Supplemental text for reply to RC2

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(2nd paragraph of Section 2.3)

Here we briefly introduce the calculation procedure of the LETKF. For a detailed formulation, we recommend that the readers refer to Hunt et al. (2007).

In the LETKF formulation, the Kalman filter calculation is performed in the ensemble space formed by the background ensemble perturbations, of which the background perturbation matrix $\mathbf{X}^b = [\mathbf{x}^{b(1)} - \overline{\mathbf{x}}^b, \ldots, \mathbf{x}^{b(K)} - \overline{\mathbf{x}}^b]$ consists, where K is the ensemble size. The background error covariance in the model space is

$$\mathbf{P}^b = \frac{1}{K-1}\mathbf{X}^b(\mathbf{X}^b)^T, \tag{1}$$

whereas the expression in the ensemble space is

$$\tilde{\mathbf{P}}^b = \frac{1}{K-1}\mathbf{I}, \tag{2}$$

then the analysis error covariance in the ensemble space is

$$\tilde{\mathbf{P}}^a = [(K-1)\mathbf{I} + (\mathbf{Y}^b)^T\mathbf{R}^{-1}\mathbf{Y}^b]^{-1} \tag{3}$$

where $\mathbf{Y}^b$ is the matrix with the columns of the background ensemble perturbations in the observation space, and $\mathbf{R}$ is the observation error covariance matrix.

For the consistency with Eqs. (1) and (3), the analysis ensemble perturbation is obtained as follows.

$$\mathbf{X}^a = \mathbf{X}^b\mathbf{W}^a \tag{4}$$

$$\mathbf{W}^a = [\tilde{\mathbf{P}}^a]^{1/2} \tag{5}$$

The analysis mean value is obtained using the Kalman filter formula.

$$\overline{\mathbf{x}}^a = \overline{\mathbf{x}}^b + \mathbf{X}^b\overline{\mathbf{w}}^a, \tag{6}$$

$$\overline{\mathbf{w}}^a = \tilde{\mathbf{P}}^a(\mathbf{Y}^b)^T\mathbf{R}^{-1}(\mathbf{y}^o - \overline{\mathbf{y}}^b), \tag{7}$$

where $\mathbf{y}^o$ is the observation and $\mathbf{y}^b$ and the background ensemble mean in the observation space.

Equation (6) is derived from the Kalman filter formulation, which calculates the Kalman gain that minimizes the trace of the analysis error covariance matrix. The analysis coincides with the maximum likelihood solution only when the background and observation error have Gaussian distributions. Therefore, when a background error is strongly non-Gaussian, the analysis increment could be different from the desirable solution.

References

Hunt, B. R., Kostelich, E. J., and Szunyogh, I.: Efficient data assimilation for spatiotemporal chaos: A local ensemble transform Kalman filter, Physica D: Nonlinear Phenomena, 230, 112–126, 2007.