Supplement of

Accelerated Permafrost Degradation in the Source Area of the Yellow River: Spatiotemporal Dynamics of Freeze-Thaw Indices Revealed by High-Resolution DEM-Corrected ERA5-Land Data (1981–2020)

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Appendix S1: Technical details of DEM-based calibration

S1.1 Monthly lapse rate derivation

Equations (S1)–(S4) correspond to the derivation of Γ_t and its intercept a_t from station data:

$$\Gamma_t = \frac{\mathrm{n}\sum T_i h_i - \sum T_i \sum h_i}{\mathrm{n}\sum h_i^2 - (\sum h_i)^2}, \quad a_t = \frac{\sum T_i - \Gamma_{month}\sum h_i}{n} \qquad \text{(S1-S2)}$$

The monthly lapse rate (Γ_t) values were screened for quality ($R^2 \geqslant 0.2$, $n \geqslant 5$); unreliable Γ_t replaced by climatological Γ_m.

The bias correction to ERA5-Land data was then applied as:

$$ST = T_{0_{1km}} + \Gamma_t(H_{DEM} - H_{ref}) \qquad \text{(S3)}$$

S1.2 Monthly surface temperature lapse rates

Table S1 lists the estimated lapse rates (Γ_t, °C/100 m) for each month, derived from the station-based regression model described above.

Table S1 Vertical lapse rate of surface temperature in the source area of the Yellow River month by month

month	Γ_{month} (°C/100m)	b	R^2	F value	p
1	- 0.90	24.751	0.955	41.626	0.003
2	- 0.769	23.667	0.948	35.205	0.004
3	- 0.68	25.390	0.945	33.582	0.004
4	- 0.62	28.614	0.863	11.694	0.027
5	- 0.54	30.123	0.860	11.363	0.028
6	- 0.46	30.005	0.852	10.563	0.031
7	- 0.42	30.504	0.849	10.364	0.032
8	- 0.41	29.634	0.856	10.942	0.030
9	- 0.47	28.325	0.924	23.242	0.009
10	- 0.64	28.836	0.962	49.665	0.002
11	- 0.78	24.194	0.961	48.877	0.002
12	- 0.88	25.430	0.944	32.550	0.005

S1.3 Accuracy validation and statistical metrics

The bias-corrected 1 km surface temperature dataset (ST) was validated using in situ surface temperature observations ($ST_{o\beta s}$) from seven meteorological stations located in the source area of the Yellow River. Four statistical indicators were used to assess model performance: the coefficient of determination (R^2), root mean square error (RMSE), unbiased RMSE (ubRMSE), and mean bias error (Bias). Their formulations are expressed as follows:

$$R^2 = \frac{\sum_i (ST_i - ST_{obs,i})^2}{\sum_i (ST_{obs,i} - ST_{obs})^2} \qquad \text{(S5)}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(ST_i - ST_{obs,i}\right)^2} \tag{S6}$$

$$ubRMSE = \sqrt{RMSE^2 - Bias^2} \tag{S7}$$

$$Bias = \frac{1}{n}\sum_i\left(ST_i - ST_{obs,i}\right) \tag{S8}$$

where n is the number of stations, ST_i and $ST_{obs,i}$ represent the modeled and observed surface temperatures, and ST_{obs} denotes the mean observed value. Validation results (Table S2) demonstrate strong agreement between ST and ST_{obs}, with $R^2 > 0.93$, $RMSE \approx 1.9$ °C, and $ubRMSE \approx 1.2$ °C. Seasonal analysis further shows that errors are substantially reduced during winter, confirming the effectiveness of the elevation bias correction for low-temperature conditions.

Table S2 Accuracy analysis of month-by-month surface temperature data before and after DEM revision

month	ERA5-Land skt				ST			
	R^2	**RMSE**	**ubRMSE**	**Bias**	**$R2_{ST}$**	**$RMSE_{ST}$**	**$ubRMSE_{ST}$**	**$Bias_{ST}$**
1	0.11	11.31	3.42	10.78	0.20	2.48	2.37	0.73
2	0.19	11.61	3.00	11.21	0.27	1.79	1.65	0.70
3	0.20	12.20	2.71	11.89	0.25	0.92	0.81	0.43
4	0.22	10.50	2.65	10.16	0.35	1.23	0.97	0.76
5	0.36	7.00	1.33	6.88	0.27	1.82	0.89	1.59
6	0.37	4.62	0.88	4.53	0.43	2.26	0.78	2.12
7	0.53	4.45	0.63	4.41	0.55	2.12	0.60	2.04
8	0.56	4.78	0.74	4.72	0.60	2.55	0.71	2.45
9	0.68	4.80	0.68	4.75	0.73	2.23	0.57	2.16
10	0.32	8.90	1.42	8.79	0.37	1.63	0.97	1.31
11	0.15	12.86	3.80	12.28	0.19	1.69	1.62	0.50
12	0.16	12.08	3.52	11.56	0.28	2.51	2.39	0.78

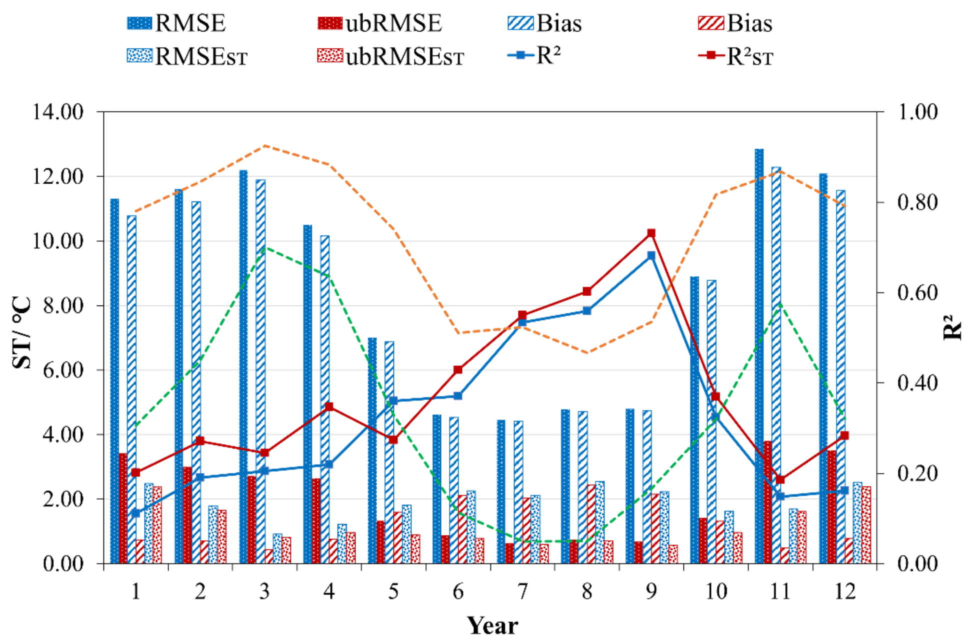

Figure S1 Changes in accuracy metrics for month-by-month surface temperature data before and after DEM revision

Appendix S2: Duration of thawing (DOT) Regression Model

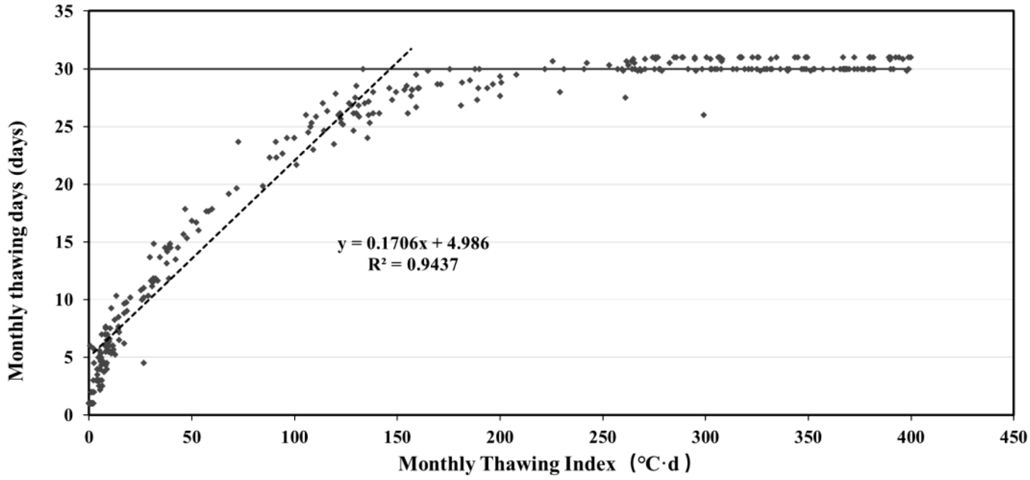

Figure S2. Regression relationship between monthly thawing index (TI) and the number of thawing days derived from meteorological station observations in the Source Area of the Yellow River. The regression model was applied to estimate the annual duration of thawing (DOT) for 1981–2020.

Appendix S3: Technical Details of the MK Test and Sen Slope

S3.1 MK test

Trend statistic S:

$$S = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} sgn(x_j - x_i) \tag{S8}$$

Where $sgn(\theta)$ is the logical discriminant function:

$$sgn(\theta) = \begin{cases} 1, & \theta > 0 \\ 0, & \theta = 0 \\ 1, & \theta < 0 \end{cases} \tag{S9}$$

Variance calculation:

$$\mathrm{var(S)} = \frac{n(n-1)(2n+5) - \sum_{i=1}^{n} t_i(i-1)(2i+5)}{18} \tag{S10}$$

t_i represents the length of the ith set of repetitions, and n is the total number of repetitions.

The standardized statistic *Zmk*:

$$Z_{mk} = \begin{cases} \frac{S-1}{\sqrt{var(S)}} & S > 0 \\ 0 & S = 0 \\ \frac{S+1}{\sqrt{var(S)}} & S < 0 \end{cases} \tag{S11}$$

S3.2 Sen slope estimation

The Sen slope β is the median slope, and it works well with data that is not normally distributed. Its confidence intervals can be calculated by the permutation test or the normal approximation (See Mann 1945; Asadieh and Krakauer 2015).

Appendix S4: Elevation–index relationship

Elevation–index relationships for freezing and thawing indices in the source area of the Yellow River. This figure provides supporting evidence for the topographic gradient effects described in the main text (Sect. 4.2).

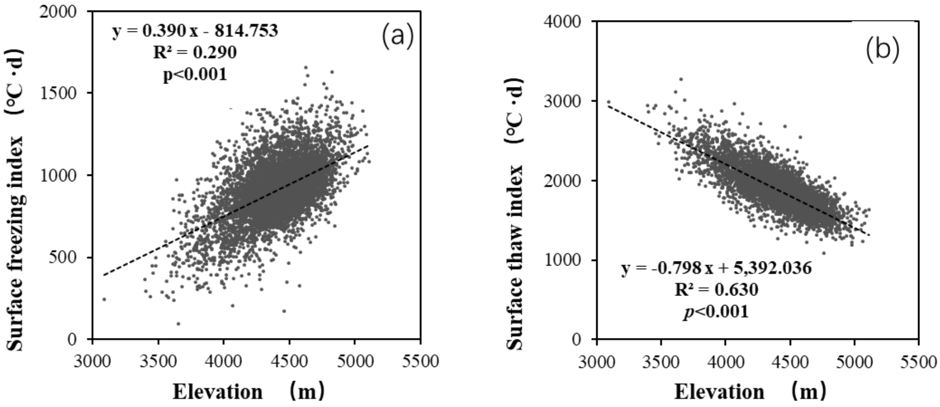

Figure S3 Relationship between elevation (m) and freeze–thaw indices: (a) SFI vs elevation; (b) STI vs elevation. Solid lines denote linear fits; reported slopes and R^2 values indicate elevation sensitivity.

References

Asadieh, B.; Krakauer, N. Y. (2015): Global trends in extreme precipitation: climate models versus observations. In Hydrol. Earth Syst. Sci. 19 (2), pp. 877–891. DOI: 10.5194/hess-19-877-2015.

Mann, Henry B. (1945): Nonparametric Tests Against Trend. In Econometrica 13 (3), p. 245. DOI: 10.2307/1907187.