



# 1 Introduction

With global warming, we anticipate more intense rainfall events and associated natural hazards, e.g., in terms of floods and landslides, in many regions of the world Lee et al. (2023). Understanding and accurately simulating precipitation is particularly important for adaptation planning and, hence, for mitigating damages and reducing risks associated with climate change. Earth System Models (ESMs) play a crucial role in simulating precipitation patterns for both historical and future scenarios. However, these simulations are computationally extremely demanding, primarily because they require solving complex partial differential equations. To manage the computational load, ESMs resort to approximate solutions on discretized grids with coarse spatial resolution (typically around 100 km). The consequence is that these models do not resolve small-scale dynamics, such as many of the processes relevant to precipitation generation. This leads to considerable biases in the ESM fields compared to observations. Moreover, the coarse spatial resolution prevents accurate projections of localized precipitation extremes. The precipitation fields simulated by ESMs can therefore not be used directly for impact assessments Zelinka et al. (2020) and especially tasks such as water resource and flood management, which require precise spatial data at high resolution Gutmann et al. (2014).

Statistical bias correction methods can be used as a post-processing to adjust statistical biases. Quantile mapping (QM) is the most common method for improving the statistics of ESM precipitation fields Tong et al. (2021); Gudmundsson et al. (2012); Cannon et al. (2015). QM reduces the bias using a mapping that, locally at each grid cell, aligns the estimated cumulative distribution of the model output with the observed precipitation patterns over a reference time period. Although QM is effective in correcting distributions of single grid cells, it falls short in improving the spatial structure and patterns of precipitation simulations Hess et al. (2022). Visual inspection shows that ESM precipitation remains too smooth compared to the observational data after applying quantile mapping.

To address these problems, deep learning methods have recently been introduced Hess et al. (2023); Pan et al. (2021); François et al. (2021); Hess et al. (2022). In these approaches, the statistical relationships between model simulations and observational data are learned implicitly. A general constraint when using machine learning methods for bias correction is that individual samples of observational and Earth System Model data are always unpaired. In this context, a sample is a specific weather situation at a specific point in time. The reason for this lack of pairs is that simulations, even with very similar initial conditions, diverge already after a short period of time due to the chaotic nature of the underlying atmospheric dynamics. Currently, one can, therefore, not utilize the wide range of supervised ML techniques that have shown great success in various disciplines in recent years and the available options are consequently restricted to self- and unsupervised machine learning methods. Recent studies Hess et al. (2023); Pan et al. (2021); François et al. (2021); Hess et al. (2022) applied generative adversarial networks (GANs Goodfellow et al. (2020)) and specifically cycleGANs Zhu et al. (2017) to improve upon existing bias correction techniques. A major limitation of GAN-based approaches is that the stability and convergence of the training process are difficult to control and that it is challenging to find metrics that indicate training convergence. In addition, GANs often suffer from mode collapse, where only a part of the target probability distribution is approximated by the GAN.

As noted above, the low spatial resolution of ESM fields prevents local risk and impact assessment, necessitating the additional use of downscaling methods. In line with the climate literature, we refer to increasing the spatial resolution as downscaling throughout our manuscript, although we are aware that, especially in the machine learning literature, the term upsampling is more prevalent. We use the term downscaling only when we want to increase the information content in an image as well as the number of pixels. When we refer to upsampling (downsampling), we only mean an increase (decrease) in the number of pixels. Statistical downscaling aims to learn a transformation from the low-resolution ESM fields to high-resolution observations. Recent developments lean towards using machine learning methods for this task Rampal et al. (2022); Hobeichi et al. (2023); Rampal et al. (2024). The potential for machine learning-based downscaling methods was already shown in Vandal et al. (2017); van der Meer et al. (2023); Doury et al. (2023, 2024); Rampal et al. (2025). The work presented a downscaling approach built on purely convolutional networks.

Recently, Hess et al. Hess et al. (2025) used an unconditional consistency model (CM) for downscaling  $3^\circ \times 3.75^\circ$  precipitation data to  $0.75^\circ \times 0.9375^\circ$ . Our work addresses the more challenging task of downscaling from  $1^\circ \times 1.25^\circ$  to  $0.25^\circ \times 0.25^\circ$  horizontal resolution, a scale essential for regional

89 impact assessments. We show that the consistency model applied to our higher resolution setting with  
90 limited amounts of training data struggles to approximate the distribution, highlighting an advantage  
91 of our conditional training approach. The analysis is further extended to out-of-distribution scenarios,  
92 particularly those involving extreme precipitation and future emission projections.

93 Diffusion models (DMs) have recently emerged as the state-of-the-art ML approach for conditional  
94 image generation Saharia et al. (2022b); Rombach et al. (2022); Saharia et al. (2022c) and image-to-  
95 image translation Saharia et al. (2022a), mostly outperforming GANs across different tasks. Diffusion  
96 models (Fig. 1 and fig. S1) avoid the common issues present with GANs in exchange for slower  
97 inference speed. A diffusion model consists of a forward and a backward process. During the forward  
98 process, noise is added to an image in subsequent steps to gradually remove its content. The amount  
99 of noise added follows a predefined equation. During the backward process, a neural network is trained  
100 to reverse each of these individual noising steps to recover the original image. The trained diffusion  
101 model can generate an image of the training data distribution, given a noise image as input. Recent  
102 work Wan et al. (2024) introduced a framework for downscaling and bias correction, combining a  
103 diffusion model that is responsible for downscaling and a model based on optimal transport responsible  
104 for bias correction Cuturi (2013). Optimal transport learns a map between two data distributions in  
105 an unsupervised setting. However, this framework is computationally expensive and has so far only  
106 been demonstrated on synthetic datasets, without evaluation on real-world observational or ESM  
107 fields. In contrast, our approach is computationally efficient by integrating computationally efficient  
108 QM for large-scale bias correction with a conditional diffusion model that performs both small-  
109 scale bias correction and downscaling by generating matching small-scale patterns. Our conditional  
110 design ensures that essential ESM properties, such as trends, uncertainties, and time consistency, are  
111 preserved. We demonstrate its effectiveness for precipitation data, highlighting its ability to correct  
112 biases, downscale accurately, and capture extremes, uncertainties, and trends. A major advantage is  
113 that our conditional training allows us to use a relatively small dataset for training and still capture  
114 the distribution accurately. In contrast, unconditional models often need considerably more data to  
115 capture the full data variability, as we also show in our comparison to Hess et al. (2025) (see fig. S2).

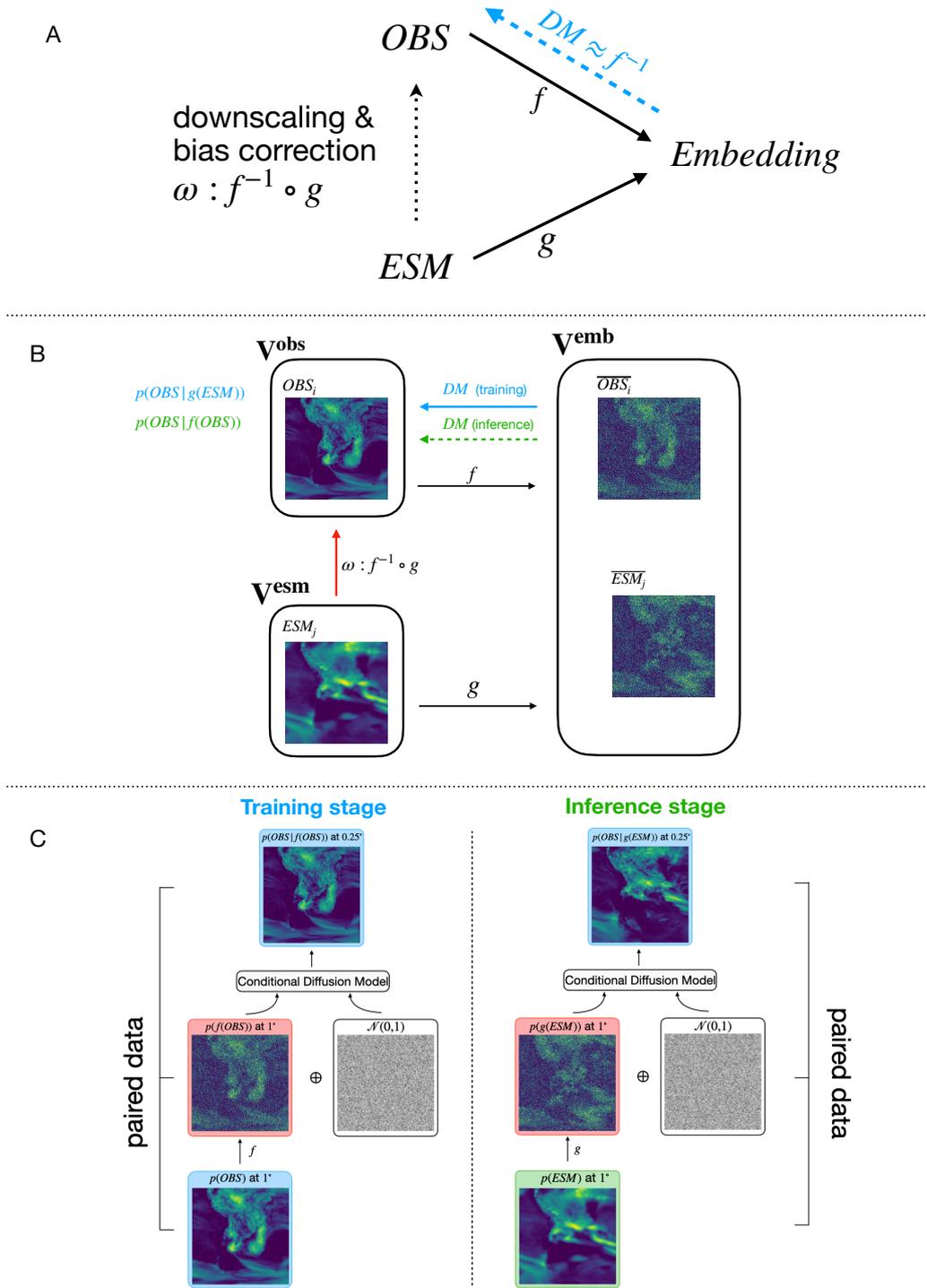
116 Existing work leveraging state-of-the-art ML methods for bias correction and downscaling does not  
117 systematically investigate out-of-distribution scenarios like future emission scenarios and especially  
118 the representation of extreme events of the generative models in detail. Understanding the generaliza-  
119 tion performance of the models under these conditions is, however, crucial for impact modelers who  
120 rely on these outputs for risk assessments under future climate conditions. We will therefore present  
121 a detailed analysis of the generalization capability of our approach, both in terms of its performance  
122 in preserving climate change trends, as well as in capturing extreme events and their trends.

123 A major challenge in bias correction and downscaling of ESMs is that the whole class of state-  
124 of-the-art supervised machine learning methods is not applicable in this setting. This is due to two  
125 fundamental issues. First, due to the chaotic nature of atmosphere and ocean dynamics, ESM simula-  
126 tions and observational data are inherently unpaired. This means that the weather on a specific day  
127 in an ESM simulation does not correspond to the observed weather on the same day, which prevents  
128 directly training a supervised ML method on the task. Second, training a ML model on observational  
129 data and applying it to ESM data is unreliable due to the substantial distribution shift between both  
130 datasets caused by systematic biases in the ESM. This violation of the assumption of independently  
131 and identically distributed (i.i.d.) data leads to poor generalization. Our proposed framework directly  
132 addresses both challenges. We reformulate the problem in a novel way, which allows us to train arbi-  
133 trary ML models in a conditional setup without the need for explicit ESM-observation pairs, while  
134 at the same time resolving the distribution shift.

135 We present a novel framework based on state-of-the-art conditional diffusion models that allows  
136 us to perform both bias correction and downscaling with one single neural network, which only takes  
137 precipitation as input and output. We use a conditional diffusion model (Fig. 1 and fig. S1) to  
138 correct low-resolution (LR) ESM fields toward high-resolution (HR) observational data (OBS). The  
139 supervised formulation of the task allows us to train a conditional diffusion model that is more data  
140 efficient (requiring less training data) than its unconditional counterpart because it is trained to only  
141 learn the small-scale precipitation patterns, given the large-scale patterns. The model then learns to  
142 copy the correct large-scale information from the condition channel. An unconditional model that  
143 learns to approximate the full distribution of precipitation at all scales is unnecessarily complex for  
144 the task. In general, our task of bias correction and downscaling can be seen as taking a field from a  
145 distribution  $p(ESM)$  and transforming it into a field from a conditional distribution  $p(OBS|ESM)$ .

146 A key idea of our framework is to reformulate the problem in a way that yields a clear training  
147 objective. A key part of it is a statistical mapping to an embedding space, which ensures that training  
148 and inference data are identically distributed. We achieve this by introducing transformations  $f$  and  
149  $g$  that map observational (OBS) and ESM data to a shared embedding space (see Methods and Fig.  
150 1A). This space is explicitly designed to solve the two fundamental issues mentioned above: it creates  
151 a valid supervised objective by providing paired samples of observational data and their perturbed  
152 embeddings (OBS,  $f(\text{OBS})$ ), and it ensures the training and inference distributions match by making  
153 the distributions of the embeddings  $f(\text{OBS})$  and  $g(\text{ESM})$  similar. On this shared embedding space,  
154 we can train a conditional diffusion model to approximate the inverse of  $f$  (Fig. 1B and Fig. 1C). The  
155 neural network is trained to predict the clean OBS data given the embedded OBS data, thereby only  
156 relying on pairs between OBS and  $f(\text{OBS})$ . For inference, the ESM data is mapped into the same  
157 embedding space using the transformation  $g$ . The statistical similarity of the resulting embeddings  
158  $f(\text{OBS})$  and  $g(\text{ESM})$  enables the diffusion model, which was trained exclusively on observational  
159 data, to generalize effectively to downscale and bias-correct the ESM fields. The diffusion model  
160 will map the embedded ESM data towards the distribution of observational data, resulting in bias-  
161 corrected and downscaled ESM fields.

162 This framework offers great flexibility as it can be applied to any ESM, with minimal adjustments  
163 in the embedding pipeline. The embedding transformation for the ESM has two key components.  
164 First, we use quantile mapping (QM) as a fast and effective method to correct large-scale biases in  
165 the ESM. Second, we introduce noise to remove small-scale information in the precipitation fields. We  
166 define large scales as those spatial scales that are effectively corrected using QM alone, while smaller  
167 spatial scales, which require additional correction, are referred to as small scales (Fig. 2). This noise  
168 selectively targets small-scale patterns, leaving intact large-scale patterns. In our approach, quantile  
169 mapping addresses large-scale biases, while the small-scale biases and downscaling are handled by  
170 our diffusion model. The task of our model is then to perform downscaling and bias correction by  
171 regenerating these small-scale features, in a way that ensures consistency with the preserved large-  
172 scale patterns. When applying our framework on a different region or ESM, it is computationally  
173 inexpensive to recompute the Quantile Mapping (QM) for the embedding transformation.



**Fig. 1 Schematic overview of our approach.** (A) Bias correction and downscaling can be formulated as a mapping  $\omega$  from the ESM data space to the data space of observations (OBS) used for training. We first map both datasets to a shared embedding space and then learn the inverse of the mapping  $f$  with a DM. We achieve a correction of the ESM data by applying  $DM \circ g$ . (B) Our framework allows to train a single model for bias correction and downscaling in a supervised way despite the unpaired nature of OBS and ESM fields. We construct functions  $f, g$  that map  $OBS \in \mathbf{V}^{\text{obs}}$  and  $ESM \in \mathbf{V}^{\text{esm}}$  fields to a shared embedding space  $\mathbf{V}^{\text{emb}}$ . Note that this embedding space does not enforce pairing between individual fields, but a similar distribution between the embedded fields. By inverting  $f$ , we can rewrite  $\omega$  as  $\omega = f^{-1} \circ g$ . We learn the inverse  $f^{-1}$  with a conditional diffusion model. This model is trained (blue arrow) on pairs of observational data to approximate the map from  $f(OBS)$  to OBS. Because  $f(OBS)$  and  $g(ESM)$  share the embedding space (and are identically distributed by construction), we can evaluate (green arrow) the DM on the embedded ESM data  $g(ESM)$  and thereby approximate the bias correction and downscaling function  $\omega = f^{-1} \circ g \approx DM \circ g$ , without the need of paired data between OBS and ESM. The indices  $i, j$  highlight that the two exemplary fields  $ESM_j$  and  $ESM_i$  are not paired.

(C) Left: Training process of the conditional DM  $DM \approx f^{-1}$ . Note that the individual samples of the input OBS and their embeddings  $f(OBS)$ , as well as the embeddings  $f(OBS)$  and the output of  $DM \approx f^{-1}$  are paired, respectively. Right: Inference process of  $DM \approx f^{-1}$ . In this case, the individual samples of the input ESM, their embeddings  $g(ESM)$ , and the output of  $DM \approx f^{-1}$  are paired, respectively. It is not necessary for the training embedding samples to be paired with the inference embedding samples. See fig. S1 for details.

## 2 Results

The ability of the diffusion model  $DM$  to approximate  $f^{-1}$  and the effectiveness of the transformations  $f, g$  will determine the overall performance of the downscaling and bias correction model  $\omega = DM \circ g$ . Therefore, we first investigate the effectiveness of the embedding transformations  $f$  and  $g$ , followed by an analysis of the downscaling and bias correction performance of the diffusion model  $DM$ , on the observational dataset. Once we have shown that both work as expected, we investigate the performance of the diffusion model in bias correction and downscaling of the ESM precipitation fields. Without loss of generality, we chose the  $0.25^\circ$  ERA5 reanalysis [Hersbach et al. \(2020\)](#) data as observational data and the state-of-the-art GFDL-ESM4 [Dunne et al. \(2020\)](#) at  $1^\circ$  as our ESM.

### 2.1 Embedding evaluation

Transformations  $f, g$  are chosen so that they map observational (OBS) and model (ESM) data to a common embedding space  $\mathbf{V}^{\text{emb}}$ , where all samples are identically distributed. For constructing  $f$  and  $g$  we need  $f(ERA5)$  and  $g(GFDL)$  to be unbiased with respect to each other. The transformations need to be chosen such that the embedded data share the same distribution and the same power spectral density (PSD). We assess if they are statistically unbiased towards each other by analyzing their histograms and latitude / longitude profiles, as well as their spatial PSDs (after applying pre-processing transformations). Figure S3 shows that  $f(ERA5)$  and  $g(GFDL)$  have the same spatial distribution (fig. S3A) with minor differences in temporal statistics shown by the histogram (fig. S3B) and latitudinal/ longitudinal profiles (fig. S3C and fig. S3D).

The individual operations that make up the transformations  $f$  and  $g$  do not change the large-scale patterns of their respective inputs, as desired for a valid bias correction. The goal of downscaling and bias correction  $\omega$  (Fig. 1) is to rely on the unbiased large-scale patterns of the ESM and correct statistics, as well as small-scale patterns. The transformation  $g$  conserves the unbiased information from the ESM by construction. Therefore, we want the diffusion model, approximating  $f^{-1}$ , to also preserve unbiased information.

The extreme case of erasing all detail with large amounts of noise (Fig. 2A) leads to learning the unconditional distribution  $p(ERA5)$ , which is then not a correction of  $GFDL$  but a generative emulation of the ERA5 reanalysis data. We tested this by adding the same amount of noise to the output of our diffusion model that was added to create  $g(GFDL)$ . This ensures that both the downscaled and bias-corrected fields, as well as the original GFDL fields, lack the small-scale details up to the same point.

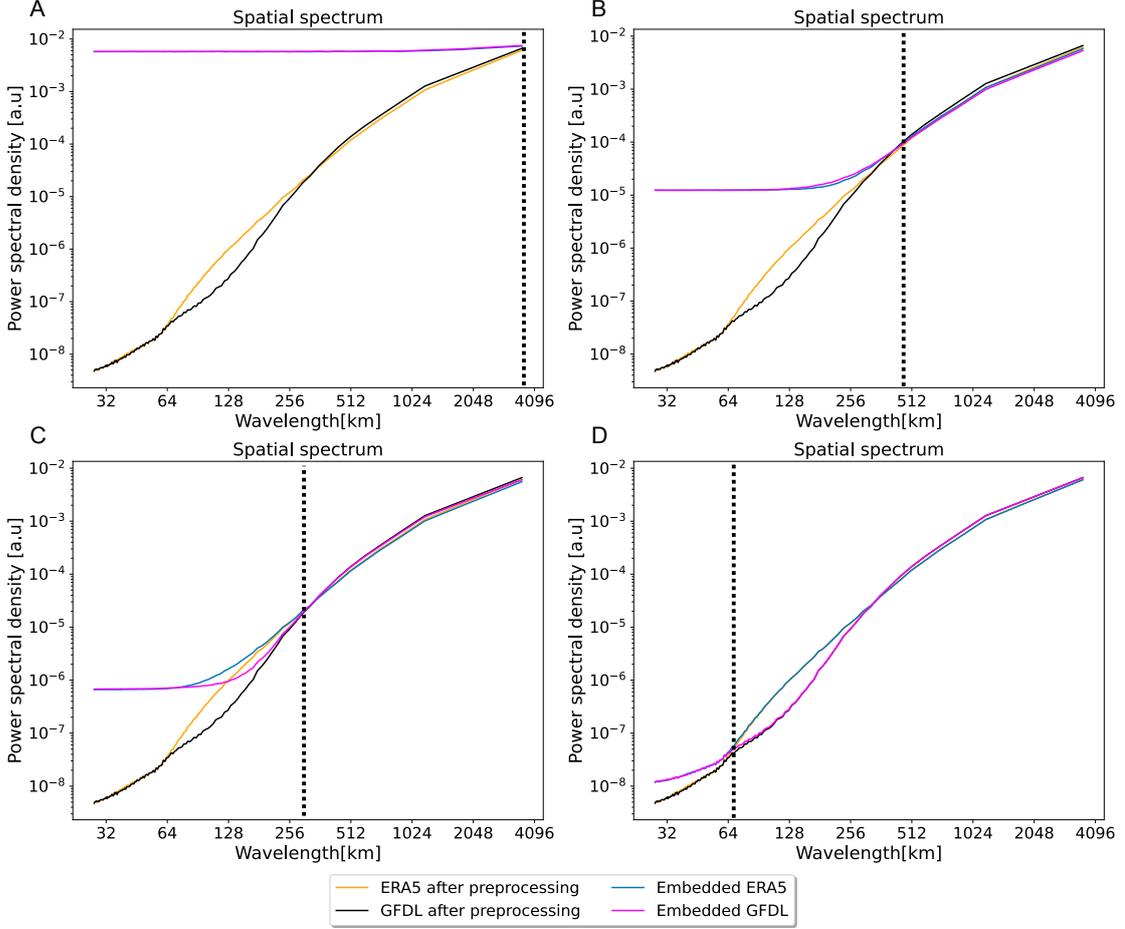
To verify that large-scale patterns are preserved by the diffusion model, we compute image similarity metrics between the low pass filtered version of the input of the diffusion model (embedded ERA5 data  $f(ERA5)$ ) and the low pass filtered output of the diffusion model  $DM(f(ERA5))$ . The output of the low pass filter leaves the large-scale features unchanged. The comparison yields an average structural similarity index (SSIM [Wang et al. \(2004\)](#)) value of 0.85 and a Pearson correlation coefficient of 0.95 for the validation dataset. This verifies that large-scale patterns are well preserved by the diffusion model.

Our diffusion model is able to reconstruct high-resolution fields following the ERA5 distribution from embedded ERA5 fields  $f(ERA5)$ , with only minor discrepancies in small-scale patterns (fig. S4A).

A comparison between the mean absolute spatial-temporal difference between the first downsampled and then bilinearly upsampled ERA5 and the ground truth ERA5 fields at  $0.25^\circ$  yields a mean bias of 0.27 mm/d. The downscaling of our diffusion model reduces this bias to 0.21 mm/d (at  $0.25^\circ$ ). Our diffusion model thus approximates  $f^{-1}$  well, and we successfully created a shared embedding space in which  $f(ERA5)$  and  $g(GFDL)$  are identically distributed.

### 2.2 Evaluation of downscaling and bias correction performance

We investigate the inference performance of our diffusion model on embedded GFDL data  $g(GFDL)$ . We compare the downscaling and bias correction performance of our diffusion model to a benchmark consisting of first applying bilinear upsampling followed by QM for bias correction.



**Fig. 2 Power spectral densities (PSDs) for different choices noising scale of the diffusion model.** The noising scale  $s$  (dashed line) is a hyperparameter that can be chosen depending on the ESM and observational datasets, as well as on the specific task. For the maximal choice of  $s$  (**A**) all information in the observations (ERA5) and model simulations (GFDL) is noised and thereby destroyed. Conditioning on pure noise makes the task equivalent to unconditional image generation. The diffusion model will learn to generate observational fields with no relation to the ESM fields. When  $s$  is chosen to be minimal, there will be no noising and the conditional generation will directly replicate the condition, i.e. the ESM field. In (**B**) we chose  $s$  as the point where the PSDs of the observational and simulated datasets intersect. We then apply sufficiently many forward diffusion noising steps to both datasets, destroying small-scale structure until they agree in the PSD. We call scales smaller than  $s$  small scales and scales larger than  $s$  large scales. In (**C**) and (**D**), the effects of choosing a smaller noise scale  $s$  are shown. Prior knowledge about the ESM or its accuracy can also guide the choice of  $s$ .

227 Figure 3 presents a qualitative comparison between the different individual precipitation fields.  
 228 The upsampled *GFDL* fields, as well as our benchmark consisting of the upsampled and QM-corrected  
 229 *GFDL* fields, are visually too smooth. They therefore appear blurry compared to the ERA5 pre-  
 230 cipitation fields despite having the same spatial resolution of  $0.25^\circ$ . Our diffusion model produces  
 231 high-resolution detailed outputs that are visually indistinguishable from the ERA5 reanalysis that  
 232 we treat as the ground truth. We also compared our diffusion model to a different state-of-the-art  
 233 diffusion model implementation, EDM Karras et al. (2022). The EDM model was trained for the  
 234 same number of epochs, while taking twice as long for one. The EDM almost perfectly corrects the  
 235 spectrum (fig. S5A). However in both the histogram (fig. S5B) as well as in latitudinal and longitu-  
 236 dinal profiles (fig. S5C and fig. S5D) the EDM model is far inferior to our proposed diffusion model.  
 237 We also compared our method against a VQ-VAE-based generative model, finding that our model  
 238 outperforms it across these metrics (for details, see SI Sec. S3 and fig. S23).

239 To further validate our choice of architecture, we also compare the diffusion model’s performance  
 240 against two other state-of-the-art deep learning models, a UNet and a Transformer, using the same  
 241 experimental setup. The results (fig. S6) show a significant advantage for the DM in reproducing  
 242 small-scale spatial patterns, by aligning better with the ERA5 reference spectrum (fig. S6A). In

243 contrast, all three models perform comparably well in correcting the overall precipitation distribution  
244 and the latitudinal/longitudinal mean profiles (fig. S6B-D). The generative process of the diffusion  
245 model is particularly well-suited for correcting the high-frequency spatial details. Another advantage  
246 over both deterministic models is the DM’s stochasticity, which allows for the generation of ensembles  
247 to quantify uncertainty.

248 The temporally averaged precipitation fields show that the climatology of the diffusion model-  
249 corrected GFDL data (Fig. 4A) and the high-resolution ERA5 data (Fig. 4C) is more accurate and  
250 less smooth than the climatology of the GFDL data (Fig. 4B). A comparison between the absolute  
251 temporally and absolute spatial-temporally averaged diffusion model corrected GFDL and ERA5  
252 fields (Fig. 4D) yields a bias of 0.32 mm/d. This is a substantial improvement over the original GFDL  
253 dataset, which yields a bias of 0.69 mm/d (Fig. 4E). Our diffusion model performs comparably with  
254 the state-of-the-art bias correction performance of our benchmark, which is by design optimal for  
255 this task, at 0.26 mm/d (Fig. 4F). For a quantitative comparison including Root Mean Square Error  
256 (RMSE) and Pearson correlation for these climatologies, see Table S1.

257  
258 There are large differences between the GFDL and ERA5 data in small-scale patterns (Fig. 5A).  
259 The histogram of precipitation intensities (Fig. 5B) also confirms that the ESM is only really accurate  
260 for precipitation events up to 40 mm/d, after which the respective frequencies diverge. The latitudi-  
261 nal and longitudinal mean profiles (Fig. 5C and Fig. 5D) indicate the presence of regional biases.

262  
263 Our framework demonstrates comparable skill to the QM-based benchmark in correcting the  
264 latitude and longitude profiles, for which QM is near optimal by construction (Fig. 5C and Fig. 5D).  
265 Comparing the histograms (Fig. 5B and fig. S7) shows that our diffusion model is superior compared  
266 to the benchmark, strongly outperforming it for extreme values, in particular.

267 For the spatial patterns and especially the small-scale spatial features, the QM benchmark shows  
268 only slight improvements over the original GFDL data (Fig. 5A). The diffusion model is vastly superior  
269 in correcting these small-scale spatial patterns (Fig. 5A and Fig. 3) and almost completely removes  
270 the small-scale biases, as seen in the spatial PSD.

271 To verify that large-scale patterns are preserved by the diffusion model, we compute image simi-  
272 larity metrics between the low-pass-filtered embedded GFDL data and the low-pass filtered output of  
273 the diffusion model. The comparison yields an average structural similarity index value (SSIM Wang  
274 et al. (2004)) of 0.77 and a Pearson correlation coefficient of 0.90, verifying that large-scale patterns  
275 are well preserved by the diffusion model.

276 We also assess our model’s performance on extreme precipitation events. For this, we use the  
277 R95p metric, which is defined as the total annual precipitation from wet days ( $PR > 1$  mm/day) that  
278 exceed the 95th percentile of our reference period. The difference between the R95p values for the  
279 ERA5 and DM corrected GFDL (fig. S8A), the ERA5 and QM corrected GFDL (fig. S8B) and ERA5  
280 and GFDL (fig. S8C), demonstrate that the diffusion model effectively corrects the bias in extreme  
281 precipitation events, performing at least as well as the quantile mapping correction. To further test  
282 the model’s performance on correcting characteristics of rainfall events in the tail of the distribution,  
283 we conduct a return-level analysis for extreme rainfall events (fig. S9). We calculated the average  
284 return periods for both moderately extreme ( $>50$  mm/day) and very extreme ( $>80$  mm/day) events.  
285 The raw GFDL model has a significant wet bias, substantially underestimating the return periods  
286 (3.33 years and 4.60 years) compared to the ERA5 reference (4.11 years and 7.38 years). Our DM  
287 successfully mitigates this bias, yielding more realistic return periods of 4.18 and 7.98 years.

288 We show that the spatial correlation between the climatologies is improved through our method by  
289 computing the Pearson correlation between the temporally averaged fields. The Pearson correlation  
290 between ERA5 and GFDL climatology is 0.83, while the correlation between ERA5 and DM-corrected  
291 GFDL is 0.98, which is the same as that for the QM-corrected GFDL data. We also investigate how  
292 our DM captures the statistics of consecutive dry days (CDD) and consecutive wet days (CWD)  
293 compared to the QM benchmark and the raw GFDL (fig. S10). Our diffusion model produces superior  
294 CDD (fig. S11A and fig. S11B) and CWD (fig. S11D and fig. S11E) statistics compared to our QM  
295 benchmark and GFDL, as shown in the difference plots of CDD / CWD.

296 Our method therefore accurately preserves the large-scale precipitation content, while successfully  
297 correcting small-scale structure of the precipitation fields, as well as statistical biases in histograms  
298 and latitude / longitude profiles (Fig. 5). Finally, we confirmed the temporal consistency of our model  
299 by analyzing autocorrelation (fig. S25) and seasonal spell duration (fig. S26). We further validated  
300 the robustness of our metrics over an extended validation period (1995-2014) (fig. S27).

301 We also test our framework on a different region of similar size over South Asia. We choose the  
302 same GFDL dataset and keep the experimental setup and evaluation identical to the South American  
303 region. The setup for quantile mapping the South Asia GFDL data and creating the benchmark data  
304 is also the same. We retrained our DM on mapping embedded ERA5 data (over South Asia) to the  
305 original ERA5 data. The noising scale in this experiment is the same as for South America, as the  
306 PSDs for both regions diverge around the same spatial scale. The evaluation (fig. S12) confirms that  
307 our DM successfully corrects precipitation biases in this new region and most notably outperforms  
308 the QM baseline in representing small-scale spatial features.

309 To further assess our framework’s robustness, we conducted an additional experiment using a  
310 different ESM. We replaced the GFDL dataset with the MPI-ESM HR model while keeping the  
311 experimental setup and evaluation protocol identical. The MPI and GFDL data diverge at a similar  
312 spatial scale in the PSD over the South American domain, allowing us to use the same noising  
313 scale hyperparameter  $s$ . Quantile delta mapping was applied in the same way as for the GFDL  
314 data. Consequently, our diffusion model did not require retraining and could be applied directly to  
315 the embedded MPI data at inference. Evaluation on our main metrics (fig. S13) demonstrates our  
316 framework’s ability to generalize to different ESMs. Our DM not only restores spatial variability  
317 across all scales significantly better than the QM benchmark (fig. S13A), but also shows superior  
318 ability to reproduce the frequency of extreme precipitation events (fig. S13B).

### 319 2.3 Evaluation of ensemble spread

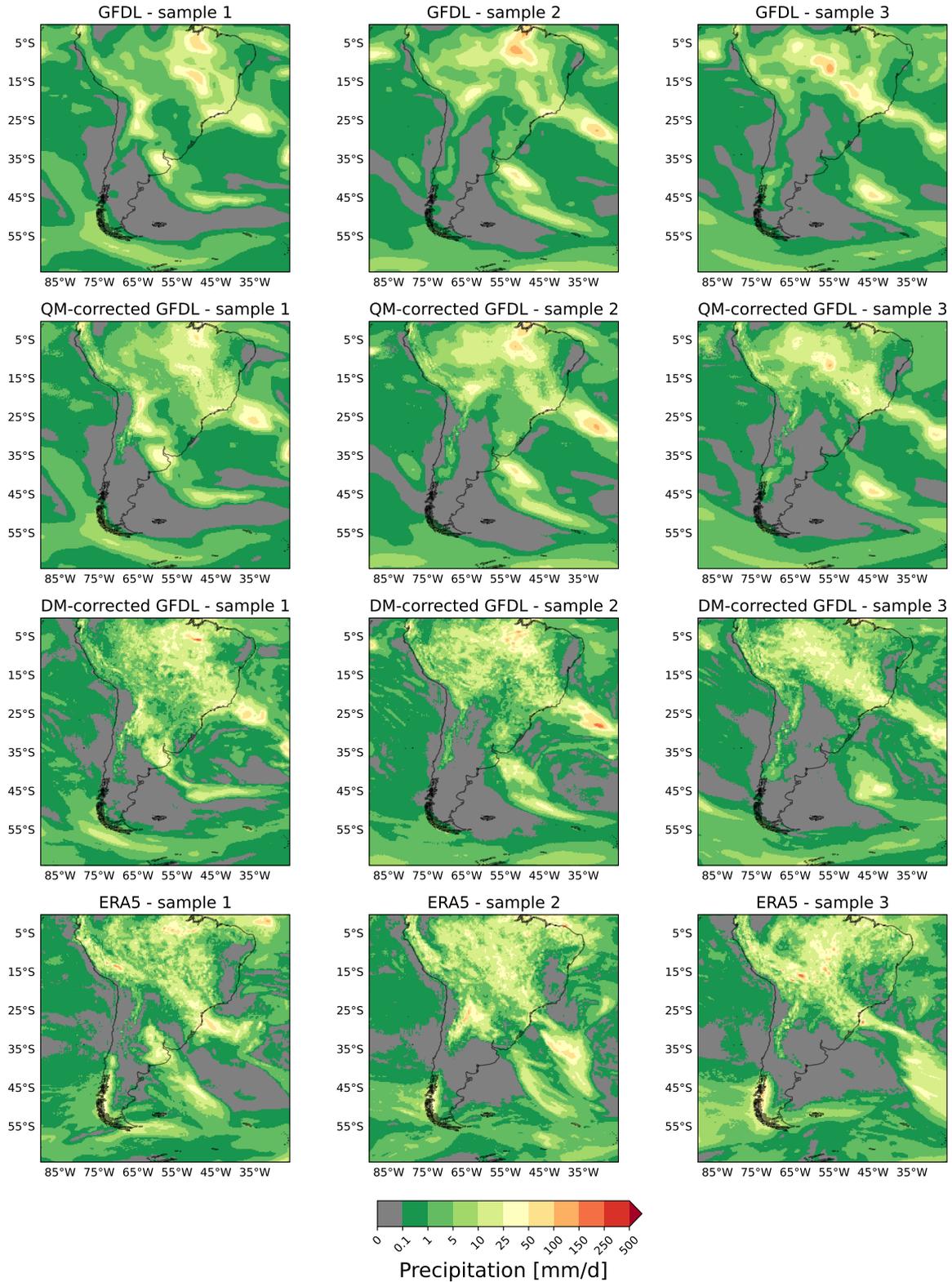
320 One of the key strengths of our method lies in its capability to generate a diverse ensemble of  
321 downscaled and bias-corrected fields from a single condition. We therefore evaluate the ability of our  
322 diffusion model to represent and produce accurate estimates of uncertainty, a critical aspect for robust  
323 climate modeling and decision-making. We generate a 50-member DM ensemble by running the model  
324 50 times, each conditioned on the same low-resolution ERA5 year, producing one-year trajectories.  
325 The corresponding high-resolution year serves as the ground truth. Our results demonstrate that the  
326 DM ensemble effectively reproduces the correct precipitation patterns, as shown by the close alignment  
327 between the ensemble mean and the high resolution ground truth of ERA5 over the annual cycle  
328 (fig. S14). Probabilistic performance, evaluated using CRPS, highlights that the DM significantly  
329 outperforms a bilinear baseline, with lower mean CRPS values (0.76 mm/day vs 0.90 mm/day), as  
330 well as better temporally and spatially averaged CRPS (fig. S15). Furthermore, we confirm that  
331 the DM ensemble produces well-calibrated uncertainty estimates with a spread-skill plot. Our model  
332 achieves near-perfect alignment with the 1:1 line, indicating an accurate representation of uncertainty  
333 (fig. S24). For more details see SI Sec. S4.1.

### 334 2.4 Evaluation on future climate scenarios

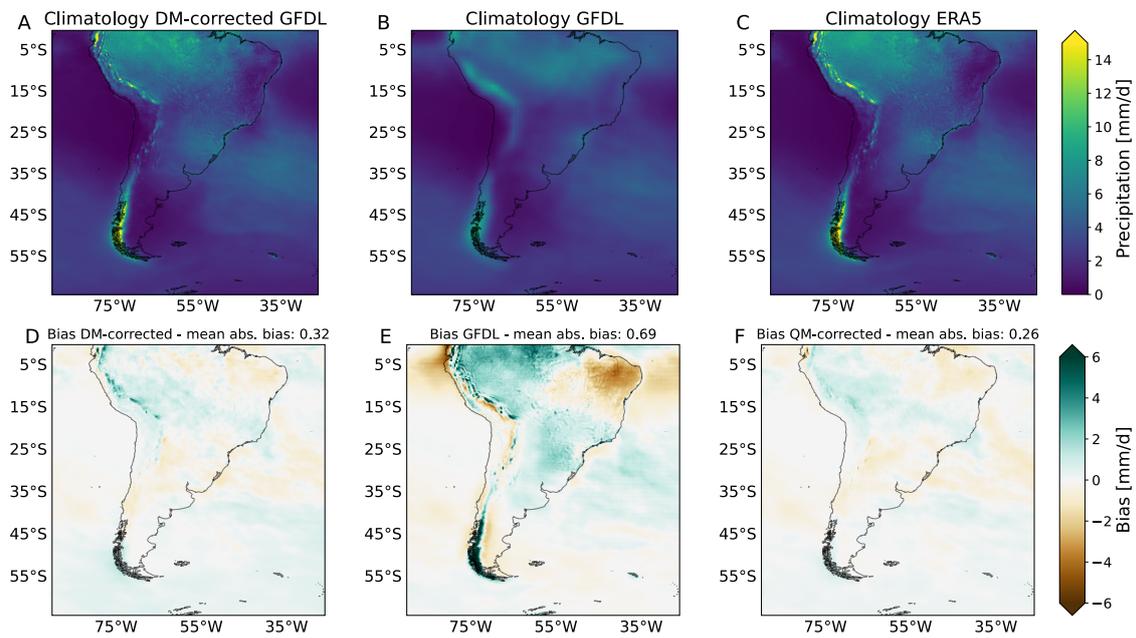
335 Evaluating the performance of downscaling models is crucial for their application in climate impact  
336 studies under future climate scenarios. We assess our diffusion model’s ability to preserve climate  
337 change signals in the underlying ESM simulations by applying it to a high-emission future scenario  
338 (SSP5-8.5). Figure 6 compares the relative climate change signal between the late 21st century (2081-  
339 2100) and the historical period (1995–2014) for annual mean and annual extreme precipitation. We  
340 find that our downscaled  $0.25^\circ$  fields successfully capture the mean precipitation change, closely  
341 matching the pattern and magnitude shown in the original  $1^\circ$  GFDL data (Fig. 6A and Fig. 6B). The  
342 diffusion model also robustly preserves the climate change signal for extreme precipitation indices,  
343 including Rx1Day (wettest day for each year) and R95p (Fig. 6C - Fig. 6F). The spatial patterns of  
344 change for the extremes are well-reproduced in the DM-corrected output compared to the original  
345 model data. Notably, slight differences are observed in the northwestern domain (Fig. 6C and Fig.  
346 6E), where the DM-correction projects a slightly stronger increase in extreme events under SSP5-8.5.  
347 A slight increase in extremes aligns with the diffusion model’s bias correction capabilities, reflecting  
348 its role in addressing the known under-representation of extreme precipitation in the original GFDL  
349 simulations.

350 Furthermore, we demonstrate that our conditionally trained diffusion model generalizes robustly  
351 to unseen future emission scenarios by accurately preserving regional precipitation trends without  
352 requiring retraining. We analyze the full annual mean precipitation timeseries from 2015 to 2100  
353 over two representative regions, one exhibiting a strong negative trend and one with a pronounced  
354 positive trend (fig. S16). For each region, we compare the annual mean precipitation from the original  
355 GFDL SSP5-8.5 data at  $1^\circ$  with the DM-corrected output at  $0.25^\circ$  resolution. The diffusion model

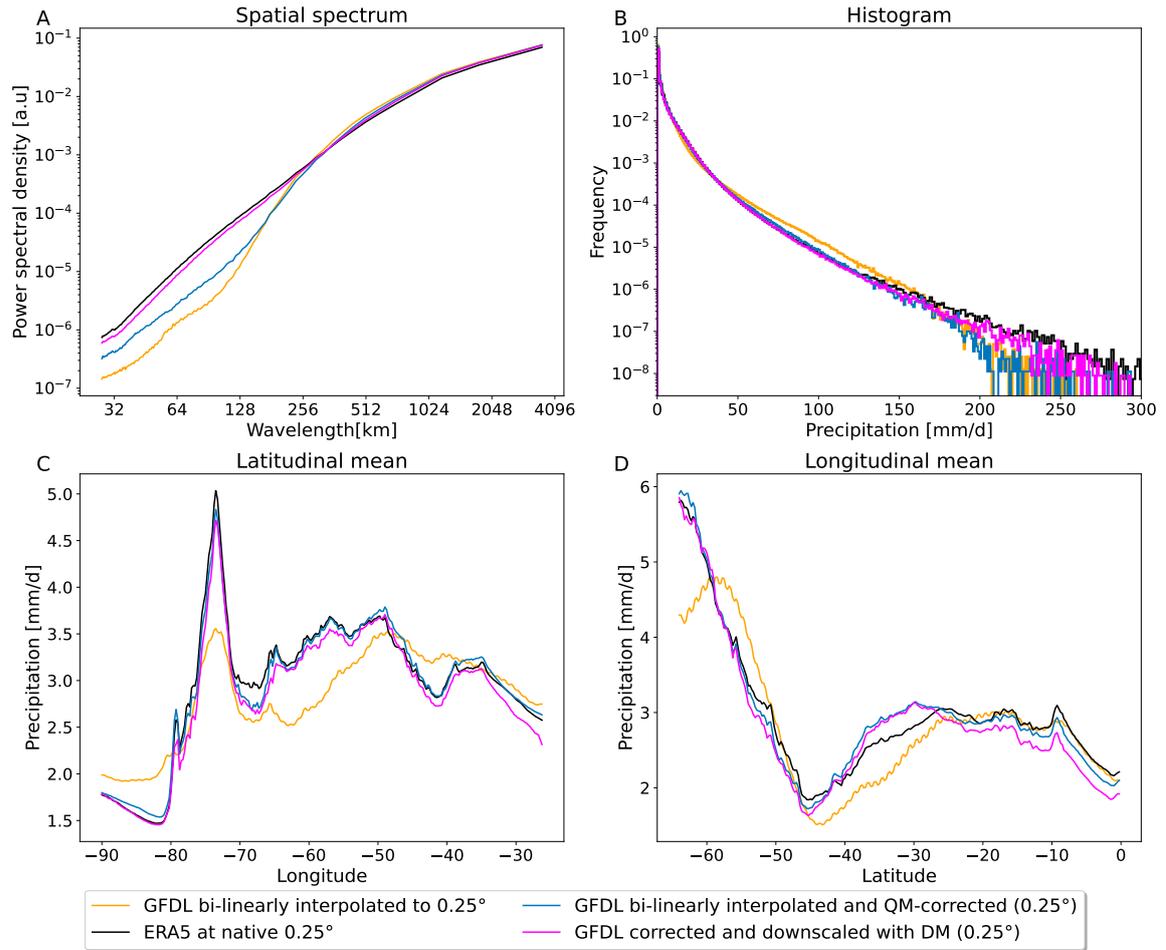
356 consistently preserves the direction and magnitude of the trends found in the original GFDL data  
357 across the entire timeseries, for both the negative (fig. S16 blue) and positive trend (fig. S16 red)  
358 regions. This demonstrates the model’s ability to maintain physically meaningful long-term changes  
359 in precipitation, further supporting its generalization capability to future scenarios. Note that the  
360 absolute values do not have to coincide, as our model corrects the bias and hence the numerical  
361 values. Our model can generalize to unseen climates, preserving the trends, since there is no decrease  
362 in performance during inference on GFDL SSP5-8.5 data. Note that our set-up generalized to unseen  
363 climate scenarios without any external constraints. The reason why our model preserves trends well  
364 is likely given by the fact that the trend is dominated by the large-scale patterns and our model  
365 learned to rely on the large-scale patterns of the condition and only generates small-scale patterns.



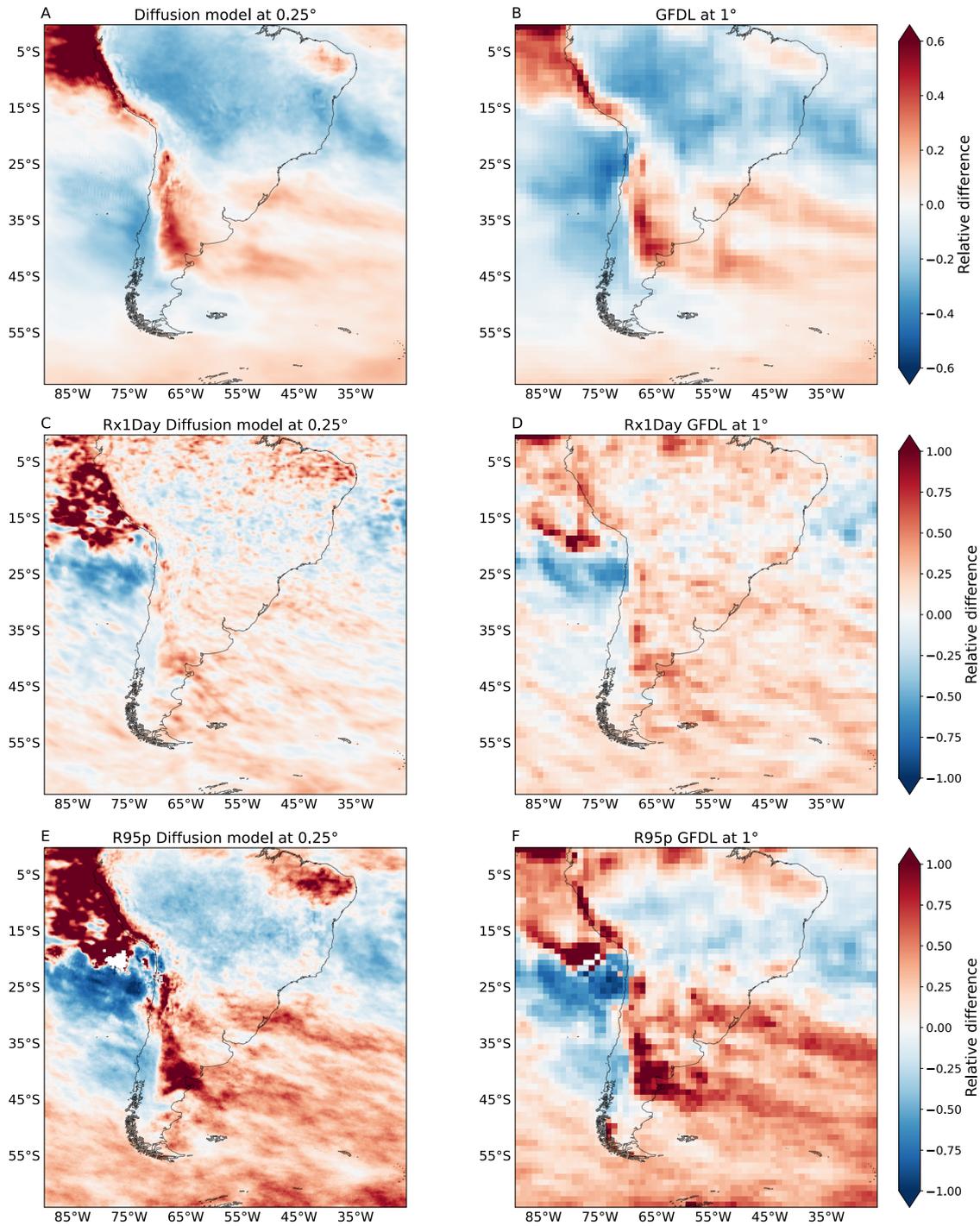
**Fig. 3 Comparative visualization of individual randomly selected samples.** Each row presents three samples of the same dataset. The top row shows GFDL ESM4 data, bi-linearly upsampled to  $0.25^\circ$  to match the other fields. The second row shows QM-corrected and the third row diffusion model-corrected GFDL fields. The bottom row shows samples of the original ERA5 data, which are unpaired to the GFDL fields above. Visual inspection shows that the diffusion model correction greatly improves upon the QM correction in terms of producing realistic spatial patterns, since the QM-corrected fields remain way too blurry compared to the HR ERA5 data. The overall large-scale patterns are preserved by the DM. There is no visual difference between the details and sharpness of diffusion model-corrected GFDL fields compared to ERA5.



**Fig. 4 Comparison of climatologies and model biases.** The first row shows the climatology of (A) the diffusion model-corrected GFDL at 0.25°, (B) the GFDL ESM4 model, upsampled to 0.25° and (C) the 0.25° ERA5 data. The second row shows the bias of the GFDL and the QM- and diffusion model-corrections, defined as the difference between long-term temporal averages of all validation samples. Specifically, the temporally averaged bias fields with respect to ERA5 are shown for (D) the diffusion model correction, (E) the uncorrected GFDL and (F) the QM correction. Results indicate a substantial improvement of our diffusion model (A) and the benchmark (C) over just upsampling GFDL to 0.25°. The absolute bias on top of each panel is given by the mean absolute value of the differences over the spatial and temporal dimension with respect to ERA5.



**Fig. 5 Evaluation of our diffusion model’s performance for downscaling and bias correction.** Comparison of GFDL (bi-linearly upsampled to 0.25°) (orange) and ERA5 (black) to diffusion model-corrected GFDL (magenta) and QM-corrected GFDL fields (blue) as our benchmark. The Power spectral density (PSD) plot (A) shows that the diffusion model corrects the small-scale spatial details far better than our benchmark. The spectrum aligns very well with the high resolution ERA5 target data. The histograms (B) as well as the latitude (C) and longitude (D) profiles show substantial improvements compared to the uncorrected GFDL data.



**Fig. 6 Comparison of relative climate change signals.** We compute the relative climate change signal between the late 21st century (2081-2100) under the GFDL SSP5-8.5 scenario and the historical GFDL period (1995–2014). In (A) and (B), we show that our diffusion model successfully preserves the mean precipitation climate change signal in the downscaled 0.25° GFDL fields, matching the change of the original 1° GFDL data. Positive values (red) indicate an intensification of precipitation, while negative values (blue) indicate reductions. In (C-D) and (E-F) we evaluate how well the DM-correction preserves the climate change signal for extreme events in historical and future scenarios. Both the Rx1Day (C-D) as well as R95p in (E-F) show that the DM-downscaling does preserve the climate change signal for extreme events. There are only slight differences over the north western part of our fields, where the DM-correction predicts more slightly more extremes for the SSP5-8.5 scenario. This is in line with the bias correction capabilities of the DM, correcting the under-representation of extreme precipitation in the original GFDL data.

### 3 Discussion

We introduced a framework based on generative machine learning that enables to do both bias correction and downscaling of Earth system model fields with a single diffusion model. We achieve this by first mapping observational fields and ESM data to a shared embedding space and then applying the learned inverse of the observation embedding transformation to the embedded ESM fields. We learn the inverse transformation with a conditional diffusion model. Although the underlying observational and ESM fields are unpaired, our framework allows for training on paired data (between observations and embedded observations, see above), and therefore any supervised machine learning method can be adopted to the task, which allows for more flexibility. Supervised methods are often superior in performance and more natural for the downscaling application. The diffusion model is trained on individual samples and has successfully learned to reproduce the statistics of observational data. For the observational ground truth, we chose the ERA5 reanalysis, and for the ESM data to be corrected and downscaled, we chose fields from GFDL-ESM4.

We demonstrated our framework’s robustness and generalizability in two additional experiments (Sec. 2.2). When applying the model to a new geographical region in South Asia with the same ESM, the DM requires retraining to adapt to the new regional characteristics. In contrast, when applying the framework to an entirely different ESM (MPI-ESM) over the South American region, the core DM did not need to be retrained since the same noising scale hyperparameter  $s$  could be used. For different ESMs a new noising scale hyperparameter could be necessary, requiring retraining of our DM with a different noising scale; however this depends on the choice of the spatial scale below which bias correction is desired, and for comparable outputs, we recommend to keep the noising scale  $s$  fixed for different ESMs. For example, to correct multiple ESMs at once, one can use the most heavily biased model to select the noising scale. A single diffusion model can then be trained to correct all ESMs at once, saving significant computational resources during inference. In general, we expect that many ESMs (like the MPI and GFDL model we use) will have similar spatial scales up to which they can capture realistic spatial precipitation features, because they have a similar resolution and have similar limitations from parameterization schemes. In all cases, readjusting the computationally inexpensive Quantile Delta Mapping (QDM) is a required step in the embedding process. The results will also depend on the specific quantile mapping scheme, QDM is chosen to preserve trends.

Our diffusion model corrects small-scale biases of the ESM fields, while completely preserving the large-scale structures, which is key for impact assessments, especially with regard to extremes and local impacts in terms of floods or landslides. The diffusion model performs particularly well for extreme events where traditional methods struggle. The method improves the temporal precipitation distribution at the grid cell level and surpasses the state-of-the-art approach (quantile mapping) in correcting spatial patterns. The downscaling performance has also been shown to be excellent. The diffusion model manages to generate small-scale details for the low resolution ESM data, that match those of high resolution observations. Our model preserves relevant information from the large scales, such as trends and extremes, and generates bias corrected and downscaled precipitation fields with adequate uncertainties.

We show that our method is robust in the out-of-distribution setting of downscaling and bias-correcting the SSP5-8.5 future emission scenario. It is critical for impact assessments that our model is able to accurately preserve the climate change signal of the original SSP5-8.5 data.

A key innovation of our approach is the embedding strategy, which makes the training process independent of the source ESM (apart from a single data-dependent hyperparameter setting the spatial scale below which the fields are corrected), which not only allows the framework to be flexibly applied to downscale and bias-correct a wide range of ESMs but also allows it to be used with different state-of-the-art machine learning backbone models. Another key advantage of our framework is its data efficiency. In our conditional approach the model only needs to learn how to generate small-scale features given the large-scale ones. The task is considerably less demanding than that of unconditional models (e.g., Hess et al. Hess et al. (2025)), which must learn the entire data distribution from scratch during training. This data efficiency makes our method applicable to datasets with shorter record lengths than ERA5, such as newer observational products.

Indeed, comparing results for generated climatologies between our conditional DM and the unconditional consistency model (CM) by Hess et al. Hess et al. (2025), it becomes apparent that

423 the CM struggles to learn the target distribution accurately, leading to blurring (fig. S2) that would  
424 hinder applications for impact assessments.

425

426 Our method is not specific to ERA5 and GFDL because the training of the diffusion model does  
427 not directly depend on the ESM choice. A specific ESM choice will only modify a hyperparameter  
428 in the embedding transformations  $f$  and  $g$ . This, however, requires almost no fine-tuning, as the  
429 temporal frequencies can always be matched with quantile mapping. The only parameter that might  
430 change for different datasets and use cases is the amount of noise that is added to the observational  
431 and ESM datasets. We choose the amount of noise such that the PSDs of the observational ground  
432 truth and the ESM fields align beyond a certain scale. This means that we have complete flexibility  
433 in deciding which patterns we want to preserve and which we want to correct. This is a major  
434 advantage over existing GAN based approaches.

435

436 We can decrease the level of detail that is preserved by the diffusion model through increasing  
437 the amount of noise added in the transformations  $f$  and  $g$ . This leads to higher uncertainties, and  
438 therefore increased variance in the outputs of the diffusion model. It is, however, not guaranteed that  
439 this increase in variance corresponds to the uncertainties of the ESM. The amount of noise added  
440 is directly proportional to the freedom the diffusion model has in generating diverse outputs and  
441 inversely proportional to the model’s ability to preserve large-scale patterns.

442

443 The downscaled and bias corrected fields will automatically inherit time consistency between  
444 different samples up to the noising scale. This means that ESM fields showing two successive days  
445 will still look like two successive days after the correction. Future work could build a video diffusion  
446 model that inputs and outputs full time series instead of single frames, in order to guarantee time  
447 consistency across all scales.

448

449 We focused on precipitation data over the South American continent, because of its heavily tailed  
450 distribution and the pronounced spatial intermittency. Especially at small scales, precipitation data  
451 is extremely challenging to model and therefore serves as a reasonable choice to show the frame-  
452 works capabilities in a particularly difficult setting. Regional data is chosen due to computational  
453 constraints, yet the diverse terrain of our study region, encompassing land, sea, and a wide range  
454 of altitudes, enables robust testing of the downscaling and bias correction performance, also given  
455 the substantial biases of the GFDL model in this region. We also conducted additional experiments  
456 for another region over South Asia, and using another ESM, namely the MPI-ESM-HR, in order to  
457 confirm the generality of our approach. The extension to global scales is straightforward and requires  
458 no major changes in the architecture. We intend to include more variables in a consistent manner  
459 on a global scale in future research. Optimizing the inference strategy, with speedup techniques such  
460 as distillation [Luhman and Luhman \(2021\)](#), to decrease the sampling time will prove helpful in this  
461 context. Taking the growing number of diffusion model variants into consideration, a comparison  
462 between different approaches can help establish the best method suited for the general task of bias  
463 correcting and downscaling ESM fields.

464

465 As for any ML model, the ability to generate the rarest extremes is limited by their frequency  
466 in the training data. Our conditional approach helps mitigate this to some extent by inheriting the  
467 large-scale patterns for these events directly from the ESM.

468

469 It is straightforward to extend our methodology to downscaling and bias correction of numerical  
470 or data-driven weather predictions on short- to medium-range or even seasonal temporal scales. This  
471 would not require any fundamental changes to the architecture. This would, however, require a target  
472 dataset with sufficiently high resolution. The ability of the diffusion model to not disturb the temporal  
473 consistency between samples can be useful in this scenario. Future work could then focus on extending  
474 this model to a multivariate setting, which would be essential for weather prediction and for assessing  
475 physical consistency between variables.

## 4 Materials and Methods

### 4.1 Data

For the study region, we focus on the South American continent and the surrounding oceans. Specifically, the targeted area spans from latitude  $0^{\circ}\text{N}$  to  $63^{\circ}\text{S}$  and from longitude  $-90^{\circ}\text{W}$  to  $-27^{\circ}\text{E}$ . For the ablation study of the South Asian region, we selected an area from  $0.75^{\circ}\text{N}$  to  $64.5^{\circ}\text{N}$  latitude and from  $42^{\circ}\text{E}$  to  $105.75^{\circ}\text{E}$  longitude. The training period comprises ERA5 data from 1992-01-01 to 2011-01-01. The range of years included for the evaluation on ERA5 and GFDL spans from 2011-01-02 to 2014-12-01. Additionally, an extended 20-year window (1995–2014) is used for analyses requiring greater statistical robustness.

#### ERA5

ERA5 [Hersbach et al. \(2020\)](#) is a state-of-the-art atmospheric reanalysis dataset provided by the European Center for Medium-Range Weather Forecasting (ECMWF). Reanalysis refers to the process of combining observations from various sources, such as weather stations, satellites, and other instruments, with a numerical weather model to create a continuous and comprehensive representation of the Earth’s atmosphere. We use the daily total precipitation data at  $0.25^{\circ}$  horizontal resolution as the target for the diffusion model.

#### GFDL

The climate model output is taken from a state-of-the-art ESM from Phase 6 of the Coupled Model Intercomparison Project (CMIP6), namely GFDL-ESM4 [Dunne et al. \(2020\)](#). We abbreviate the model with GFDL throughout the paper. The dataset contains daily precipitation data of the first ensemble member (r1i1p1f1) of the historical simulation (esm-hist). The data is available from 1850 to 2014, at  $1^{\circ}$  latitudinal and  $1.25^{\circ}$  longitudinal resolution and a daily temporal resolution.

GFDL-ESM4 [Dunne et al. \(2020\)](#) SSP5-8.5 represents a high-emission future pathway. We use daily-resolution data from the CMIP6 archive, provided at  $1^{\circ}$  latitude and  $1.25^{\circ}$  longitude spatial resolution, covering the period from 2015 to 2100.

#### MPI

For our ablation study, we repeat our experiments for the MPI-ESM HR model [Gutjahr et al. \(2019\)](#). We abbreviate MPI-ESM-HR with MPI in the paper.

The data has  $0.9375^{\circ} \times 0.9375^{\circ}$  spatial resolution. We use daily data from 1992 to 2014 with using data from 1992-2011 for training and 2011 to 2014 for inference.

The climate model output is taken from a state-of-the-art ESM from Phase 6 of the Coupled Model Intercomparison Project (CMIP6), namely GFDL-ESM4 [Dunne et al. \(2020\)](#). The dataset contains daily precipitation data of the first ensemble member (r1i1p1f1) of the historical simulation (esm-hist). The data is available from 1850 to 2014, at  $1^{\circ}$  latitudinal and  $1.25^{\circ}$  longitudinal resolution and a daily temporal resolution.

#### Benchmark dataset

In order to benchmark our method, we first apply bilinear interpolation to increase the resolution of the GFDL fields from  $1^{\circ}$  to  $0.25^{\circ}$ . After that, we apply Quantile delta mapping [Cannon et al. \(2015\)](#) to fit the upsampled GFDL data to the original  $0.25^{\circ}$  ERA5 data. QM is fitted on past observations and can then be used to correct the statistics of any (past/present) ESM field towards that reference period. We use quantile delta mapping (QDM) and chose the ERA5 training period from 1992-01-01 to 2011-01-01 as the reference period to fit the GFDL to ERA5. The benchmark dataset to evaluate our approach is then constructed by applying QM to the GFDL validation period (2011-01-02 to 2014-12-01). Some analyses required a longer evaluation period (1995–2014). To create a fair benchmark for these specific cases QDM was also recalibrated, it was both fitted and applied using data exclusively from this 1995–2014 window. For the SSP5-8.5 data, we use the 1995 to 2014 period of ERA5 as reference data and the historical GFDL data as the model input to fit the QDM. We then apply this mapping to the full time period of the GFDL SSP5-8.5 data (2015–2100).

## 524 Data pre-processing

525 The units of the GFDL data and MPI data are  $\text{kg m}^{-2}\text{s}^{-1}$ , and for ERA5 m/h. For consistency, both  
526 are transformed to mm/d.

527 Our pre-processing pipeline consists of:

- 528 • Only GFDL: rescaling the original  $1^\circ \times 1.25^\circ$  GFDL data to  $1 \times 1^\circ$  ( $64 \times 64$  pixel).
- 529 • Only MPI: rescaling the original  $0.9375^\circ \times 0.9375^\circ$  GFDL data to  $1 \times 1^\circ$  ( $64 \times 64$  pixel).
- 530 • Add +1 mm/d precipitation to each value in order to be able to apply a log-transformation to the  
531 data.
- 532 • Apply the logarithm with base 10 in order to compress the range of values.
- 533 • Standardize the data, i.e. subtract the mean and divide by the standard deviation to facilitate  
534 training convergence.
- 535 • Transform the data to the range  $[-1, 1]$  to facilitate the convergence of the training.

536 An ablation study (fig. S28) confirms the choice of our precipitation pre-processing pipeline,  
537 showing that omitting the log-transformation or the final range scaling leads to spectral discrepancies  
538 or distributional biases. As part of the transformation  $g$ , the  $1^\circ$  GFDL data is bilinearly upsampled.  
539 This and the downsampling and upsampling of ERA5 data, which is part of  $f$ , are already done during  
540 pre-processing. The downsampling of  $0.25^\circ$  ERA5 data ( $256 \times 256$  pixel) to  $1^\circ$  ( $64 \times 64$  pixel) is done  
541 by only keeping every fourth pixel in each field. For the just mentioned upsampling, we apply bilinear  
542 interpolation to increase the resolution from  $1^\circ$  to  $0.25^\circ$ . Note that bilinear interpolation to  $0.25^\circ$  does  
543 not increase the amount of information in the images compared to the  $1^\circ$  fields. After preprocessing  
544 the data as described, the embedding transformation  $f$  is applied. The diffusion model is trained with  
545 the preprocessed  $f(\text{ERA5})$  as a condition and the original  $0.25^\circ$  ERA5 data as a target. Before we  
546 apply the embedding transformation  $g$  we first pre-process the  $1^\circ$  GFDL data by applying quantile  
547 delta mapping (QDM Cannon et al. (2015)) with 500 quantiles. The bilinear upsampling is then used  
548 to increase the resolution to  $0.25 \times 0.25^\circ$  ( $256 \times 256$  pixels). The preprocessed data are used as input  
549 to the embedding transformation  $g$ . The corresponding output serves as the condition during the  
550 inference process of the diffusion model

## 551 4.2 Embedding framework

552 Our framework introduces transformations  $f$  &  $g$  that map OBS and ESM data to a shared embedding  
553 space  $f : \mathbf{V}^{\text{obs}} \rightarrow \mathbf{V}^{\text{emb}}$  and  $g : \mathbf{V}^{\text{esm}} \rightarrow \mathbf{V}^{\text{emb}}$ . The goal is to do bias correction and downscaling  
554 of ESM fields, i.e., to obtain samples from the conditional distribution  $\omega = p(\text{OBS}|\text{ESM})$ . Training  
555 a conditional model to approximate this distribution directly is not possible because OBS and ESM  
556 are unpaired. Therefore, we will train the model without the ESM data, only using OBS data and  
557 utilize a trick to enable transfer learning and inference on the ESM data. We apply transformations  
558 on ESM and OBS such that the resulting datasets are similarly distributed and therefore allow for  
559 generalization. The arrows in the diagram of Figure 1 show that we can represent the mapping that  
560 achieves the bias correction and downscaling as  $\omega = f^{-1} \circ g$ . Our idea is to approximate  $f^{-1}$  with a  
561 neural network  $f^{-1} \approx \epsilon$ . We chose a conditional diffusion model (DM), denoted by the conditional  
562 distribution  $p(\text{OBS}|f(\text{OBS}))$ , to approximate  $f^{-1} = \text{DM} : \mathbf{V}^{\text{emb}} \rightarrow \mathbf{V}^{\text{obs}}$ . The diffusion model (Fig.  
563 1C) is only trained on pairs  $(\text{OBS}, f(\text{OBS}))$ . The shared embedding space allows us to evaluate the  
564 trained model on ESM embeddings  $p(\text{OBS}|g(\text{ESM}))$ , as all embeddings are identically distributed.

### 565 4.2.1 Constructing the embedding space

566 The goal of  $f$  and  $g$  is to map OBS and ESM to a shared embedding space, where  $f(\text{OBS})$  and  
567  $g(\text{ESM})$  are identically distributed (Fig. 1). To achieve this, both embedded datasets need to be  
568 unbiased towards each other. OBS and ESM are biased towards each other in terms of statistical biases  
569 between distributions and biases between small-scale patterns visible in the spatial power spectral  
570 density (PSD) (fig. S4A).

571 As mentioned earlier, the input for the embedding transformation  $f$  is  $0.25^\circ$  ERA5 data, which is  
572 first downsampled, then upsampled and preprocessed. The input to the embedding transformation  $g$   
573 is the upsampled and preprocessed  $0.25^\circ$  GFDL data. By first downsampling ERA5 to  $1^\circ$  and then  
574 upsampling it to  $0.25^\circ$  we ensure that the fields match the information content of the original  $1^\circ$   
575 GFDL fields.

576 To remove small-scale pattern bias, we apply a noising procedure analogous to the forward diffusion  
577 process as part of  $f$  and  $g$ . Gaussian noise contains all frequencies in equal measure and the Fourier  
578 transform of Gaussian noise is itself Gaussian noise, so its power must be equal across all frequencies  
579 in expectation. The power spectrum of pure Gaussian noise corresponds to a horizontal line in the  
580 spectrum of Fig. 2A, reflecting the fact that it contains all frequencies in equal amounts. Adding  
581 noise to an image results in a hinge shape in the PSD of the noisy images (Fig. 2B, 2C and 2D).  
582 Increasing the variance of the noise increases its power and, as a result, its PSD will shift upward.  
583 Adding noise hence acts as a low-pass filter, while the variance of the added noise determines the cut-  
584 off frequency. Increasing variance leads to higher cut-off points as the power of the noisy frequencies  
585 increases. Both ERA5 and GFDL data are noised up to the cutoff frequency, denoted by  $s$ . The scale  
586  $s$  is determined by the point where ERA5 and the ESM data (in our case GFDL) start to disagree in  
587 their spatial PSDs (Fig. 2), i.e., the intersection between the two. Adding noise in this way ensures  
588 that  $f(ERA5)$  is unbiased compared to  $g(GFDL)$  in the PSD by erasing all information beneath  $s$ .  
589 In our implementation, the transformations  $f$  and  $g$  utilize the same cosine scheduler as the forward  
590 diffusion process to add Gaussian noise to the data. ERA5 data undergoes 50 noise steps within  
591  $f$ , while  $g$  applies the same 50 noise steps to the GFDL data. We ensure that the observational  
592 and ESM data have aligned distributions by incorporating Quantile Mapping (QM) directly into the  
593 transformation  $g$ . It is directly applied as pre-processing and only needs to be included in  $g$ . The  
594 quantile-mapped and bilinearly downsampled data is then noised as described above, as part of the  
595 embedding transformation. It is important to clarify that QM is not included because the diffusion  
596 model is unable to do bias correction. QM is only used as a tool in our framework to ensure that in  
597 the embedding space  $f(ERA5)$  and  $g(GFDL)$  are identically distributed, such that  $g(GFDL)$  can  
598 be used for the inference of the diffusion model.

#### 599 4.2.2 Determining the noising scale

600 The choice of the spatial scale  $s$  influences up to which scale we correct the spatial PSD. We note that  
601 the PSD shows spectral distributions normalized to 1; therefore, we can still observe slight changes  
602 above  $s$  when small-scale patterns are corrected. The point  $s$  is a hyperparameter chosen before  
603 training and purely depends on the datasets ESM and OBS and can be adjusted to the specific needs  
604 in a given context and task.

605 In the extreme case, where  $s$  is maximal, the conditional images will contain pure noise (Fig. 2A).  
606 In this case, the diffusion model is equivalent to an unconditional model. As an unconditional model,  
607 the diffusion model will correct all biases at all spatial scales, however, at the expense of completely  
608 losing any pairing between the condition and the output. We chose  $s$  to be at the intersection of the  
609 ERA5 and GFDL spectrum around 512 km (Fig. 2B). Thereby, we trust in the ESM’s ability to model  
610 large-scale structures above the point  $s$ , which we do not want to correct with the diffusion model.

#### 611 4.3 Network architecture and training

612 The general architecture of our diffusion model  $DM$  consists of a Denoising Diffusion Probabilistic  
613 Model (DDPM) architecture Ho et al. (2020) conditioned on low resolution images. For details about  
614 diffusion models and conditional diffusion models, see SI Sec. S1.1 and SI Sec. S1.2. This is similar  
615 to a single block of a cascading superresolution model. We employ current state-of-the-art techniques  
616 to facilitate faster convergence and find the following to be important for convergence and sample  
617 quality Saharia et al. (2022b): The memory efficient architecture, "Efficient U-Net", in combination  
618 with dynamic clipping and noise conditioning augmentation Ho et al. (2022) turned out to be effective  
619 for our relatively small dataset. We adopt the Min-SNR Hang et al. (2023) formulation to weight  
620 the loss terms of different timesteps based on the clamped signal-to-noise ratios. The diffusion model  
621 architecture utilizes a cosine schedule for noising the target data and a linear schedule for the condition  
622 during noise condition augmentation with 100 steps each. The diffusion model is trained to do  $v$ -  
623 prediction instead of noise prediction. The U-Net follows the  $64 \times 64 \rightarrow 256 \times 256$  Efficient U-Net  
624 architecture Saharia et al. (2022b). The diffusion model has 730.95 million trainable parameters and  
625 is trained for 100 epochs using the ADAM optimizer Kingma and Ba (2015) with a batch size of 2  
626 and a learning rate of  $1e^{-4}$ . Note that in the case of fig. S4, where the inference data is also embedded  
627 OBS data and there is no ESM data present, the model performs better when being trained and  
628 evaluated with 1000 denoising steps, instead of the 100 steps that we used in all our experiments that  
629 include ESM data. The model with 100 steps is far superior in training and inference speed and also  
630 in correcting the histograms, when correcting ESM data. We also compared the effect of not adding

<sup>631</sup> noise (SI Sec. S2.1) and the effect of not applying QM (SI Sec. S2.3) as shown in Figures S17, S18, S19,  
<sup>632</sup> S20, S21, as well as different noise choices (SI Sec. S2.2, fig. S22) during both training and inference.

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645 Conceptualization: MA, PB, YH, NB

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649 Writing—review & editing: MA, SB, PH, BP, YH

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654 Validation: NB, MA, YH

655 Funding acquisition: NB

656 Project administration: NB

657 Visualization: MA

658 Resources: YH

659

### 660 Data and Materials Availability

661 All data needed to evaluate the conclusions in the paper are present in the paper and/or the  
662 Supplementary Materials.

663 The ERA5 reanalysis data is available for download at the Copernicus Climate Change Service  
664 (<https://cds.climate.copernicus.eu/cdsapp#!/dataset/reanalysis-era5-single-levels?tab=overview>).

665 The CMIP6 GFDL-ESM4 is available at <https://esgf-data.dkrz.de/search/cmip6-dkrz/>.

666

667 The code will be available on GitHub ([https://github.com/aim56009/ESM\\_cdifffusion\\_](https://github.com/aim56009/ESM_cdifffusion_downscaling.git)  
668 [downscaling.git](https://github.com/aim56009/ESM_cdifffusion_downscaling.git)) and Zenodo (<https://doi.org/10.5281/zenodo.14849615>) Aich (2025).

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### 670 Competing interests

671 The authors declare no competing interests.

### 672 List of Supplementary Materials

673 Supplementary Text

674 Figures S1 to S28

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