Response to the reviews (manuscript egusphere-2025-2839)

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We are extremely grateful and sincerely thank the two Reviewers for their exceptionally thoughtful, careful and constructive comments on our manuscript. We feel priviledged to have benefited from such high-quality reviews. We also thank the Editor for ensuring such an efficient and effective review process. Below, we explain how we have addressed the Reviewers' comments and questions in the revised manuscript.

Point-by-point response to Reviewer 2's comments

We thank Reviewer 2 for their very careful reading of the whole manuscript, their throughtful comments and their help in ensuring a better readibility of the paper.

Summary

In this study, the authors examine the behaviors of thermals, and the cumulus clouds they support in the trade-wind region. They use a combination of observations taken from multiple platforms during the EUREC4A campaign to examine the interplay between thermals and clouds. Specifically, they find that their size distribution can be explained by the sum of two exponentials which they further determine to be related to "merged" and "unmerged" object (i.e., cloud or thermal) populations. This theoretical framework describing the relationship between merged and unmerged populations controlling the total behavior of trade thermals and clouds comes from extensive analytical calculations that are further validated through comparison with a one-dimensional statistical model. The attraction between clouds due to their convective circulations, helping this merging to happen, is also mathematically detailed. Once tested, the authors show the strength of this merging population framework through interpreting the EUREC4A (and later the deeper convective MAESTRO) observations. They find that the behaviors can be described well as interactions between merged and unmerged populations of clouds and thermals. Such interactions appear to shape the characteristics of the mesoscale cloud patterns observed in the tropics (e.g., gravel vs. Flower behaviors) and set the upper bound on the amount of cloud cover, which has crucial implications for trade cumulus feedback.

This is an exceptionally well-crafted and thoughtful study, expertly grounded in the literature and employing a wealth of cutting-edge observations as well as classical theory and statistical models. The results have the potential to revolutionize how the field thinks about tropical cumulus clouds; their development from and interaction with thermals; and how this interplay shapes cloud organization and ultimately influences the climate. In particular, the detailed analytical calculations and elegant theoretical framework developed in this work is a significant step forward in describing how cumulus, and potentially deeper convective clouds, develop, organize, and persist. This will likely be a major contribution to the field, advancing the fundamental understanding of cloud behaviors with critical implications for how they will respond under future climates and how we can improve their representation in high resolution models. I have noted places where analysis details could be clarified and otherwise urge prompt publication of this excellent manuscript.

Detail Comments

General: Please provide R2 values for scatter plots and be consistent about using R2 throughout (later figures use R instead). Assume these are all Pearson (Line 443) correlations at 95% confidence? Worth mentioning that somewhere as well.

Done.

Main Text

Figure 2, 5: Following the Q-Q plot in Figure 1, please provide the R2 values for the individual flight fits.

Done.

Line 50: It might be worth noting that many clouds are not classified into any of these four patterns (e.g., Schulz 2022)

True, but this sentence is about the 'most prominent patterns'.

Line 132: What instrument are you getting the vertical velocity from? Apologies if I missed this in the methods.

We added in section 2: "The aircraft measured turbulence (including horizontal and vertical velocity, inferred from a five-hole nose radome measurements) and humidity..."

Figure 4, Lines 159-161: It looks like the figure described in the text and the caption do not match the actual figure. However, it sounds like a very intriguing result, so please include.

We added to the Supplementary material (Fig. S3) a figure similar to Fig. 5 but considering non-drizzling clouds only. In that case, $L_1^{CLD} \approx L_2^{CLD}$ and the distributions are well fitted by a single exponential.

Figure 5: It might be helpful to indicate the heights (or at least the dominant types of legs that were flown) in each flight. It could help to illustrate your point about change in exponential behavior with height (i.e., one to two populations).

All panels of this figure show the cloud chord length distributions at cloud base (this level was sampled several times during each flight). The PDF is fitted by one or two exponentials depending on the cloud types in presence.

Figure 6: Please describe in more detail what is being shown in the different panels in the text. I think I follow your logic, but it would help to have your reasoning more explicitly stated. Also please label the panels (a-c).

Done.

Line 193: Just to be clear, the betaD0L0 is the ratio to determine if there is merging?

Yes. We added: "These physical arguments thus suggest that the product $\beta\mathcal{D}_0 L_0$ describes a merging efficiency."

Line 217-220: where are you looking in Figure 6? Would be helpful to describe in a bit more detail how you see this.

We now write: "These calculations, illustrated by Fig. 6, thus show that the merging of thermals that are characterized initially by an exponential size distribution of length scale L_0 produces a second population of thermals, and that the size distribution of the thermal population after merging can be represented by the sum of two exponential functions, characterized by two length scales L_1 and L_2. In the absence of merging ($\beta\mathcal{D}_0 L_0 = 0$), $L_1 = L_2 = L_0$ (Fig. 6a) and $p_1 = p_2 = 0.5$ (Fig. 6b): the size distribution can be represented by a single exponential." and later: "There is therefore a critical merging efficiency of thermals beyond which the merging becomes so efficient in producing larger but fewer thermals that the densities of thermals before and after merging become anti-correlated (Fig. 6c)."

Line 224: I don't understand how you got this result, could you please add a little more detail?

The value of $\beta\mathcal{D}_0 L_0$ that maximizes $\mathcal{D}$ is found by solving the equation $\frac{\partial \mathcal{D}_1}{\partial(\beta\mathcal{D}_0 L_0)} = 0$ (added to the text).

Line 232, eq. 9: Would you please explain how you get to this in more detail? And why you are using a Lambert W function here?

We obtain this equation by combining equations 3 and 4 to eliminate L_0 and obtain an implicit equation for $\beta\mathcal{D}_0 L_0$.

Line 234, eq. 10: How do you get to the L0=sqrt(L1L2) result here?

We added: the second expression for L_0 is obtained after a multiplication of equations 3 and 4, followed by a first order Taylor expansion of the exponential function.

Line 259-261: Do you think the merging here is assisted by mesoscale circulations in the real world? E.g., Janssens et al. 2023, 2024

In the real world yes (it is discussed later in the paper), but by construction it is not the case in these simulations with a homogeneous $\mathcal{D}_0$.

Line 268, Figure 7c: Please add the statistical comparison here, such as reporting the R2. It looks like theory is underpredicting the medium to larger sizes a bit. I would have thought it should be better at the larger sizes based on the assumptions, which focuses on capturing the behavior in the larger size tail (if I understood correctly). This is particularly true for the merged case. Is there a reason we would expect this disagreement?

R^2 added. In figure 7c, the theory is overpredicting L_2 for large values of $\mathcal{D}_0 L_0$. For a given size distribution, we indeed expect the model to better match observations for large lengths. However, here, it is not the same comparison : each dot on the figure corresponds to different parameters. And in the computation of L_2, we have used an approximation based on equation S73. This equation is a strong constraint for small values of $\beta\mathcal{D}_0 L_0$ (see eq S74) and so we expect the theory to work better in this case. This corresponds to the situation when L_2 is relatively small and $L_2 \approx L_0$. On the contrary, when $\beta\mathcal{D}_0 L_0$ is very large, equation S73 is a very loose inequality and so does not constrain L_2 well. So it is not surprising that we observe a worse agreement with the numerical statistical model in that case. Solving analytically the exact equations for L_2 would be great to alleviate this problem, but we were not successful in this.

Line 293-294: Please add R2 to know how much variance is explained with this linear proxy.

Done.

Line 296: Please report the R2 for this correlation

We did not report it because since the relationship is non monotonic, we would need to add four R2 values (for L, for B, for weak merging, for large merging), with the risk of distracting the reader.

Figure 8: Please add R2 for all these plots. Is the dashed line in d the best fit line? Otherwise, it looks like the dashed lines are 1:1? Are the error bars here and throughout (e.g., Figure 11) for a single standard error (or deviation), so 68% confidence?

Done.

Figure 9, Lines 308-321: It would be helpful to provide a little more detail in how to interpret this figure. It looks like the caption is inconsistent with what is being shown as well, please clarify. Specifically, for 9a, LTH1 and LTH2 are mentioned but not shown and the colors are green and brown in the caption but gray in the figure.

Sorry for the confusion with the colors (corrected). Regarding the caption of panel (a): the mention of L_1^{TH} and L_2^{TH} was just a reminder of how L_0^{TH} was computed, but since it was misleading, we don't mention it anymore. Figure 9b has been simplified: we indicate more clearly (with a number) whether a flight was associated with one or two cloud populations, and we don't mention the range of L_1^{TH} and L_2^{TH} in the subcloud layer (we show it only at cloud base). Finally, we added a Figure to the SI (Fig. S4) showing more explicitly the relationship between L_0^{CLD} and L_1^{TH} in the presence of a single cloud population, and between L_0^{CLD} and L_2^{TH} in the presence of two cloud populations. The caption has been rewritten so as to clarify the interpretation of the figure.

Line 344-346: It's exciting to have two observations of the mass flux to compare here. Would it be possible to show the scatter between Mb estimates from the two different methods in the appendix? Or at least report the R2? Both would be preferable if doable.

A detailed comparison between the two Mb estimates is available in the supplementary material of Vogel et al. (2022, Fig S2d). We added the R^2 values for each relationship.

Figures 10 and 13: These are very helpful and elegant diagrams, please make sure to reference in the text somewhere and thank you for including.

Sorry about that. Done.

Figure 11b: why are some of these scatter points in gray?

We added to the caption: grey markers correspond to the flights whose mixed layer depth suggests that they were influenced by cold pools.

Line 346-348: Are you inferring this from comparing 11a to b? Might be helpful to color the points by Mb in 11b or something so it is more explicit.

We prefer keeping the figure as it is for the sake of homogeneity with the other panels. Since M_b and W_b are shown side by side, and the flight numbers are indicated on each figure, the reader can easily check the correspondance between M_b and W_b.

Line 348-350: Are you referring to the weak positive relationship here? Would be helpful to add the R2

Given the small sampling size (18 flights at most) and the many factors that can affect the correlation between quantities derived from observations from two different aircraft (with different flight patterns) and from different days (associated with a lot of meteorological variability), this figure intends to show relationships that are more qualitative than quantitative. Therefore, we feel that reporting R^2 values is unnecessary and potentially misleading for this figure.

Line 356-357, 11b,d: Please consider adding the R2 here as there is a fair amount of variability in this relationship.

Same response.

Line 363: Also, Janssens et al. 2024?

Added.

Line 368-369: This is quite an interesting hypothesis; I hope you pursue this idea further.

Thank you.

Line 390: Please indicate which panel in Figure 11 you are highlighting here.

Done.

Line 430-436: This is very interesting. Do you think that the merging helps to transition the organization types? i.e. from something short lived, like Sugar and Gravel, to something long lived like Flowers? This is what happens in Narenpitak et al. 2021 and you discussed seeing something similar between flights 15 and 16. Might be worth mentioning the Lagrangian evolution implications somewhere (if not already in the discussion).

Yes, it is exactly what we think, and it is discussed in the Summary section.

Figure 14b: Please report R2 to be consistent.

Done.

Appendix A

Line 606-607: Is this consistent with the discussion in the main text of having the thermals merging closer to the cloud base and not at the surface? This would seem to merge at the base but still have separate updrafts aloft.

Our model should be valid when thermals are extremely coherent structures with $T_{life} \gg T_{transit}$. This is a simplification especially for thermals, for which T_{merge} and T_{life} are in general of the order of $T_{transit}$. Therefore, as the thermals approach each other they probably also rise and may even detach from the ground, which is not accounted for in our model. As a result we expect to find more merged thermals higher in the subcloud layer. Also, thermals grow by entrainment which we have neglected in our model and can lead to more merging in the upper layers. So to summarize, this mathematical model is a very simplified view, which probably deserves some further investigation.

Line 627-628: I don't understand what this is saying, please clarify

We noticed that there was an error in the original manuscript (the distance that needs to be considered is between the centers of the updraft and not only between the edges); the correction results in a slightly different expression for β (the 1/2 factor disappears in the different equations defining β), and in a different γ value in Figure 14b (3/2 instead of 3), but it does not change anything else in the manuscript, nor in the supplement, because to estimate β from observations we use the coverage fraction (eq 11).

We also hope the revised text clarifies the calculation: "In other words, taking into account the influence of the thermal-induced circulations on merging amounts to replace the actual updrafts by effective objects whose size is the actual size of the updrafts multipled by β."

Supplemental Material

S11: Please explain why this is the "actual" coverage?

We added: "By definition, the size of effective updrafts is larger than their actual size by a factor of β. The actual coverage fraction is thus obtained by dividing the coverage fraction of effective updrafts by β."

Section S2.1: I think I am missing the definition of P0 and P0eff that are used in these equations. I couldn't find them here or in the main text, apologies. Would be very helpful to have explicitly stated somewhere to follow the substitutions made later in the calculations.

We added: "We will note $\mathbb{P}_0^{eff}$ the associated probability function for the exponential population of effective updrafts before merging: $\mathbb{P}_0^{eff}(x) = \mathcal{D}_0\mathbb{S}_0(x)$".

S13: Please explain how Pmerging becomes P0eff?

We now write: "Now, we compose this equation by the logarithm function. We use the fact that dy and dz are infinitely small, as well as the properties of the logarithm function, to transform the left member as follows:

$$\ln(1 - \mathbb{P}_{merging}(x,y)\mathrm{d}y) = -\mathbb{P}_{merging}(x,y)\mathrm{d}y \tag{1}$$

and the right member as follows :

$$\ln\left(\prod_{z=-y}^{x}(1 - \mathbb{P}_0^{eff}(y)\mathrm{d}y\mathrm{d}z)\right) = \sum_{\text{all}\,\mathrm{d}y\,\mathrm{d}z}\ln(1 - \mathbb{P}_0^{eff}(y)\mathrm{d}y\mathrm{d}z) = -\int_{z=-y}^{x}\mathbb{P}_0^{eff}(y)\mathrm{d}y\mathrm{d}z \tag{2}$$

We therefore obtain the following equation:...

S16-20: Please share a little more detail in how you do this. I think it is the same strategy you apply later as well, right?

The development added as an answer to the previous question probably has already clarified how the product transforms into an integral. We also added a step between the (original) equations S16 and S17 to make the computation even clearer ($= \int_0^\infty -\mathbb{P}_{merging}(x,y)\mathrm{d}y$).

S22-S24: I don't follow how S22 collapses into 23 and 24, please share a little more detail

We have now added an equation after the (former) equation S22 making it clear that the result is an exponential size distribution: $= \frac{\beta\mathcal{D}_0}{L_0}e^{-\beta\mathcal{D}_0 L_0}e^{-\left(\mathcal{D}_0+\frac{1}{\beta L_0}\right)x}$ and have clarified the derivation of the following equations.

Above S25, S65: It would be very helpful to discuss how you used homothety here and potentially include a diagram. I am not familiar with this technique, and it seems crucial for this calculation and how you connect it to the framework used for the observational analysis.

The homothetic transformation of a size distribution is now more detailed (between the former equations S24 and S25, which now correspond to S28 and S30).

S27: Please clarify how you derive the density.

We added: "By comparing with the expression of $\mathbb{P}_1^{eff}$ we also obtain the density of non merged updrafts" (just before equation S32, which was formally numbered S27).

S28: Please say a little more about the "simple arithmetic calculations" here

We now write: "If we solve for the equation $\frac{\partial \mathcal{D}_1}{\partial(\beta \mathcal{D}_0 L_0)} = 0$ we find that..."

S41: Please clarify how you get to this, it seems like we have lost some constants.

Sorry, this equation is obviously wrong as you noted. This is only a writing mistake because the other equations are right after. It is now corrected (new eq S46-S47).

S60-62: Please clarify how you get these approximations.

We only use the approximation once between (former) equations S60 and S61 (now S66 and S67). For clarification, we haved replaced "for which $\mathcal{D}_0 x \gg 1$" by "for which $e^{\alpha \mathcal{D}_0 x} \gg 1$". Note that another typo was corrected in (former) equation S60 (now S66): $e^{-\frac{x}{L_0}}$ should have been $e^{-\frac{x}{\beta L_0}}$.

Discussion of S69: Please consider including this figure showing the simulations match the equation well, would be very valuable to have. Maybe add in an appendix for the main text?

We now write: "Moreover, this expression has been validated against numerical experiments (section 5.2 of the main manuscript) and equation S75 matches well the simulations. Proving mathematically equation S75, by relaxing the hypothesis of a constant α, remains a perspective for future work."

Typographical Comments

Main Text

Figures 4, 7d, 9a, 11a,c: The edge labels/legends have been cutoff for these.

Corrected.

Line 48: "(George et al., 2023)"

Done.

Line 69: "the HALO"

Done.

Line 101: "anomalies called thermals"

Unchanged.

Caption for Figure 2: "shown on Fig. 1c" rather than 1a?

Corrected.

Line 204 and elsewhere: suggest saying "is written as" instead of "writes" for describing these equations.

Done.

Line 256-258: Suggest splitting into two sentences.

Done.

Figure 7: colors appear different between the caption and figure: blue ¿ purple and red ¿ pink

Done.

Figure 12: consider reordering panels so they are discussed in order in the text. Also, is the dashed line in b the critical merging efficiency?

Unchanged.

Line 448: “Feb”

Corrected.

Appendix A

Line 629: “its”

Done.

Line 648: “as is the”

Done.

Supplemental Material

Above S5: “is” instead of “writes”

Done.

Above S57: only one “that”

Done.

Above S58: “updraft sizes to the”

Done.

Above S64: “to an”

Done.

Above S67: “L2 is not”

Done.

Below S67: “strong constraint”

Done.