long-Term Variations of Caloric Insolation Resulting from the Earth's Orbital Elements. *Quaternary Research*, *9*, 139–167, 1978b.
- Berger A., Loutre M.F., Tricot C.,: Insolation and Earth's orbital periods. *Journal of Geophysical Research*, 98 (D6), 10,341-10,362, https://doi.org/10.1029/93JD00222, 1993.
- Berger A., Yin Q., Wu Z.: Length of astronomical seasons, total and average insolation over seasons. *Quaternary Science Reviews,* 334, 108620. https://doi.org/10.1016/j.quascirev.2024.108620, 2024.

as well as references to the code from Berger (1978) and Laskar (2004):

- Berger, A. : Long-term variations of daily insolation and Quaternary climatic change, Journal of the Atmospheric Sciences, 35, 2362–2367, https://zenodo.org/records/7198109, 1978a.
- Laskar, J., Robutel, P., Joutel, F., Gastineau, M., Correia, A. C. M., & Levrard, B. : A long-term numerical solution for the insolation quantities of the Earth. Astronomy & Astrophysics, 428, 261–285 – doi: 10.1051–0004–6361:20041335, http://vo.imcce.fr/insola/earth/online/earth/online/index.php, 2004.

I also provided a table to summarize different available software packages in Appendix D.

Name	Reference	code	interface	link to software	Daily inso	Integrated inso	Caloric seasons	Min/Max	above threshold
	Berger, 1978	Fortran		https://zenodo.org/records/7198109	x				
	Laskar et al, 2004	Fortran	web interface	http://vo.imcce.fr/insola/earth/online/earth/online/index.php	x				
AnalySeries	Paillard et al, 1996	C++	GUI on old macOS	... outdated	x	x	(*)		
PyAnalySeries	Hevia-Cruz et al, 2025	python	GUI multiplatform	https://github.com/PaleoIPSL/PyAnalySeries	x	x	(*)		
palinsol	Crucifix, 2023	R code		https://doi.org/10.5281/zenodo.14893715	x	x	(*)		x
DInsol v1.0	Oliveira, 2023	Fortran	GUI Windows/Linux		x				
orbitalchime	Lougheed, 2025	python		https://github.com/bryanlougheed/orbitalchime	x	x	(*)		x
inso	*this paper*	python		https://github.com/dpaillard/Insolation	x	x	x	x	x

Table A1: A short, non-exhaustive list of available software for insolation computations. The (*) symbol in the 'caloric season' column is meant to highlight that the software might not be able to compute the insolation over caloric seasons for all possible orbital configurations (in particular at low-latitudes with multiple maxima) but was designed mostly for the ice age problem (high latitudes and Earth-like parameters) as discussed in the main text (see for instance Figure 10).

But an exhaustive review on the subject would be quite a difficult endeavour, and certainly would go far beyond this paper. In particular, it would likely require some testing or benchmarking of these different packages.

These packages are never explicitly cited and the phrasing be read as casting doubt on their reliability: l. 10: "some people might not be aware of" ; l. 49 : "A frequent mistake" ; l. 204 : "as some people tend to believe".

It was not in my intention to cast doubt on existing softwares. I have changed or deleted some of these phrasings. Not the first one *"some people might not be aware of"* since this is obviously the case. *"A frequent mistake"* is now changed into:

« A mistake frequently made by students or non-specialists » [line 49]

and *"as some people tend to believe"* has been deleted. Into order to dissipate any misunderstanding, I further specify that computing integrated insolation:

« ... requires dedicated methods and softwares (see for instance table A1 in Appendix D). » [line 49]

In addition, the description of the incorrect procedure ("add daily insolation at some different positions") could be made more explicit to avoid ambiguity. Indeed, the time integral of any quantity can be well approximated with the rectangle or trapeze rule, over equally spaced intervals in time. That is: $\int_{\lambda 1}^{\lambda 2} W(\lambda)dt$ is correct; One can also replace the increment by $d\lambda * dt/d\lambda$. By contrast, it would be incorrect to simply substitute $d\lambda$ by dt . This is certainly what the author implies.

This is indeed what I meant. I now make this more explicit in the revised manuscript:

[lines 219-223] :

« It is therefore not correct to add the daily insolation taken at some different positions nor to take the corresponding average. In contrast, it is necessary to perform some specific computation using suitable software packages (see Table A1 in Appendix D). In other words, $\int_{\lambda_1}^{\lambda_2} W_D dt$ should not be confused with $\int_{\lambda_1}^{\lambda_2} W_D d\lambda$.

Indeed, here again it is critical to account for the non-uniform motion of Earth on its orbit since $d\lambda/dt$ is not constant. »

Note that neither palinsol nor, to my knowledge, DINSOL make this mistake.

I am sorry that the manuscript could potentially be misinterpreted in this direction: this was not my intention. I hope that the new formulation, as cited above, is much more explicit.

Reference to previous work is also lacking in the introduction of the elliptic integrals. Berger et al., 2010 introduces these equations (also reproduced in palinsol) and also introduces the history of their usage in the context of insolation calculations, citing works in German, e.g. Fempl.

Indeed, the elliptic integrals have already been described before and the formulas are now classical. In this revision, I am now referring to Berger et al (2010) and also to Loutre et al. (2004) [line 240].

My recommendation here would be to provide a more extensive review of previous work, outlining the specific needs addressed by this new package and acknowledge what has already been done successfully; and focus on the more specific contribution of identifying minima and maxima of daily insolation.

An "extensive review" is difficult to do and will certainly not be an "exhaustive" one. In particular, it would likely require some testing or benchmarking of these different packages, something entirely out of the scope of this paper. As mentioned above, I have provided a small table to summarize the characteristics of different software packages available on the internet, in Appendix D. This should help to better identify and explain the specificities of this new package, which is based on the possibility to compute minima and maxima of insolation, as mentioned in the abstract and in the introduction.

 # Incorrect definition of caloric insolation
The definition of caloric insolation provided by Milanković (1941; 2002), in his paragraph 87, "the half year that comprises all the days of stronger radiation", and therefore "experiences the greatest possible irradiation. The boundaries of this half-year are determined by solving a differential equation (his eq. 138), whose solution is used by Berger (1978) in his "Long-term variations of caloric insolation resulting from the earth's orbital elements", and not in general time-centred on the solstices as implied by eq. l. 215.

This is indeed true and is now explained in this revised version. This problem was also noted by Marie-France Loutre.
(therefore the same reply below).
I changed the corresponding sentence in the introduction into:

[lines 41-42] :
More precisely, he [Milankovitch] defined the "caloric summer" as being the half-year during which the insolation, for any specific day, is larger that the insolation during the other half, the 'caloric winter'.

It happens that the computation of minima and maxima of daily insolation, developed later in the paper, allows for a very generic way to compute 'caloric insolation' following the correct definition given by Milankovitch for all latitudes or orbital parameters, even when the caloric half years are not continuous. I have therefore decided to include new functions in the library for this purpose. The corresponding 'true' caloric insolation is called WcalS (for caloric summer) while the former one computed as an approximation is now called WcalSSol (for caloric summer solstice-centered). The corresponding paragraph has been entirely rewritten:

[lines 205-218]

« In fact, Milankovitch used another possibility, which is to integrate the daily forcing over a fixed given duration, exactly half a year. More precisely, it is possible to find a threshold value such that the insolation exceeds it during half a year (the "caloric summer") while remaining below it during the other half (the "caloric winter") and to compute the corresponding integrated insolation over these two periods (J.m^{-2}), or equivalently the corresponding averaged insolation W_{CalS} and W_{CalW} (in W.m^{-2}) since the duration is by definition a constant (half a year). It is critical to note that W_S and W_{CalS} are in fact entirely different quantities as illustrated below on figure 2.

Though this definition is quite simple, the precise computation of W_{CalS} is difficult in the generic case, in particular when there are several maxima or minima of insolation during the annual cycle. This will be discussed later on in details in paragraphs 4 and 5 below. Still W_{CalS} is most often used in the context of Quaternary climates and ice ages, for Earth-like situations at rather high latitudes. Then, the "caloric summer" is extremely close to the half-year time period centered at the summer solstice as will be shown later (see figure 10). For such situations, we can therefore compute a very good approximation of W_{CalS} as $W_{CalS} \approx W_{CalSSol} = \frac{1}{T/2}\int_{\lambda_{Summer-year/4}}^{\lambda_{Summer+year/4}} W_D dt$. Similarly, we can also define the winter counterpart as $W_{CalW} \approx W_{CalWSol} = \frac{1}{T/2}\int_{\lambda_{Winter-year/4}}^{\lambda_{Winter+year/4}} W_D dt$. »

The new computation is presented in paragraph 5, since it requires the calculation of minima and maxima:

[lines 458-467]:

« But instead of using a fixed threshold, we can also use the same strategy in order to compute the Milankovitch insolation W_{CalS} (the integral over a 'caloric season') for all possible orbital situations, even when the daily insolation has several maxima during the annual cycle, as this is the case in low latitudes today. As explained in paragraph 3.1, we first need to find the threshold value such that the insolation exceeds it during exactly half a year (the "caloric summer"). Since, for any threshold $W_{Threshold}$, we can obtain as above the value(s) λ_{T1} and the value(s) λ_{T2} for which $W_D(\lambda_T)$ crosses it upwards and downwards, we can also compute the associated mean longitudes L_{T1} and L_{T2}, and the corresponding duration(s) $L_{T2} - L_{T1}$, and finally sum up all these durations in case there are several such intervals. In other words, it is quite straightforward to define a function $f(W_{Threshold})$ that computes the time during which the insolation remains above the threshold. By solving $f(W_{Threshold}) = 1/2$, we get the desired value of $W_{Threshold}$ and the caloric summer mean insolation W_{CalS} can then be computed as the integral above this threshold, using the above procedure. »

Finally, the two quantities are now compared on the new Figure 10:

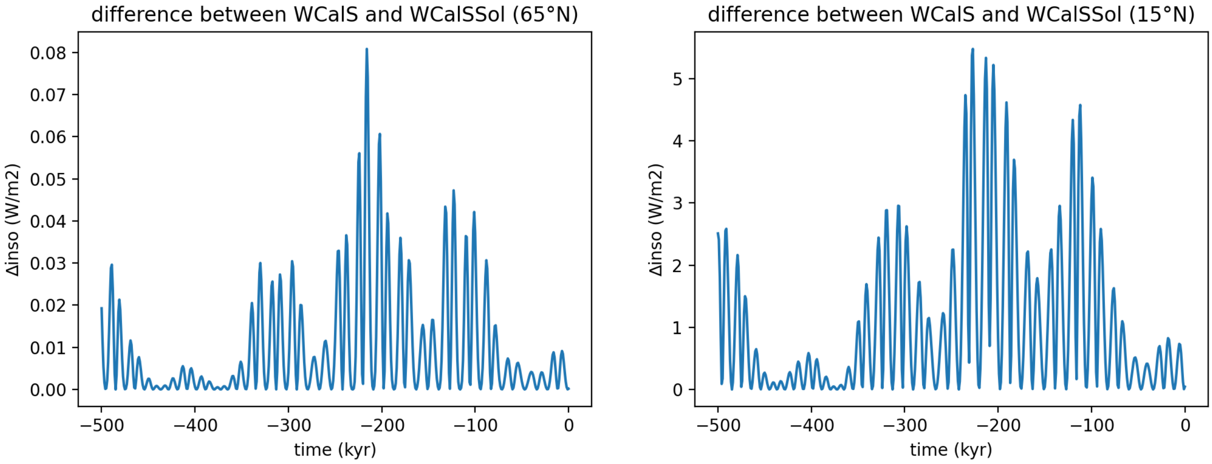

Figure 10: Comparison of W_{CalS} and $W_{CalSSol}$ at high northern latitudes (65°N) and at lower latitudes (15°N). The difference is negligible at 65°N (about 0.01%) but becomes significant at lower latitudes. The approximation $W_{CalSSol}$ in the context of Quaternary glaciations is therefore very good and much faster to compute, while it might fail in other contexts, like lower latitudes or other planetary settings.

Admittedly, the definition of Milanković is slightly ambiguous, since he does not explicitly mention that the half-year is meant to be continuous, though his Figure 42 implies this. This is a subtlety that I realised while doing the present review, and lead me to conclude that the brute-force approach in palinsol is not applicable in the tropics (this will be corrected in a next version).

I thank Michel Crucifix for these comments, which stimulated the addition of the new function described above, based first of all on the computation of minima and maxima, then on the computation of the threshold separating both caloric seasons. My understanding of Berger (1978) and Milankovitch (1941) is that they are both using an approximation similar to mine, as shown on formulae (165) to (167) in Milankovitch 1941 (§89) or on Berger (1978b) formula (3) to (6), that treat the start and end of caloric seasons in a strict symmetric way, something which is linked to the approximations used (first order in eccentricity). This is clearly stated in Milankovitch (1941):

> "this formula [166] is valid (...) on the condition that the term $2e(\lambda' - \lambda")\cos\varpi$ (where e as well as λ' and λ" are very small) is neglected as it is of a higher order of smallness".

In the end, the Milankovitch approximation of caloric insolation does not depend on λ' or λ", the true longitudes defining the caloric seasons, but only on their sum: $\lambda' + \lambda" \approx 4e\sin\varpi$. In other words, this approximation considers that the caloric seasons are symmetrically centered around the solstices. This is pointed out in the new paragraph:

[lines 473-478]:

When comparing W_{CalS} (insolation over the caloric summer) to its approximation $W_{CalSSol}$ (insolation over the half year centered at the solstice) as shown on Fig.10, we note that the difference is very small for the Earth at high northern latitudes. It is therefore fully acceptable to approximate W_{CalS} with $W_{CalSSol}$ and the approximations used by Milankovitch were in fact quite similar to the ones used here in $W_{CalSSol}$. Interestingly, the difference (about 0.01%) is much smaller than the one between maximum insolation and insolation at the solstice, as shown on figure 8 (about 0.3%), an approximation that many people are using without further caution.

Outside the tropics, I would consider that the computations of Berger (1978), and the brute-force approach in palinsol are, in principle, closer to the intended definition than in the current contribution.

I believe that the Berger (1978b) computation is similar to my definition of WcalSSol since it is symmetric (as explained above) while the brute force should be equal to WcalS (the true 'caloric insolation'). In any case, the difference is shown on figure 10 and is very small at high latitudes, for Earth-like situations.

 # Minor remark on geocentric vs heliocentric

The words "geocentric" and "heliocentric" are swapped ll. 125-126. Indeed, the equation coming just after eq 1, $\sin\delta = \sin\varepsilon\ \sin\lambda$ *emerges from the resolution of a spherical triangle on the celestial sphere (geocentric), with, thus* δ *positive when* λ *is between 0 and* π*.* λ *is in that case the true longitude of the Sun, and is 0 when the Sun is aligned with the first point of Aries.* ϖ *is, indeed, commonly supplied in e.g., Laskar, in heliocentric coordinates, thus* $\varpi = 0$ *when the perihelion in reach in September.*

Indeed. This is corrected.[lines 129-130]

Minor typographic / others

Use English quotes (") rather than guillemets(«).
l. 282 : for "a" generic planet.
l. 265 : "sun hour angle" rather than "time-step". ;

Thank you. This is now corrected.

l. 640 : hyperbolic functions: use LaTeX straight characters (as for $\cos$, etc.).

Since there is no hyperbolic in my formulae, I believe the confusion arises from 'cos*h*' or 'cot*h*' (with '*h*' italized) that stand for 'cos(*h*)' or 'cot(*h*)'. I have added the parenthesis in order to avoid confusion.

l. 252 : The sentence "This is a pity" is probably unnecessarily colloquial; yes analytical solutions need to be preferred to numerical approximations when necessary, however assuming convergence checks are made a numerical approximation is not necessarily inaccurate ;

I have removed this phrasing and added a short comment instead:

[lines 270-272]

« ... using simply a "mid-point value" may lead to wrong interpretations, since the actual forcing applied is not the same. Specifically, this might be the case at high latitudes near the polar night limit, like for instance in the context of Quaternary glaciations. »

Note that Eq. A2a -> A3 have not been thoroughly checked.

(I have double-checked them many times over several years… before deciding to publish this paper).

One point also that I forgot to mention: other state-of-the art solutions now compete with La04 and I would encourage the authors to consider orbital solutions by the group of R. Zeebe. This is specially relevant for studies beyond the Pleistocene.

I have added the old Stockwell-Pilgrim 1904 solution (the one used by Milankovitch in his 1920 publication). I will also gladly add more recent 'state-of-the-art' astronomical solutions in the future.

Response to comments from B. Lougheed

« I have read your manuscript with great interest, as I am myself have written a Python package for calculating past irradiance (https://github.com/bryanlougheed/orbital chime/), although I haven't gotten around to finishing a manuscript yet due to my current employment situation. Therefore, I had a look through your package and found that it works as described and will be helpful for researchers using Python. In addition to the helpful comments from reviewers Marie-France Loutre and Michel Crucifix, I have some further minor comments: »

These comments are indeed useful and I thank Bryan Lougheed for taking the time to look at the software package and write his comments.

« On referring to as "climatic precession" (ϖ).
In your manuscript, longitude of perihelion (ϖ) is referred to as "climatic precession" (line 23, 92 and 472). However, ϖ is not in itself precession (climatic or otherwise), but rather the orbital angle corresponding to longitude of perihelion, which is governed by changes in general precession (the combination of axial and apsidal precession), as you note in the second half of line 23. I would suggest to refer to ϖ as "longitude of perihelion", as is usually done in the literature. »

As always, the vocabulary might be problematic and may differ from authors to authors. Using usual notations (eg. Berger 1978a), we have: $\varpi = \psi + \Pi$
where indeed ϖ is the "longitude of perihelion" but only with respect to the moving vernal point. Π is also the "longitude of perihelion", but this time with respect to the fixed stars. In the astronomical community, "longitude of perihelion" refers unambiguously to this angle Π, not to ϖ. The terminology "longitude of perihelion" is therefore unsufficient and likely to be ambiguous. Besides, ψ is also usually called the "general precession" and corresponds to the precession of equinoxes. In order to avoid confusion (many students do have difficulties to understand the difference between general precession, precession of the perihelion and climatic precession), it is therefore critical to specify that climatic precession is associated to the angle ϖ (and combines both the precession of the perihelion measured by Π and the precession of equinoxes measured by ψ). And to be more precise, "precession" is the phenomenon associated with the change of these different angles. So, indeed, it would be important to add the word "angle". I should therefore state either :
- ϖ is the longitude of perihelion with respect to the moving vernal point.
or :
- ϖ is the angle measuring climatic precession.
I am now specifying "climatic precession angle" everywhere in the revised manuscript.

*« The term "climatic precession" has typically been used in the palaeoclimate literature for e*sin(ϖ), an index that visualises both eccentricity and longitude of perihelion in tandem (Imbrie and Imbrie, 1982; Vernekar, 1972; most likely also earlier works), which you have described in line 476 as the "climatic precession parameter", but can be simply described as "climatic precession". »*

I disagree, since e*sin(ϖ) is a mathematical mixture of two quantities: eccentricity *e* and "the angle measuring climatic precession" ϖ and it is important to identify correctly the individual astronomical parameters. In mechanics, "precession" is always a moving angle, like for instance the 3 quantities ϖ, ψ and Π mentionned above. When taking the sine and multiplying by eccentricity, this cannot be called "precession" anymore. Besides, it is quite a convention to take the sine (taking the cosine is also perfectly acceptable), therefore the word "parameter" seems useful to me to underline that this is a rather arbitrary choice. I did not invent the terminology "climatic precession parameter" and it is also widely used in the community, though indeed many people working on paleoclimates tend to use "climatic precession" and even more often just "precession" as a shortcut. The aim of this manuscript is to explain a few subtleties of insolation quantities and I therefore want to be as precise as possible in the terminology.

« Elliptical vs numerical integration

Regarding your text that I have quoted below:
The simplest way to produce the corresponding time series for Quaternary climates is to compute the daily insolation for every day of the year, then select and sum the ones above the chosen threshold. This might not be the most efficient way. Besides, discretizing the year into an integer number of days (360 or 365 days) is not entirely rigorous. Discretization itself might lead to slight numerical errors.
Here, you are discussing the disadvantages of using an a numerical integration instead of an elliptical integration. A numerical integration would indeed involve calculating irradiance across discrete intervals, but typically this does not involve breaking down the year into 360 or 365 discrete days. Mean daily irradiance can be calculated at any desired discrete interval resolution of the year (e.g. 1/10000th of a year resolution, or better), after which numerical integration can be carried out on the intervals using, e.g., a midpoint, trapezoidal, etc., approach. Berger et al. (2010) compare the results of elliptical and numerical integration for various irradiation quantities in their Tables 1a-c, 2a-c, where differences between the two methods are found to be near-negligible.

Therefore, I would argue that the "numerical errors" or lack of "rigour" are not practical reasons for using elliptical integration instead of numerical integration. In any case, there is already a very small loss of precision in palaeo-irradiance calculations due a number of approximations, including, for example, the assumption of the orbital paramaters remaining constant throughout the tropical year (in particular in the case of ϖ). Therefore, precision becomes a somewhat irrelevant reason for choosing either elliptical or numerical integration. »

I agree that numerical integration can be as precise as desired when increasing the time step and that is is indeed possible to used "as many days as necessary" to get a good precision when, for instance, computing integrated insolation. Still, when computing the sum of daily irradiance, people generally use the actual duration of actual days, ie. 86400 seconds (eg. see Huybers, 2006: $J = \sum_i W_i * 86400$) since it looks much more natural, even if summing up 36500 days of 864 seconds would indeed be more precise. What I had in mind is not so much the possible computation methodologies, but much more the actual usual practice.
I changed somewhat the formulation in the paper, which now reads:

[lines 434-437] :

« Besides, discretizing the year into an integer number of days (360 or 365 days) as in Huybers 2006 (using the sum $J = \sum_i W_i * 86400$) is not entirely satisfying. Indeed, the number of days is not an exact integer and such a discrete sum will change discontinuously when the threshold is modified continuously. »

« You mention efficiency, and I agree that this could be a decisive motivation for using elliptical integration, depending on computational resources and the size of the dataset (e.g. number of kyr and/or latitudes) needing to be computed. »

Efficiency is indeed important in many cases and algorithms for computing elliptic integrals are very efficient. But having an analytical expression for insolation series in general has also many other benefits, like a better theoretical understanding, or the possibility to apply other analytical methods (equations, derivatives, integrals, ...). I certainly prefer to express results as analytic expressions, whenever possible, not only for efficiency.

« Approach used to solve the Kepler equation for E
When solving the Kepler equation for E, there is no closed-form algebraic solution for E, so other methods must be used, typically involving an iterative approach that converges on the correct value for E to within a desired precision. I saw that in your py files you make use of the Halley's method option within SciPy's optimize.newton function. Perhaps you could consider citing the necessary literature as well as the SciPy package itself, seeing as solving the Kepler equation is of such importance for many irradiance calcuations. »

I am not sure that citing Edmund Halley would be very useful for the reader:
Halley (1694) "Methodus nova accurata & facilis inveniendi radices æqnationum quarumcumque generaliter, sine praviæ reductione". Philosophical Transactions of the Royal Society (in Latin).
I am now citing the SciPy package since, indeed I rely on it for several computations.

Pauli Virtanen, et al. (2020) SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python. Nature Methods, 17(3), 261-272. DOI: 10.1038/s41592-019-0686-2.

Python documentation strings
You have provided some documentation for your python functions as commented out text (using #) in your py files, above the def line of the functions. This documentation can only be accessed by the user by opening the py file in a text editor. You could instead include the documentation as a python docstring block (bookended using """) just below the def line, which would also make your documentation callable to the user's python environment using the help(functionname) or ?functionname commands.

Thank you for the suggestion!
I will gladly implement this type of comments in future releases of the software.

I look forward to seeing the work published in Climate of the Past and hope that my comments have been of some help.

Yes, they are certainly useful. Thanks again.