Response on RC2

Meng Sun (correspondent author) September 16, 2025

First and foremost, the authors want to express our sincere gratitude to the reviewer for taking the time to review our manuscript titled "Analytical approaches for wave energy dissipation induced by wave-generated turbulence and random wave-breaking". We are grateful for your thoughtful feedback and constructive suggestions in improving the quality of our work.

The authors thank the reviewer for the overall very comprehensive summary of our work. And we fully agree with the reviewer's suggestion that our manuscript should be edited by a native English speaker or a professional editor.

We apologize for our ambiguous expressions for lacking clarity and will rephrase, extend and supplement assumptions and some critical derivation steps to improve the manuscript. In order to evaluate the new dissipation formulations due to wave-generated turbulence and wave-breaking, which are focal points of the present study, the numerical tests in section 3 consists of scaling behavior only for the duration-limited growth and decay. Simple numerical experiments were performed and model results were compared to the original MASNUM wave model and WAM wave model (Janssen et al., 1994). For duration-limited growth, no good observations lead us to compare to the original MASNUM wave model and WAM wave model (Janssen et al., 1994), to evaluate the stable and reliable performance of the new dissipation formulations, and furthermore, to perceive directly the different effects between the two new dissipation formulations due to wave-generated turbulence and wave-breaking. In addition, in our series of works, Wang et al. (2024) used the new model for global hindcast and forecast to evaluate the modeling performance by comparing to the Jason-3 satellite altimeter observation data and four non-nearshore NDBC buoys data. We are also conducting studies for uniform wind, turning wind and rotatory wind in fetch-limited conditions which are stated in Section 5. These model results and verifications with abundant fetch-limited observation data will be part of scaling behaviors in our future series papers (due to the length limit of the journal).

Key references

Wang, F., Yang, Y., Yin, X., Jiang, X., and Sun, M.: Improving wave modeling performance by incorporating wave-generated turbulence dissipation and improved post-breaking spectrum, Ocean Modell., 188 (2024) 102311, https://doi.org/10.1016/j.ocemod.2023.102311, 2024.

Some major revisions are listed below, following the guidance of the reviewer.

- 1. Abstract
- L6: Here "improved" means the new dissipation coefficient (26) that we present in section
 2.2, For easily displaying its relation to the previous formulation (24) (Yuan et al., 1986;
 Yuan et al., 1993; Donelan and Yuan, 1994), the new dissipation coefficient is rewritten as

(27) by introducing the ratio of the kinetic energy loss to the potential one due to wave-breaking, which was first proposed by Yuan et al.(2009), inspected by Wang et al. (2017, 2018) and validated by Shi et al. (2025). Comparison to the previous one (24) indicates the new one varies with the ratio of the kinetic energy loss to the potential one due to wave-breaking, and it aligns with the previous one under certain conditions. The constant coefficient in the previous one comes originally from the complicated 0-1 st order asymptotic expansions of the covariance of surface elevation, which served as the foundation of the wave-breaking dissipation source function of the original MASNUM wave model. That is to say, it is more reasonable physically to introduce the kinetic energy loss and the potential one to the dissipation coefficient. We apologize for our ambiguous expressions for lacking clarity and will rephrase and/or extend to improve the text.

Key references

- Donelan, M. A., and Yuan, Y.: Wave dissipation by surface processes, in: Dynamics and Modelling of Ocean Waves, edited by: Komen, G. J., Cavaleri, L., Donelan,
 M., Hasselmann, K., Hasselmann, S., and Janssen, P. A. E. M., Cambridge University Press, Cambridge, UK, 143-155, ISBN 0-521-47047-1, 1994.
- Yuan, Y., Han, L., Hua, F., Zhang, S., Qiao, F., Yang, Y., and Xia, C.: The statistical theory of breaking entrainment depth and surface whitecap coverage of real sea waves, J. Phys. Oceanogr., 39, 143-161, https://doi.org/10.1175/2008JPO3944.1, 2009.
- Wang, H., Yang, Y., Sun, B., and Shi, Y.: Improvements to the statistical theoretical model for wave breaking based on the ratio of breaking wave kinetic and potential energy, Sci. China Earth Sci., 60(1), 180-187, https://doi.org/10.1007/s11430-016-0053-3, 2017.
- Wang, H., Yang, Y., Dong, C., Su, T., Sun, B., and Zou, B.: Validation of an improved statistical theory for sea surface whitecap coverage using satellite remote sensing data, Sensors, 18, 3306, https://doi.org/10.3390/s18103306, 2018.
- Shi, Y, Yang, Y., Qi, J., and Wang, H.: Adaptability assessment of the whitecap statistical physics model with cruise observations under high sea states, Front. Mar. Sci. 12:1486860, https://doi.org/10.3389/fmars.2025.1486860, 2025.
- O L8: We agree that there is considerable noise in the observation data in Figs. 5 and 6, which Young and Babanin (2006) also stated, Some Comparisons may yield valuable insights and needs further interpretations. This sentence is changed to: "Their comparisons with laboratory observations or comprehensive measurements were provided and validated tentatively, and applications on simple duration-limited growth and decay experiments were implemented."

Key references

Young, I. R., and Babanin, A. V.: Spectral distribution of energy dissipation of wind-generated waves due to dominant wave breaking, J. Phys. Oceanogr., 36, 376-394, https://doi.org/10.1175/JPO2859.1, 2006.

- 2. The analytical derivation processes in section 2
- Eq. (1): We apologize for a discrepancy in the cited date of the reference "Yang et al. (2019)", which should be revised to Yang et al. (2022). Equation (1) was provided in the latter reference. As per the reviewer's suggestions, the description to outline the derivation process is supplemented briefly in Appendix A of the revised manuscript.

Appendix A

In the comprehensive framework of the ocean dynamic system comprised of wave-like motions, eddy-like motions and circulation, which are controlled by dynamic gravity balance, static gravity balance and geotropic balance respectively, ocean turbulence, highly random perturbations due to strong nonlinear advections in the foregoing three sub-systems, interacts with larger scale motions including the advection transport and shear instability generation of large-scale dynamic processes as well as the mixing effect in the form of its transport flux residual on the latter (Yuan et al., 2012). More comprehensive governing equations for wave motion were derived by Yuan et al. (2012) and formulated in tensor expression as follows:

$$\frac{\partial u_{\text{SM}j}}{\partial x_j} = 0 , \qquad (A1)$$

$$\frac{\partial u_{\text{SM}i}}{\partial t} + \hat{U}_{j} \frac{\partial u_{\text{SM}i}}{\partial x_{j}} + u_{\text{SM}j} \frac{\partial \hat{U}_{i}}{\partial x_{j}} + \frac{\partial}{\partial x_{j}} \left(u_{\text{SM}j} u_{\text{SM}i} - \left\langle u_{\text{SM}j} u_{\text{SM}i} \right\rangle_{\text{SM}} \right) - 2 \varepsilon_{ijk} u_{\text{SM}j} \Omega_{k}
= -\frac{1}{\rho_{0}} \frac{\partial p_{\text{SM}}}{\partial x_{i}} - g \frac{\rho_{\text{SM}}}{\rho_{0}} \delta_{3i} + \frac{\partial}{\partial x_{j}} \left(v_{0} \frac{\partial u_{\text{SM}i}}{\partial x_{j}} \right) + \frac{\partial}{\partial x_{j}} \left[-\left(\left\langle u_{\text{SS}j} u_{\text{SS}i} \right\rangle_{\text{SS}} - \left\langle \left\langle u_{\text{SS}j} u_{\text{SS}i} \right\rangle_{\text{SS}} \right\rangle_{\text{SM}} \right) \right] ,$$
(A2)

$$\frac{\partial \rho_{\text{SM}}}{\partial t} + \hat{U}_{j} \frac{\partial \rho_{\text{SM}}}{\partial x_{j}} + u_{\text{SM}j} \frac{\partial \hat{\rho}}{\partial x_{j}} + \frac{\partial}{\partial x_{j}} \left(u_{\text{SM}j} \rho_{\text{SM}} - \left\langle u_{\text{SM}j} \rho_{\text{SM}} \right\rangle_{\text{SM}} \right)
= \frac{\partial}{\partial x_{j}} \left(K_{0} \frac{\partial \rho_{\text{SM}}}{\partial x_{j}} \right) + \frac{\partial}{\partial x_{j}} \left[-\left(\left\langle u_{\text{SS}j} \rho_{\text{SS}} \right\rangle_{\text{SS}} - \left\langle \left\langle u_{\text{SS}j} \rho_{\text{SS}} \right\rangle_{\text{SS}} \right\rangle_{\text{SM}} \right) \right] + Q_{\text{SM}}$$
(A3)

where $u_{\mathrm{SM}i}, i=1,2,3; \rho_{\mathrm{SM}}$ denote the ocean wave components , $\hat{U}_i, i=1,2,3; \hat{T}, \hat{s}, \hat{p}, \hat{\rho}$

denote the background current components, ρ_0 is the basin mean water density;

 v_0, K_0, D_0 denote the molecular viscosity, thermal and diffusion coefficients; $Q_{\rho \rm SM}$ denotes the thermal source due to temperature and salinity perturbations; $\langle \cdot \rangle_{\rm SS}$, $\langle \cdot \rangle_{\rm SM}$ denote the Reynolds averages on the turbulence and wave motions respectively. Hereafter, other symbols have their usual meaning.

Multiplying Eq. (A2) by $\rho_0 u_{\text{SM}i}$ and Eq. (A3) by ρ_{SM} , after some manipulation, yields the unit volume mechanical kinetic energy and potential energy respectively. Sum of both energy terms and Reynolds averaged on the wave motion reduce to

$$\begin{split} &\frac{\partial}{\partial t} \left\langle \frac{\rho_{0} u_{\rm SMi}^{2}}{2} + \frac{g^{2} \rho_{\rm SM}^{2}}{2 \rho_{0} \hat{N}_{3}^{2}} \right\rangle_{\rm SM} + \left\langle \left(\hat{U}_{j} + u_{\rm SMj} \right) \frac{\partial}{\partial x_{j}} \left(\frac{\rho_{0} u_{\rm SMi}^{2}}{2} + \frac{g^{2} \rho_{\rm SM}^{2}}{2 \rho_{0} \hat{N}_{3}^{2}} \right) \right\rangle_{\rm SM} + \frac{\partial}{\partial x_{i}} \left\langle p_{\rm SM} u_{\rm SMi} \right\rangle_{\rm SM} \\ &= -\rho_{0} \left\langle u_{\rm SMi} u_{\rm SMj} \right\rangle_{\rm SM} \frac{\partial \hat{U}_{i}}{\partial x_{j}} + g \left\langle \rho_{\rm SM} u_{\rm SM\beta} \right\rangle_{\rm SM} \frac{\hat{N}_{\beta}^{2}}{\hat{N}_{3}^{2}} + \frac{g^{2}}{\rho_{0} \hat{N}_{3}^{2}} \left\langle \rho_{\rm SM} Q_{\rho \rm SM} \right\rangle_{\rm SM} \\ &+ \left[\left\langle \rho_{0} u_{\rm SMi} \frac{\partial}{\partial x_{j}} \left(v_{0} \frac{\partial u_{\rm SMi}}{\partial x_{j}} \right) \right\rangle_{\rm SM} + \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0} \hat{N}_{3}^{2}} \frac{\partial}{\partial x_{j}} \left(K_{0} \frac{\partial \rho_{\rm SM}}{\partial x_{j}} \right) \right\rangle_{\rm SM} \\ &+ \left\langle \rho_{0} u_{\rm SMi} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} u_{\rm SSi} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} + \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0} \hat{N}_{3}^{2}} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} \rho_{\rm SS} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} \\ &+ \left\langle \rho_{0} u_{\rm SMi} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} u_{\rm SSi} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} + \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0} \hat{N}_{3}^{2}} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} \rho_{\rm SS} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} \\ &+ \left\langle \rho_{0} u_{\rm SMi} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} u_{\rm SSi} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} + \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0} \hat{N}_{3}^{2}} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} \rho_{\rm SS} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} \\ &+ \left\langle \rho_{0} u_{\rm SMi} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} u_{\rm SSi} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} + \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0} \hat{N}_{3}^{2}} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} \rho_{\rm SS} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} \\ &+ \left\langle \rho_{0} u_{\rm SMi} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} u_{\rm SSi} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} \\ &+ \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0}} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} u_{\rm SSi} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} \\ &+ \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0}} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} u_{\rm SSi} \right\rangle_{\rm SS} \right) \right\rangle_{\rm SM} \\ &+ \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0}} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SS}_{j} u_{\rm SMi} \right\rangle_{\rm SM} \right) \\ &+ \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0}} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SM}_{j} u_{\rm SMi} \right\rangle_{\rm SM} \right) \right\rangle_{\rm SM} \\ &+ \left\langle \frac{g^{2} \rho_{\rm SM}}{\rho_{0}} \frac{\partial}{\partial x_{j}} \left(-\left\langle u_{\rm SM}_{j} u_{\rm SMi} \right\rangle_{\rm SM} \right) \\ &+ \left$$

Based on closure assumptions of the second-order turbulence moments equations with high certainty (Baumert et al., 2005; Yuan et al., 2013), Eq. (A4) can be written as

$$\begin{split} &\frac{\partial}{\partial t} \left\langle \frac{\rho_0 u_{\mathrm{SM}i}^2}{2} + \frac{g^2 \rho_{\mathrm{SM}}^2}{2 \rho_0 \hat{N}_3^2} \right\rangle_{\mathrm{SM}} + \left\langle \left(\hat{U}_j + u_{\mathrm{SM}j} \right) \frac{\partial}{\partial x_j} \left(\frac{\rho_0 u_{\mathrm{SM}i}^2}{2} + \frac{g^2 \rho_{\mathrm{SM}}^2}{2 \rho_0 \hat{N}_3^2} \right) \right\rangle_{\mathrm{SM}} + \frac{\partial}{\partial x_i} \left\langle p_{\mathrm{SM}} u_{\mathrm{SM}i} \right\rangle_{\mathrm{SM}} \\ &= \left(-\rho_0 \left\langle u_{\mathrm{SM}i} u_{\mathrm{SM}j} \right\rangle_{\mathrm{SM}} \frac{\partial \hat{U}_i}{\partial x_j} + g \left\langle \rho_{\mathrm{SM}} u_{\mathrm{SM}a} \right\rangle_{\mathrm{SM}} \frac{\hat{N}_a^2}{\hat{N}_3^2} \right) + \frac{g^2}{\rho_0 \hat{N}_3^2} \left\langle \rho_{\mathrm{SM}} Q_{\rho \mathrm{SM}} \right\rangle_{\mathrm{SM}} \\ &+ \left\{ \frac{\partial}{\partial x_j} \left\langle \left(v_0 + \frac{k^2}{\pi^2 \varepsilon} \right) \frac{\partial}{\partial x_j} \left(\frac{\rho_0 u_{\mathrm{SM}i}^2}{2} \right) \right\rangle_{\mathrm{SM}} + \frac{\partial}{\partial x_j} \left\langle \left(K_0 + \frac{1}{\sigma_0} \frac{k^2}{\pi^2 \varepsilon} \right) \frac{\partial}{\partial x_j} \left(\frac{g^2 \rho_{\mathrm{SM}}^2}{2 \rho_0 \hat{N}_3^2} \right) \right\rangle_{\mathrm{SM}} \\ &+ \left\{ -\left\langle \rho_0 \left(v_0 + \frac{k^2}{\pi^2 \varepsilon} \right) \left(\frac{\partial u_{\mathrm{SM}i}}{\partial x_j} \right)^2 \right\rangle_{\mathrm{SM}} - \left\langle \frac{g^2}{\rho_0 \hat{N}_3^2} \left(K_0 + \frac{1}{\sigma_0} \frac{k^2}{\pi^2 \varepsilon} \right) \left(\frac{\partial \rho_{\mathrm{SM}}}{\partial x_j} \right)^2 \right\rangle_{\mathrm{SM}} \right\} \end{split}$$

where $\hat{N}_{i}^{2} = -g \frac{\partial}{\partial x_{i}} \left(\frac{\hat{\rho}}{\rho_{0}} \right)$, i = 1, 2, 3 denote the Brunt–Väisälä frequency components; k, ε

the kinetic energy and its dissipation rate of ocean turbulence, which is generated by shear instability of background current, Stokes drift and wave orbital motions in the upper layers. Here in this study, only the wave-generated turbulence is considered. Its analytical mixing coefficients were proposed through equilibrium solutions of the second-order turbulence closure model between the wave motion shear instability generations and the TKE dissipations (Yuan et al., 2013).

Key references

- Baumert, H. Z., Simpson, J. H., and Sündermann, J. (Eds.): Marine turbulence: theories, observations, and models, Cambridge University Press, Cambridge, UK, 630pp., ISBN 978-0-521-15372-0, 2005.
- Yang, Y., Sun, M., Sun ,L., Xia, C., Teng, Y., and Cui, X.: A Characteristics Set Computation Model for Internal Wavenumber Spectra and Its Validation with MODIS Retrieved Parameters in the Sulu Sea and Celebes Sea, Remote Sens., 14, 1967, https://doi.org/10.3390/rs14091967, 2022.
- Yuan, Y., Qiao, F., Yin, X., and Han, L.: Establishment of the ocean dynamic system with four sub-systems and the derivation of their governing equation sets, J. Hydrodyn., 24, 153-168, https://doi.org/10.1016/S1001-6058(11)60231-X, 2012.
- Yuan, Y., Qiao, F., Yin, X., and Han, L.: Analytical estimation of mixing coefficient induced by surface wave-generated turbulence based on the equilibrium solution of the

second-order turbulence closure model, Sci. China Earth Sci., 56, 71-80, https://doi.org/10.1007/s11430-012-4517-x, 2013.

 Eq. (12): We add information, as well as underlying assumptions in the preceding paragraph of Eq. (12), and the paragraph is rewritten as

Yuan et al. (2013) proposed a parameterization of the mixing coefficient $\left\langle \frac{\overline{k}^2}{\pi^2 \overline{\varepsilon}} \right\rangle_{\text{SM}}$ through equilibrium solutions of the second-order turbulence model with high certainty closure assumptions, in which the power function relationship between turbulent dissipation rate and shear instability generation of wave motion was fitted by observation data in deep ocean.

Here we choose a generic representation of the mixing length of $\bar{l}_D = \frac{\bar{k}^{\frac{3}{2}}}{\pi^{\frac{3}{2}}\bar{E}}$ (Baumert et al.,

2005), which is appropriate for deep and shallow water conditions, and the mixing coefficient is formulated conveniently as

Key references

Baumert, H. Z., Simpson, J. H., and Sündermann, J. (Eds.): Marine turbulence: theories, observations, and models, Cambridge University Press, Cambridge, UK, 630pp., ISBN 978-0-521-15372-0, 2005.

Yuan, Y., Qiao, F., Yin, X., and Han, L.: Analytical estimation of mixing coefficient induced by surface wave-generated turbulence based on the equilibrium solution of the second-order turbulence closure model, Sci. China Earth Sci., 56, 71-80, https://doi.org/10.1007/s11430-012-4517-x, 2013.

Eq. (19): There is no problem in Eq. (19), we carefully checked again. There is a clarification should be provided prior to Eq. (19) about an approximate coefficient 7/8. So the sentence and the equation are revised to

In consideration of an approximate coefficient 7/8 introduced from the minimization relation for the equilibrium solutions (Yuan et al., 2013), the TKE dissipation rate ε_{dis} can be derived as

$$\varepsilon_{\rm dis} \approx \alpha_{\rm wt} \frac{7}{8} \left\langle \frac{\overline{k}^2}{\pi^2 \overline{\varepsilon}} \right\rangle_{\rm SM} \left\langle \left(\frac{\partial u_{\rm SM}i}{\partial x_j} \right)^2 \right\rangle_{\rm SM} = \frac{7\sqrt{7}}{16} \alpha_{\rm wt} A^5 \omega^3 K^3 \exp\left\{ 5K_{\bar{x}_3} \right\}$$
 (19)

For monochromatic non-breaking waves, the derivation processes are similar to those for wave spectrum in the manuscript. For deep water depth,

$$\left\langle \frac{\partial u_{\text{SM}i}}{\partial x_j} \frac{\partial u_{\text{SM}i}}{\partial x_j} \right\rangle_{\text{SM}} = 2A^2 \omega^2 K^2 \exp(2Kx_3)$$

and

$$\left\langle \frac{\overline{k}^2}{\pi^2 \overline{\varepsilon}} \right\rangle_{\text{SM}} = \frac{\sqrt{7}}{4} A^3 \omega K \exp\left\{ 3Kx_3 \right\}$$

(In fact, let $H_S = 2\sqrt{2}A$, here A represents the root-mean-square wave amplitude, Eq. (14) for wave spectrum in the manuscript can be reduced to the above equation.) The two equations yield the above Eq. (19).

Key references

- Yuan, Y., Qiao, F., Yin, X., and Han, L.: Analytical estimation of mixing coefficient induced by surface wave-generated turbulence based on the equilibrium solution of the second-order turbulence closure model, Sci. China Earth Sci., 56, 71-80, https://doi.org/10.1007/s11430-012-4517-x, 2013.
- L348: We thanks for the reviewer's insightful comments. The transforms here are also valid approximately under a narrow spectrum assumption for theoretical arguments applied widely in the breaking wave statistics (Yuan et al., 1986, 2009; Donelan and Yuan, 1994). For practical numerical modeling, the wave spectrum is not actually narrow; thus, it is not precise enough to use on the results based on a narrow spectrum. We supplement the underlying assumption in the sentence.

Key references

- Donelan, M. A., and Yuan, Y.: Wave dissipation by surface processes, in: Dynamics and Modelling of Ocean Waves, edited by: Komen, G. J., Cavaleri, L., Donelan,
 M., Hasselmann, K., Hasselmann, S., and Janssen, P. A. E. M., Cambridge University Press, Cambridge, UK, 143-155, ISBN 0-521-47047-1, 1994.
- Yuan, Y., Han, L., Hua, F., Zhang, S., Qiao, F., Yang, Y., and Xia, C.: The statistical theory of breaking entrainment depth and surface whitecap coverage of real sea waves, J. Phys. Oceanogr., 39, 143-161, https://doi.org/10.1175/2008JPO3944.1, 2009.
- Eq.(26): We apologize for lacking clarity of the assumption and derivation process from Eq. (21) or (24) to Eq.(26). Yuan et al. (2009) derived some basic statistics of wave breaking for a narrow spectrum, especially the breaking kinetic and potential energy loss which add up to deduce the breaking mechanical energy loss formulated by introducing the ratio of the former to the latter. Wang et al. (2017, 2018) concluded that the ratio is mainly within the range 3-30, which indicates that there is a disproportion feature between the wave kinetic energy loss and potential one due to wave-breaking. Shi et al. (2025) validated the statistical wave-breaking model across multiple sites from the High Wind Speed Gas Exchange Study (HiWinGS), and concluded that the model is highly effective in capturing the dynamics of whitecap coverage across a range of high sea states. Based on the latest findings, an improved attenuation coefficient by introducing the breaking kinetic energy loss is proposed as follows:

$$\alpha_{b}' = 1 - \rho \frac{\omega^{2} \mu_{0}^{1/2}}{g} \frac{1}{L} \left[L \int_{-\infty}^{-L} \exp\left\{-\frac{1}{2}Z^{2}\right\} dZ + \exp\left\{-\frac{\rho^{2}}{8} \frac{g^{2}}{\mu_{0} \omega_{z}^{4}}\right\} \right]$$
(26)

where the first and second terms in the bracket on the right-hand side are related to the dimensionless breaking kinetic and potential energy loss respectively,

$$L = \frac{g}{2\mu_4^{1/2}} = \frac{\rho}{\pi\lambda} \left(\frac{H_s}{\overline{L}}\right)^{-1} = \frac{\rho}{2} \frac{g}{\mu_0^{1/2} \omega_z^2}, \text{ mean wavelength } \overline{L} = \lambda \frac{g}{2\pi} T_z^2, \lambda = \frac{2}{3} \text{ or } 0.86 \text{ (Kinsman, } \frac{1}{2} \frac{g}{2} \frac{g}{2\pi} T_z^2 = \frac{1}{2} \frac{g}{$$

2012; Yuan et al., 2009; Xu and Yu, 2001).

Key references

- Yuan, Y., Han, L., Hua, F., Zhang, S., Qiao, F., Yang, Y., and Xia, C.: The statistical theory of breaking entrainment depth and surface whitecap coverage of real sea waves, J. Phys. Oceanogr., 39, 143-161, https://doi.org/10.1175/2008JPO3944.1, 2009.
- Wang, H., Yang, Y., Sun, B., and Shi, Y.: Improvements to the statistical theoretical model for wave breaking based on the ratio of breaking wave kinetic and potential energy, Sci. China Earth Sci., 60(1), 180-187, https://doi.org/10.1007/s11430-016-0053-3, 2017.
- Wang, H., Yang, Y., Dong, C., Su, T., Sun, B., and Zou, B.: Validation of an improved statistical theory for sea surface whitecap coverage using satellite remote sensing data, Sensors, 18, 3306, https://doi.org/10.3390/s18103306, 2018.
- Shi, Y, Yang, Y., Qi, J., and Wang, H.: Adaptability assessment of the whitecap statistical physics model with cruise observations under high sea states, Front. Mar. Sci. 12:1486860, https://doi.org/10.3389/fmars.2025.1486860, 2025.
- 3. The comparison of growth and decay rates in L223-250:
- O The wave orbital velocity at sea surface u_{w0} varies with the wave amplitude A, $u_{w0} = A\hat{\omega}$. The vertical axis in Fig. 2 represents the changes of u_{w0} , which indicates that the amplitude A varies correspondingly.
- L244-245: We agree with the reviewer's point that there is no rigorous relation between u^* and u_{w0} . For wind waves, the two variables exhibit a positive qualitative relationship. Furthermore, our main topic is to show the difference of the growth and dissipation rate between Fig.1 and Fig.2, for u^* and u_{w0} varies in a range across normal and extreme sea conditions (the horizontal axis variable K needs to be considered together).
- Fig 2: We thanks for the reviewer's insightful comments. We redraw Fig. 2 again in consideration of the constraint. Figure 1 is also redrawn together.

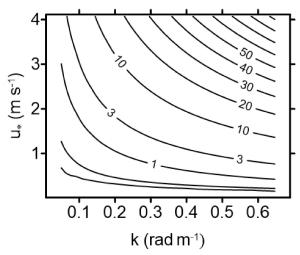


Figure 1. Distribution of growth rate γ as a function of wavenumber and friction velocity (Unit: s⁻¹)

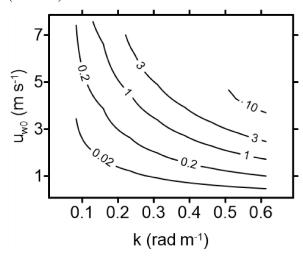


Figure 2. Distribution of dissipation rate γ_{tid} as a function of wavenumber and wave orbital velocity at sea surface (Unit: s⁻¹)

- 4. The comparison of measured TKE dissipation and wave KE dissipation in L286-310:
- We agree with the reviewer's point that the TKE production by breaking waves, wind-driven shear turbulence and the nonlocal transport of TKE through Langmuir turbulence are likely significant for the TKE balance. In this study only the wave-generated turbulence is concerned for simplicity, in addition, the nonbreaking waves observed were unforced (no wind), mechanically generated, deep-water, two-dimensional wave trains in Fig. 3 and sources of the shear production were carefully eliminated, so the turbulence observed must have been directly generated by the waves themselves(Babanin and Haus, 2009). The TKE dissipation rate $\varepsilon_{\rm dis}$ can be virtually identical to $e_{\rm tid}$ numerically, but it should be noted that the two physical quantities have different physical meanings.

• We apologize for the confusions between observations for experiment 1 and observations for experiment 2, between Line 1 with Line 2 of model results in Fig.4. Thanks for the reviewer's insightful comments, and we redraw the figure as follows:

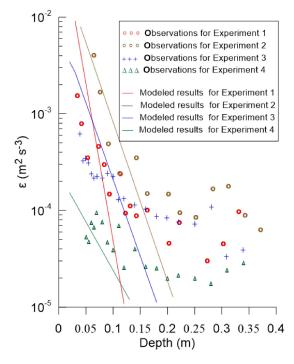


Figure 4. Dependence of TKE dissipation rate \mathcal{E}_{dis} (denoted as \mathcal{E} in the figure) versus layer depth. Observation data (circles, pluses and triangles) are digitalized from Wei et al. (2018). Dependence (19) is shown with solid lines. Data are plotted in linear/logarithmic scale on the horizontal/vertical axis.

5. Section 3: The purpose and context

- O We agree with reviewer's opinions that this section consists of scaling behavior only for the duration-limited growth and decay in order to evaluate the new dissipation formulations due to wave-generated turbulence and wave-breaking and it is not appropriate to refer to this as a "validation" (L416). So the sentence is revised to To evaluate the scaling behavior of the new dissipation formulations due to wave-generated turbulence and wave-breaking proposed in Section 2, as well as their different effects, several numerical experiments are carried out (Table 2).
- We agree the reviewer's opinion that it is possible to tune the models to get similar results by using free tuning parameters. Indeed in the new proposed dissipation formulations, there are still two undetermined parameters. Their valid ranges are inferred individually from the independent observation verifications. We discussed more concentrately about the two undetermined parameters in "Section 4 Dicussions". We think we pursued an attempt in numerical modeling not to tune parameters freely but to inspect their physics based on observations.

6. Section 4:

- In Section 4, we concentrate on the undetermined parameters in the new dissipation formulations due to wave-generated turbulence and wave-breaking, including their physical meanings, respective origins, underlying challenges and possible future solutions. Although their valid ranges are inferred individually from independent observation verifications in Section 2 and 3, they are still the remain uncertainty issues which concern the main topics of the present study. The other purpose we discussed the undetermined parameters is to propose a potential way not to tune free parameters, which was commonly used previously for dissipation terms to balance the input ones in numerical modeling. We agree the reviewer's constructive suggestions to clarify the effects of the gradient Richardson number R_g on the turbulence and the modification to the $\alpha_{\rm wt}$. In this study, the undetermined $\alpha_{\rm wt}$ implies the quasi-equilibrium level of wave-generated turbulence, originated from the equilibrium solutions of the second-order turbulence model with high certainty closure assumptions. It is qualitatively related to the instability area proportion as shown in Fig. 10 or the instability volume proportion beneath the wave surface, albeit it does not ensure the areas satisfying the criterion of the critical value R_g^c are turbulent. This is a challenging problem and our current arguments are still tentative.
- We agree with the reviewer's opinions that he stability criterion based on the gradient Richardson number is only a necessary condition for instability and the evaluations do not ensure that the areas satisfying the criterion are turbulent. We remove the not-so-rigorous claim "the perturbation must be amplified" (L504) and supplement the qualitative relationship between the undetermined $\alpha_{\rm wt}$ and the gradient Richardson number $R_{\rm g}$ as:

The coefficient $\alpha_{\rm wt}$ is qualitatively related to the two-dimensional instability area proportion as shown in Fig. 10 or the instability volume proportion beneath the wave surface in real scenarios, which needs further studies.

7. Section 5

In this study, our main topics are the analytical dissipation source functions induced by wave-generated turbulence and wave-breaking, and their validations by comparing to the independent laboratory or in-lake site measurements. For the latter, the valid range of the undermined parameter P was inferred from validations of the sea surface whitecap courage obtained from the statistical wave-breaking model with the satellite-derived data (Wang et al. 2017, 2018), and its different effects are shown in Fig. 9. We agree with the reviewer's point this is still tentative and requires further elaboration, especially its challenge that we discussed in section 4. We modify the sentence at L543 rigorously to "Exploratory comparative analysis of the paper reveals that the wave energy loss only induced by wave-breaking appears to inadequate, which indicates that it may be somewhat overestimated in the previous studies." For duration-limited growth, no good observations lead us to compare to the original MASNUM wave model and

WAM wave model (Janssen et al., 1994), to evaluate the performance of the new dissipation formulations, and to perceive the different effects between the two new dissipation formulations due to wave-generated turbulence and wave-breaking. This is the weakest part of the manuscript, and we are conducting these works in fetch-limited conditions to compare with abundant fetch-limited observation data, which will be part of scaling behaviors in our future series papers.

Specific comments and minor issues

- 8. We revised to "a second-order turbulence closure model between the wave shear instability generations and the turbulent kinetic energy (TKE) dissipations with high certainty closure assumptions" for terminological uniformity in the manuscript.
- 9. We supplement the definitions of these mathematical symbols in the revised manuscript.
- 10. We apologize for the writing error for definition of the kinetic energy at L157, the power of 2 should be deleted.
- 11. We carefully check again the dimensions of KE and PE at L162, both have consistent dimensions, in consideration of the definition of the Brunt–Väisälä frequency

$$\hat{N}_{3}^{2} = -g \frac{\partial}{\partial x_{3}} \left(\frac{\hat{\rho}}{\rho_{0}} \right) \text{ at L 157.}$$

12. This expression follows that in Chapter "7.3 THE WAVE SPECTRUM" of the book "Wind Waves" by Kinsman (2012). This expression is not entirely rigorous, but convenient for the following derivation processes, we adopt this simplified notation and there is no problem for the derived formulations.

Kinsman, B. (Eds.): Wind Waves: Their Generation and Propagation on the Ocean Surface, Dover Publications, Inc., New York, USA, 676 pp., ISBN 978-0-486-64652-7, 2012.

13. this transformation is used to introduce the following unified mean $\hat{\omega}$, \hat{K} for various integral mean variables, in order to obtain the convenient approximation (15) at L220. The unified mean $\hat{\omega}$, \hat{K} are introduced for various integral mean variables for practical numerical applications, which satisfy

$$\iint_{k} E(k_1, k_2) \exp\{2Kx_3\} dk_1 dk_2 \approx \exp\{2\hat{K}_1 x_3\} \iint_{k} E(k_1, k_2) dk_1 dk_2 \Rightarrow \iint_{k} \omega^2 K^2 E(k_1, k_2) \exp\{2Kx_3\} dk_1 dk_2 \approx \hat{\omega}^2 \hat{K}_2^2 \exp\{2\hat{K}_2 x_3\} \iint_{k} E(k_1, k_2) dk_1 dk_2$$
and
$$\iint_{k} \frac{1}{2K + 2\hat{K}_1 + \hat{K}_2} \omega^2 K^2 E(k_1, k_2) dk_1 dk_2 \approx \frac{1}{5\hat{K}_3} \iint_{k} \omega^2 K^2 E(k_1, k_2) dk_1 dk_2 \text{ . Here we assume that } \hat{K}_1 \approx \hat{K}_2 \approx \hat{K}_3 \approx \hat{K}$$
approximately.

14. The definitions of τ_w , τ at L243 are supplemented in the revised manuscript.

- 15. The wave-induced stress $\tau_{\rm w}$, is less than the total stress $\tau = u_{\rm s}^2$ with $u_{\rm s}$ the friction velocity (Janssen 1991), so we set $\frac{\tau_{\rm w}}{\tau} = 0.5$ for simplicity, which means the wave-induced stress accounts for a large proportion. $\alpha_{\rm wt}$ implies the quasi-equilibrium level of wave-generated turbulence, here we set $\alpha_{\rm wt} = 1$ which means a state of balance between the wave motion shear instability generations and the TKE dissipations is achieved (Yuan et al., 2013).
- 16. The significant TKE dissipation rates \mathcal{E}_{dis} were retrieved from the measurements at the 30 mm layer from the still surface for all recorded nonbreaking waves, which is stated at L269. So the layer depth x_3 varies for different waves, and we explicitly account for this variable during our model estimation of the TKE dissipation rates \mathcal{E}_{dis} .
- 17. We thanks for the reviewer's advice and we change the parameter $\,\mathcal{E}\,$ at L335 and L337 to $\,\mathcal{E}_{\rm sp}\,$.
- 18. The model covers a 23°×27° geographic region with 0.25°×0.25° horizontal resolution (The modeling spectral space was set as 24 directions with intervals of 15° and 25 wavenumbers spaced exponentially from the minimum wavenumber of 0.0071 m⁻¹ up to 0.6894 m⁻¹ with intervals of *K_i*/*K_{i-1}*=1.21, i=2,3,...,25). Model results in the middle of the geographic region are selected for numerical comparisons. In fact, the model exhibits spatially uniform performance for the simple duration-limited growth and decay experiments, except for a confined area near boundaries. So it appears to be not necessary to describe the model settings or the simulated domain in the manuscript. We will clarify them in our future studies in fetch-limited conditions, in which this issue is of critical importance.