Response Letter (Minor Revisions)

We thank the reviewers and the editor gracefully for their second round of comments and are glad that our manuscript has improved since the initial submission.

Comments from the Editor:

Please also add a proper data citation / reference to your list -- currently you state in the Dava Availability section: "The Argo data were collected and made freely available by the International Argo Program and the national programs that contribute to it. (https://argo.ucsd.edu, https://www.ocean-ops.org https://doi.org/10.17882/42182#110199). " -- in addition, this needs to be a proper citation, as stated on that website, as stated there "Please use the same DOI and citation as the global Argo data snapshot." -- under "how to cite" they provide the correct reference/citation: In the text "Argo (2024)", and in the reference list "Argo (2024). Argo float data and metadata from Global Data Assembly Centre (Argo GDAC) - Snapshot of Argo GDAC of May 09st 2024. SEANOE. https://doi.org/10.17882/42182#110199".

- **Response**: We thank the editor for their comment. We added the reference to Argo snapshot accordingly and cited this reference in the introduction as well as in the data availability statement.

Comments from Reviewer #1:

On L186 it is stated that: “Additionally, we provide the stream function below 2000 meters calculated from virtual RAPID moorings as an input to the reconstruction. Presumably, this is the term “MO” in table 1. It is not clear what data was used to calculate “the stream function”. I presume only the data below 2000m was used to calculate the streamfunction?

- **Response**: We understand the comment of the reviewer as more details are missing at this point in the manuscript. For these virtual moorings, which indeed are the 'MO' in table 1, we use the same level of no motion (4820 meters) as for the IGT, and aggregate the transport from this level up to 2000 meters. This ensures that no information shallower than 2000 meters is included in the virtual mooring input. We changed the manuscript accordingly by adding the following sentence: "For the calculation of the stream function we used a level of no motion at 4820 meters, similar to the RAPID calculation, and integrated the transport from this bottom level up to 2000 meters depth." (Line 185)

Other minor comments:

- **Response**: We thank the reviewer for their careful reading. We have changed all minor comments accordingly. We will address the following non-trivial changes from the minor

comments:

- In section 4.4 maybe useful to say that even if deep mooring data does not improve the AMOC estimate in these experiments we do expect that it has useful information about the stream function at deeper levels
    - **Response**: We agree with the reviewer that highlighting this hypothesis is reasonable and will be a better wrap-up to this section. The manuscript already contained some parts of this idea in the sentence (Sec 4.4. Line 523): "These findings suggest that while deeper observations can improve reconstructions, their benefit depends on having sufficient representation in the training data.", which we extended now with a second sentence making this more explicit, as suggested by the reviewer: "Although not shown in this experiment, we do expect the deeper observation to contain useful information about the stream function at depth."(L. 524)
- There is an error in equation (A4). I think that “1000” in the last term on the right should be “0”.
    - **Response**: We thank the reviewer for their comment, as we agree this equation contains a typo. But what we meant by this equation was to establish a layer of no transport at 4820 meters. Therefore, the right term should be $V(-4820m)|_{0m}$. During the discussion among the authors, we found that we could increase the readability of the equations if we use a positive upward direction, and explicitly distinguish between the reference level for velocities Z_M and the level of no transport Z_T. This leads us to the more general set of equations:

$$v(z)|_{Z_M} = \int_z^{Z_M} \frac{\partial v}{\partial z_i}\,\mathrm{d}z'$$

$$V(z)|_{Z_M}^{Z_T} = \int_z^{Z_T} L_x(z')v(z')|_{Z_M}\,\mathrm{d}z'$$

Calculating the IGT at 1000 meters depth, a level of not motion at the surface, and a level of no transport at 4820 meters, with the general equation above, shows why the right term should be $V(-4820m)|_{0m}$:

$$\begin{aligned} V(-1000m)|_{0m}^{-4820m} &= \int_{-1000m}^{-4820m} L_x(z')v(z')|_{0m}\,\mathrm{d}z' \\ &= -\int_{-4820m}^{-1000m} L_x(z')v(z')|_{0m}\,\mathrm{d}z' \\ &= \int_{-1000m}^{0m} L_x(z')v(z')|_{0m}\,\mathrm{d}z' - \int_{-4820m}^{0m} L_x(z')v(z')|_{0m}\,\mathrm{d}z' \\ &= V(-1000m)|_{0m}^{0m} - V(-4820m)|_{0m}^{0m} \end{aligned}$$

For the manuscript we do not add the general form because the reference level of the velocities is set to the surface in all our calculations. Nevertheless, we swapped the boundaries of the integral to ensure a positive upward direction and a positive transport in the northward direction for the upper branch. The new equations in the

manuscript are:

$$v(z)|_{0m} = \int_{z}^{0m} \frac{\partial v}{\partial z_i}\,\mathrm{d}z'$$

$$V(z)|_{0m} = \int_{z}^{0m} L_x(z')v(z')|_{0m}\,\mathrm{d}z'$$