Dear Reviewer

Thank you for your thorough review of our manuscript. Your comments have helped us to improve the manuscript. Below we provide a point-by-point response to your comments.

Comment 1: The authors claim that recurrence analysis has not been applied to polar cap indices (PCN/PCS) in previous literature. In my opinion, this should be demonstrated more explicitly, for example: a short discussion stating what has been done with PC indices, what has been done with RQA in geomagnetism, and where the gap lies.

Response to Comment 1: In the revised manuscript (lines 55–100), we have added text to address this suggestion.

For instance, lines 58 of the revised manuscript now read: Several studies have investigated hemispheric differences in the Earth's polar caps using PC indices.

In addition, in lines 78–102 of the revised manuscript we now state that "Additionally, the application of recurrence analysis to geomagnetic activity during geomagnetic storms has been extensively investigated. Oludehinwa et al. (2018) used recurrence analysis to examine nonlinear effects in the disturbance storm time (Dst) index for various categories of geomagnetic storms. They reported that the Dst signals behave in a stochastic manner during minor geomagnetic storms, whereas more deterministic behavior is exhibited during periods of stronger geomagnetic activity. Donner et al. (2018) applied recurrence analysis to reveal the nonlinear features of the hourly Dst index during intervals with magnetic storms and normal variability. They found that recurrence quantification analysis (RQA) distinguishes between storm and non-storm periods even better than other nonlinear characteristics such as symbolic-dynamics-based entropy. Donner et al. (2019) further used RQA to obtain complementary measures that serve as markers of different physical processes underlying quiet- and storm-time magnetospheric dynamics, demonstrating that RQA can discriminate magnetospheric activity in response to solar-wind forcing. Oludehinwa et al. (2021) applied RP and RQA to investigate nonlinear interdependence among solar-wind parameters influencing geomagnetic activity during geomagnetic storms. Solar-wind parameters, including proton density, solar-wind dynamic pressure, IMF Bz, and geomagnetic indices AE and SYM-H, were considered during the pre-storm, storm, and post-storm phases of intense, major, moderate, and minor geomagnetic storms. They found that the RP of the solar-wind parameters display a strong deterministic structure during storms, indicating strong interdependence, whereas during pre-storm and post-storm periods the RP exhibit only rare deterministic structure, signifying weak interdependence.

However, repetitive patterns in polar cap signals in response to varying geomagnetic-storm intensities have not yet been systematically investigated. In particular, to the best of our knowledge, the concept of nonlinear dynamics has not been applied to examine recurrent patterns in the polar cap indices PCN and PCS. Therefore, this study aims to identify recurring patterns in the polar cap north (PCN) and south (PCS) indices in response to different geomagnetic-storm intensities. We implement recurrence analysis, a nonlinear-dynamics technique that quantifies the times at which a system revisits similar states in its phase-space trajectory. Recurrence plots (RPs) reveal how a complex system returns to similar states over time, whereas recurrence quantification analysis (RQA) provides a set of quantitative measures derived from the RPs to characterize the underlying system dynamics

Comment 2: The present form of the manuscript reads more as a data-driven analysis with limited physical interpretation. The authors do not clearly state why recurrence analysis is the appropriate tool, nor what physical processes they expect to diagnose. In other words, what specific physical mechanisms might produce recurrence in PC indices?

Response to Comment 2: In the revised manuscript (lines 42–55), we have added text explaining the physical mechanisms that produce recurrence in the PC indices as follows:

The recurrence observed in the polar cap indices arises from recurring solar-wind input that enhances solar-wind–magnetosphere coupling through magnetic reconnection at the dayside magnetopause. In particular, southward interplanetary magnetic field (IMF Bz) conditions facilitate reconnection, open geomagnetic field lines, and drive enhanced ionospheric convection in the polar caps. These processes increase energy transfer to the magnetosphere and lead to repetitive geomagnetic disturbances that are reflected in elevated PC index values. The resulting rapid variations in the polar cap signals contain coherent structures that are complex, multiscale, and chaotic. These coherent structures are self-organized patterns that emerge from nonlinear interactions between the solar wind, ionosphere, and magnetosphere. They appear in the polar cap index time series as complex fluctuating signatures that exhibit recurrent patterns during different categories of geomagnetic storms, which motivates our analysis. Consequently, complex-systems methods such as recurrence plots (RPs) and recurrence quantification analysis (RQA), which characterize nonlinear processes in dynamical systems, are well suited to capture these recurring patterns in the polar cap signals.

Comment 3: The authors state that the recurrence analysis was computed using the embedding dimension (m=15) and time delay (\tau=6). Without justification, the results may not be meaningful. It should be clarified why/how such values have been chosen, especially the utilized such a high embedding dimension. Did the authors employ a standard method (e.g., False Nearest Neighbors for m or Average Mutual Information for \tau) to justify these specific values? Justifying these core parameters is essential for the reproducibility and validity of the nonlinear analysis.

Response to Comment 3: In the revised manuscript lines 156-163, we included statement that explicitly explain how the recurrence analysis was computed and also show the figures of false nearest neighbour and average mutual information as:

"Our recurrence analysis was computed using a threshold distance $\varepsilon = 1.5$, embedding dimension $m = 15$, and time delay $\tau = 6$, to reveal the polar cap recurrence pattern during different categories of geomagnetic storms shown in Table 1. The embedding dimension m and time delay

τ were determined using the false nearest neighbors and average mutual information methods, respectively, as shown in Figures 2 and 3. The optimal embedding dimension was chosen where the fraction of false nearest neighbors reaches a stable low value as m increases, whereas the time delay was selected at the first local minimum of the average mutual information curve. Recurrence quantification analysis (RQA) was then used to measure the structure and patterns in the RPs. RQA provides information on the number and duration of recurrences in order to characterize the behavior of the polar cap indices. In this study, we focus on two RQA measures to describe the recurrence features of the PC indices: the recurrence rate (RR) and the average diagonal line length (L)."

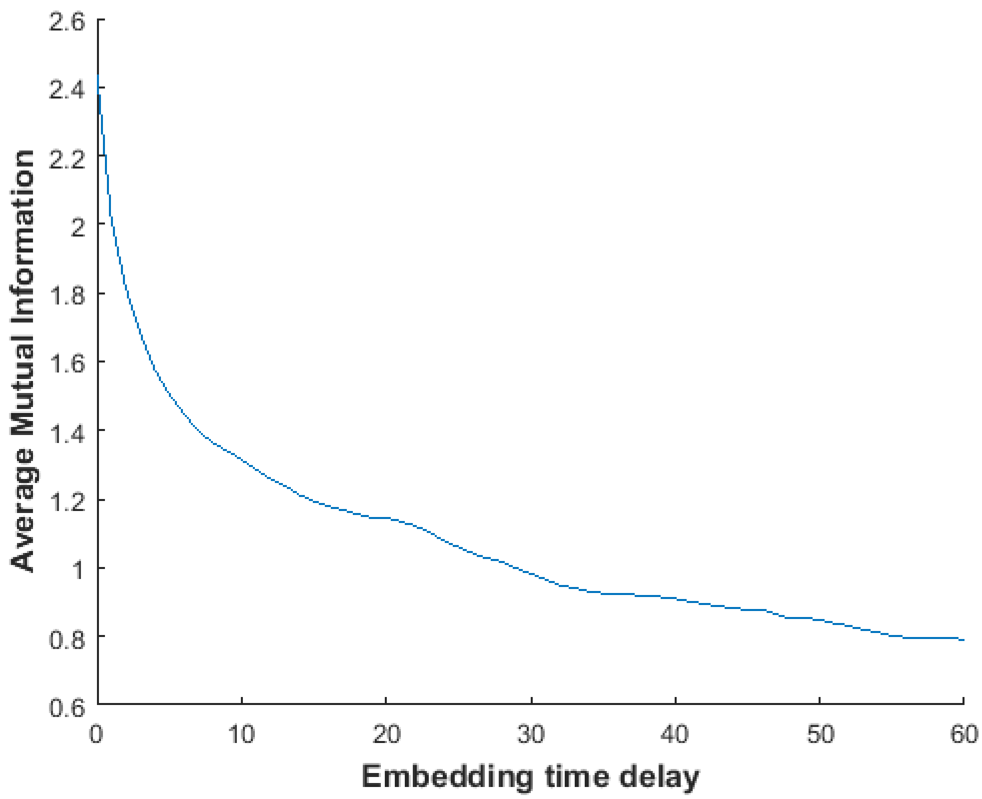

Figure 2: The plot of Average Mutual Information for the PCN and PCS index

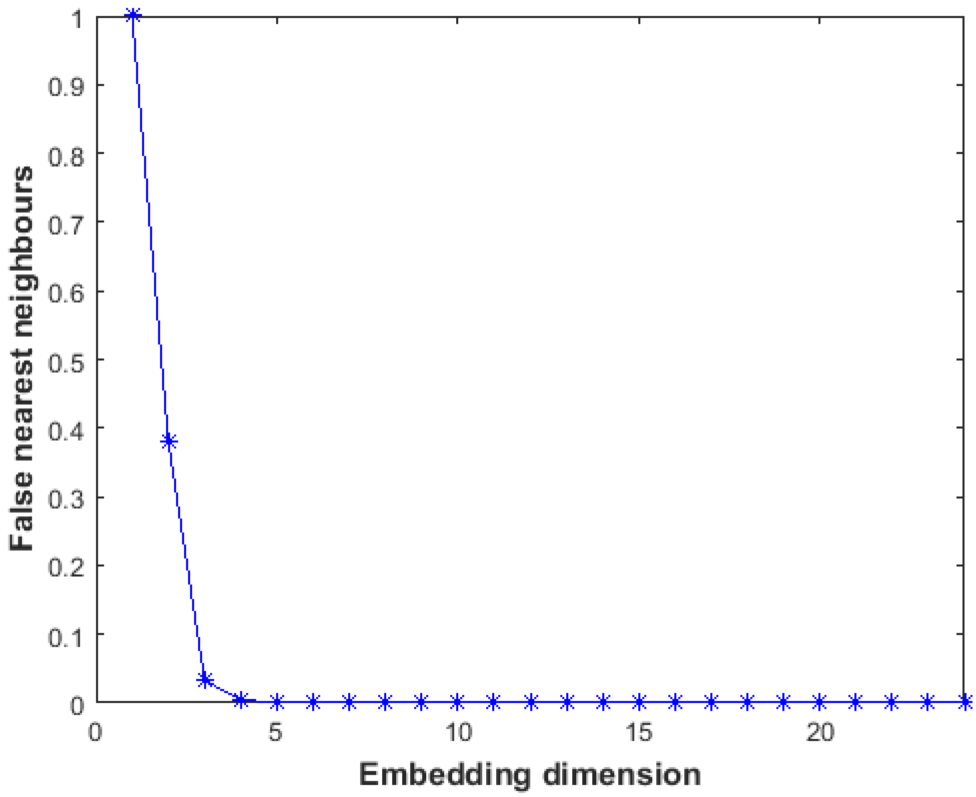

Figure 3: The plot of False Nearest Neighbors for the PCN and PCS index

Comment 4: A significant clarification is required regarding other parameters used in the recurrence analysis. For example, threshold (\epsilon) choice strongly influences RP density, RR, L, and all subsequent claims. Additionally, how robust are the obtained results to changes in these parameters (m, \tau, \epsilon, N)?

Response to Comment 4: We have included the value of the threshold ε in the revised manuscript. Specifically, our recurrence analysis was computed using a threshold distance $\varepsilon = 1.5$, embedding dimension $m = 15$, and time delay $\tau = 6$ to reveal the polar cap recurrence patterns for the different categories of geomagnetic storms, as summarized in Table 1.

Comment 5: PCN/PCS amplitude increases during stronger storms, which automatically increases recurrence density unless the time series is normalized. Thus, the manuscript risks conflating amplitude effects with dynamical structure. I would recommend to normalize the PCN/PCS time series to avoid amplitude-driven RR inflation.

Response to Comment 5: In the revised manuscript, we have normalized the PCN/PCS time series and then recalculated the recurrence analysis of the PC indices. We found that the recurrence-analysis results remain essentially the same. The PCN/PCS time series was normalized using the expression given below and has been included in line 135 of the revised manuscript

$$x_i = \frac{x - x_{mean}}{x_{std}}$$

Below we show bar charts of the recurrence-analysis results for the normalized PCN/PCS time series:

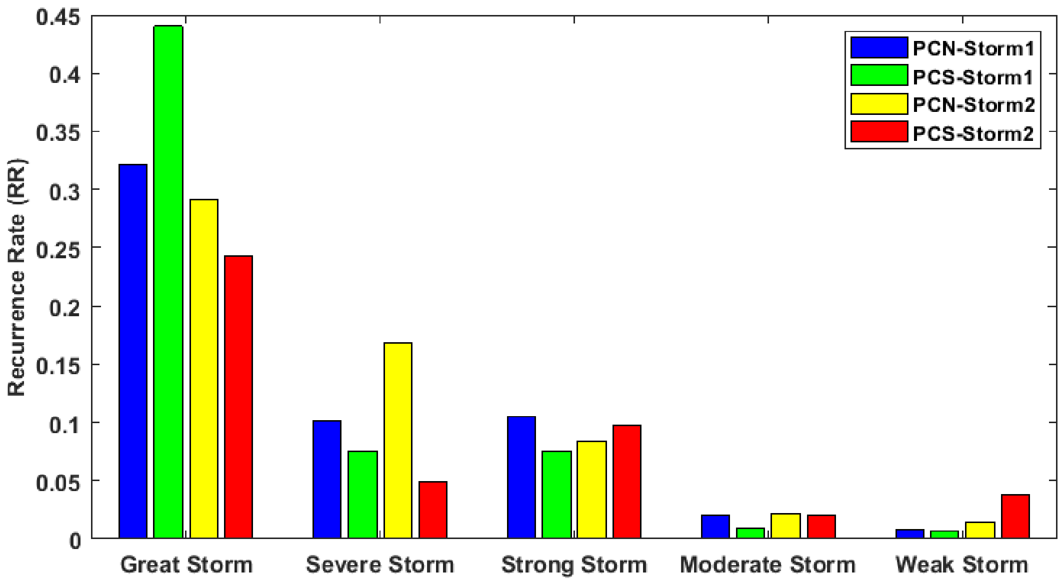

Figure A: Bar chart of recurrence rate (RR) for the normalized PCN and PCS time series during great, severe, strong, moderate, and weak geomagnetic storms

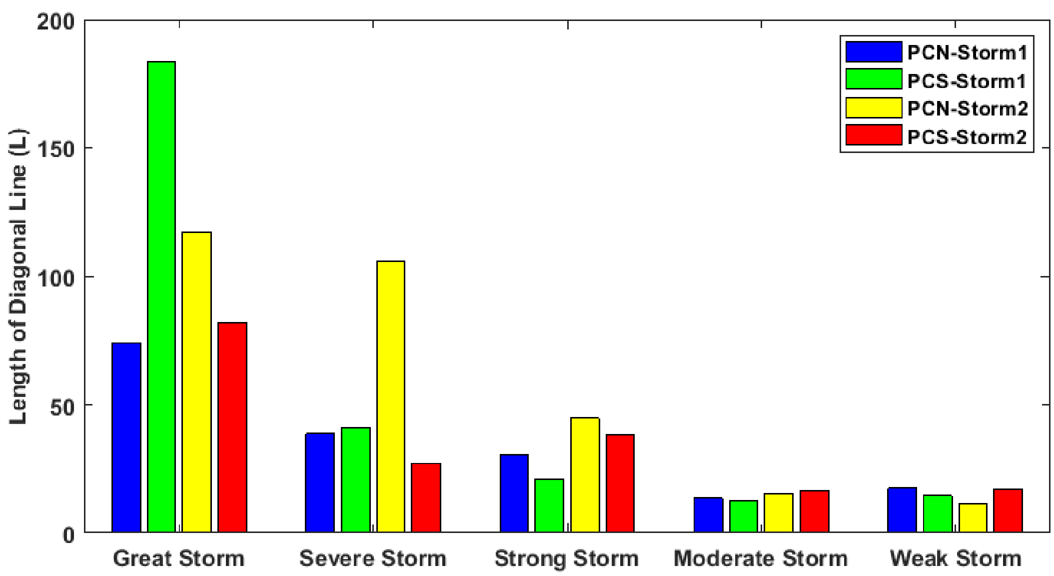

Figure B: Bar chart of average diagonal line length (L) for the normalized PCN and PCS time series during great, severe, strong, moderate, and weak geomagnetic storms

For comparison, we also show bar charts of the recurrence-analysis results for the original (unnormalized) PCN/PCS time series:

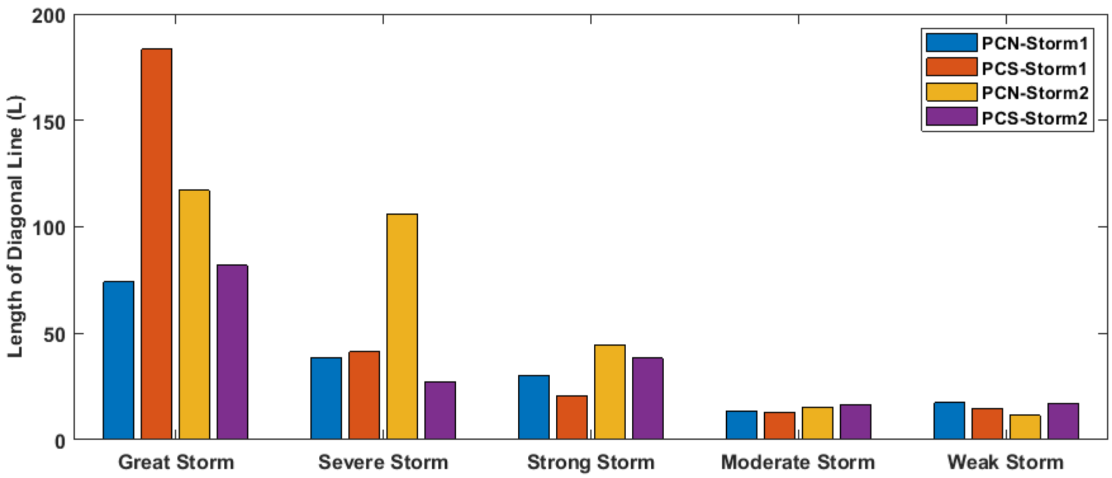

Figure 9: The Bar Chart of Length of Diagonal Line (L) for PCN and PCS at great storm, severe storm, strong storm, moderate storm and weak geomagnetic storm.

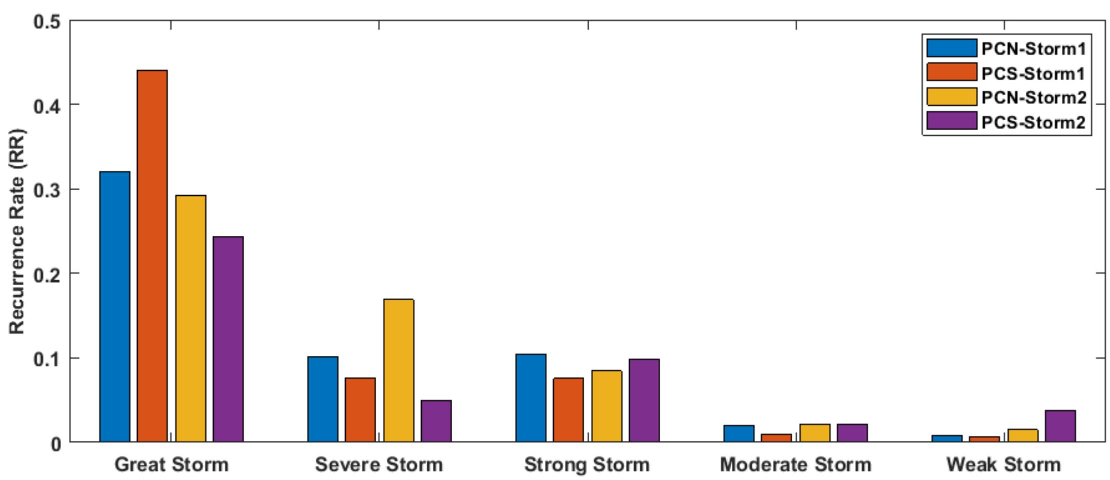

Figure 10: The Bar Chart of Recurrence Rate (RR) for PCN and PCS at great storm, severe storm, strong storm, moderate storm and weak geomagnetic storm.

This confirms that the differences in RR and L across the different categories of geomagnetic storms are not dominated by amplitude effects but instead reflect differences in the underlying dynamical structure

Comment 6: L110: *(i,l) -> (i,j)*

Response to Comment 6: In the revised manuscript (line 140), we have corrected the expression as: The RPs is based on recurrence matrix where each element $(\boldsymbol{i}, \boldsymbol{j})$

Comment 7: L131: "It is mathematically express as" ---> "… expressed as"

Response to Comment 7: In the revised manuscript (line 140), we have corrected the sentence to: 'It is mathematically expressed as:'

Comment 8: L214: "The RP results... reveals distinct patterns" ---> "... reveal ..."

Response to Comment 8: In the revised manuscript line 269, we have corrected the comment as: The RP results of the polar cap activities during geomagnetic storms **reveal** distinct patterns tied to storm intensity.

Comment 9: L214: "Great storms, severe storms, and strong storms reveals..." ---> "… reveal ..."

Response to Comment 9: In the revised manuscript lines 269-270, we have corrected the comment as: RP of PCN and PCS corresponding to great storms, severe storms, and strong storms **reveal** a robust deterministic structure in RPs.

Comment 10: L234: "...categories of suggests that..." The noun is missing after "categories of"

Response to Comment 10: In the revised manuscript lines 291-293, we have corrected the comment as: This observation of L values in the RPs during different categories of **geomagnetic storms** suggests that the polar cap activities are highly chaotic during moderate and weak storms compared to strong, severe and great storms.

Comment 11: L138 (Eq.5): Shouldn't be *P(l)* not *P(i)* ?

Response to Comment 11: Thank you for the observation. The equation has been cross-checked; the expression is correct.

Comment 12: Several references are duplicated (e.g., Oludehinwa 2018, Donner 2019).

Response to Comment 12: The references have been cross-checked. In some cases (e.g., Oludehinwa et al., 2018; Donner et al., 2019), the same authors have multiple publications, but these entries correspond to different papers with distinct years and titles.