Manuscript ID: *egusphere-2025-2542*

Journal: ***EGUsphere***

Improvement of the **Rnnmm-type** climate index approach with a spatio-temporal model based on the Hawkes process

We thank the reviewer for the careful reading and constructive suggestions. Below we respond point by point. Reviewer comments are in *italics*; our replies are in blue. All mentioned edits have been incorporated into the revised manuscript.

General comments

The paper proposes a geostatistical model based on self-exciting Hawkes processes for modeling the Rnnmm-type extreme climate index. It provides a detailed introduction to the proposed Hawkes process model. The performance of the model is evaluated through extensive cross-validation, with comparisons to Poisson models both with and without seasonality. The results are valuable for advancing research in the analysis and forecasting of climate extremes. The overall logic of the paper is systematic and complete. I provide several suggestions for improvement in the following sections. I recommend the paper for acceptance after minor revisions.

We appreciate the positive assessment. We implemented all minor revisions suggested below and clarified the presentation where indicated.

Specific comments

1. *Page 14, Line 306: MAD and MSE first appear in Sect. 5.1.1, but their definitions are only provided in Sect. 5.1.2 (page 18, lines 316–319). I suggest moving the definitions to Sect. 5.1.1 where these abbreviations first appear. If the meanings of MAD and MSE in Sect. 5.1.1 differ from those in Sect. 5.1.2, I recommend using different abbreviations to avoid confusion.*

   We moved the formal definitions of MAD and MSE to Section 5.1.1, where they first appear, and kept a short forward reference in Section 5.1.2. The meanings are identical across sections, so no distinct abbreviations are needed.

2. *Page 14, Lines 308–309: In the sentence "The results indicate that the combination of R3 with P1 yielded the lowest MAD and MSE values in most cases," Although this conclusion can be quantitatively supported by Tables 1–3, I further suggest including statistical values across all stations in the Tables or in the text. This would allow readers to more easily compare the performance of model A with different radius using a single representative value (e.g., mean or median MAD and MSE for all stations) that reflects overall performance across all stations.*

   We revised Tables 1–3 to include summary rows reporting the median MAD and MSE across all stations for each radius–weight combination. In Section 5.1.1, we added a brief explanatory note using these medians as representative statistics for model comparison, which reinforces the overall advantage of R_3 with $\mathbf{P}_1$.

3. *Page 19, Table 4: I suggest presenting the statistical data from Table 4 in the form of a map plot. The statistical values could be displayed using colored scatter points overlaid on the map, enabling readers to intuitively observe the spatial performance of the proposed model. You could also add a discussion on whether the model performs better at lower or higher elevations, or over flat versus complex terrain. The additional figure could include six subplots arranged in a 3 × 2 layout: subplot (3,2,1) showing MAD for Model A, subplot (3,2,2) showing MSE for Model A, subplot (3,2,3) showing MAD for Model B, subplot (3,2,4) showing MSE for Model B, subplot (3,2,5) showing MAD for Model C, and subplot (3,2,6) showing MSE for Model C.*

   We thank the reviewer for this helpful suggestion. However, instead of creating exactly a 3×2 panel figure, as suggested, we created a 1×2 one in which subplot (1) shows MAD for Models A, B and C, and subplot (2) shows the MSE. (Figure 1 below presents the subplot showing MSE for Models A, B and C). Each subplot overlays colored circles representing the stations on the study-area map with a radius length scale representing MAD/MSE measurement for comparability purposes. Section 5.1.3 now includes a discussion of performance by elevation and terrain complexity; we observe larger errors over complex terrain, consistent with expectations.

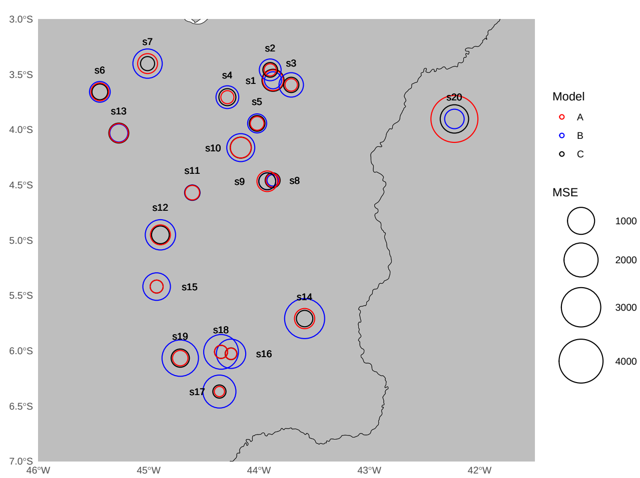

Figure 1: Spatial distribution of the Mean Squared Error (MSE) estimated by the competing models across monitoring stations.

4. *Finally, from a practical application perspective, I suggest presenting results that demonstrate when extreme precipitation is likely to occur in Maranhão based on your modeled Rnnmm-type extreme climate index. For example, you could indicate which months, seasons, or years tend to experience extreme precipitation. If this is not feasible, you could alternatively provide a time series plot showing both the Rnnmm-type extreme climate index and the observed number of extreme precipitation events, with the left y-axis representing the Rnnmm-type extreme climate index and the right y-axis representing the observed extreme precipitation events counts, x-axis*

represent the time series. I believe this would add significant value by highlighting the practical contributions of your work to improving climate extremes forecasting.

We added a new time-series figure with the R20mm-type index on the left y-axis and observed extreme-event counts on the right y-axis, sharing the time x-axis. We also summarize seasonal patterns (months/seasons with higher likelihood) in Section 5.2 and highlight how these results support practical anticipation of extreme precipitation in Maranhão.

Technical corrections

5. *Page 1, Manuscript Title: Consider changing "Rnnmm type" to "Rnnmm-type" for consistency with "Rnnmm-type extreme climate index" as shown in line 2 of the same page.*

   Done.

6. *Page 2, Line 48: The text does not provide the full form of the abbreviation IMERG. The authors should also include the full form of IMERG when it first appears on this line.*

   We now expand to "Integrated Multi-satellite Retrievals for GPM (IMERG)" at first use and use IMERG thereafter.

7. *Page 16–17, Tables 2–3: The abbreviation MSA in Tables 2 and 3 appears to be a typographical error; please correct it to MAD.*

   Corrected "MSA" to "MAD" in both tables.

8. *Page 20 Figure 2, you should clearly point it out that which axis is the Estimated function $\Lambda_3(t)$, since your plot does not show title and unit for x-axis and y-axis. I suggest you add a title and unit (if possible) for the x-axis and y-axis. Addtionaly, the fontsize for xticks and yticks is too small, you should increase the fontsize.*

   Figure 2 now includes explicit axis titles and units where applicable (time on x; estimated cumulative intensity $\Lambda_3(t)$ on y), a clear panel title, and increased tick/label font sizes for readability.

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Minor comments

1. *In the title, "Rnnmm type" should be "Rnnmm-type".*

   Done.

2. *The full form of IMERG is not introduced.*

   We now expand to "Integrated Multi-satellite Retrievals for GPM (IMERG)" at first use and use IMERG thereafter.

3. *I feel that presenting some tables in a horizontal fashion rather than in a vertical fashion (like now) would look better.*

   As suggested by the reviewer, we changed some of the tables to a horizontal format.

4. *Although the authors claim to provide an extensive cross-validation analysis, my concern is that they draw this conclusion based on a very limited dataset. Given that no extensive simulation has been shown, I think the word "extensive" should be toned down unless a larger number of spatial locations or a larger spatial domain is used.*

   We thank the reviewer for this helpful suggestion. We agree that the term "extensive" may be too strong given that the application is based on 20 monitoring stations. We have therefore revised the wording throughout the manuscript (Abstract and Conclusions), replacing "extensive cross-validation" with "leave-one-out cross-validation" (or equivalent wording that more precisely reflects the scope of the analysis).

   We emphasize that our evaluation is based on a leave-one-out cross-validation design across all available stations (20 folds), in which each station is sequentially excluded and predicted using the remaining data. The proposed model is compared against two Poisson-based alternatives, and predictive performance is assessed using MAD and MSE metrics.

   Regarding the number of locations, expanding the monitoring network is constrained both by data availability and by the computational cost of MCMC estimation, which increases substantially with the number of spatial locations. In the Brazilian environmental context, rain-gauge coverage is often limited in several regions, and a set of 20 quality-controlled stations already represents a realistic and informative regional network for applied studies.

Major comments

5. *While the article addresses an environmental statistics problem, the main contribution appears to be on the statistical side. However, the introduction cites only a few papers, mostly by the authors, and attempts to convince the reader that the existing literature is inappropriate for the problem discussed in the paper. Near the end of the Section, the authors simply claim to introduce an innovative self-exciting Hawkes process, without citing any papers or providing a proper literature review of their proposal. The introduction suggests the authors are introducing the self-exciting Hawkes process for the first time. The approach was proposed by Hawkes in the Biometrika paper "Spectra of some self-exciting and mutually exciting point processes" in 1971, more than fifty years ago. A clearly written paragraph including an appropriate literature survey on the self-exciting Hawkes process must be provided in the introduction section. Not only the Hawkes process, but also its spatio-temporal versions are common in the literature. For example, a review article on this topic is "A Review of Self-Exciting Spatio-Temporal Point Processes and Their Applications", by Alex Reinhart, published in Statistical Science in 2018. The authors should clearly indicate what their novel contribution is from a statistical perspective, or they should simply demonstrate the usefulness of an existing statistical method in the context of climate extremes.*

   We thank the reviewer for this comment. We agree that the introduction did not provide an adequate literature review on the self-exciting Hawkes process, and we appreciate the opportunity to improve this aspect of the manuscript. In the new version, we included in the introduction a clear paragraph providing an appropriate literature survey on the self-exciting Hawkes process and its extensions, in particular, we refer the papers "Spectra of some self-exciting and mutually exciting point processes", written by Hawkes (1971), and "A Review of Self-Exciting Spatio-Temporal Point Processes and Their Applications", by Alex Reinhart (2018), as suggested by the reviewer. We have also revised the text to clearly state the contribution of our work. We thank the reviewer again for this important suggestion, which has significantly improved the clarity and positioning of our contribution within the existing literature.

6. *Overall, the proposed model is a latent Gaussian model, where separate self-exciting Hawkes processes are used to model individual time series across locations, and then the potentially transformed spatially varying coefficients are modeled using Gaussian processes. In this approach, conditioning on the model coefficients, the data are modeled spatially independently. However, a convolution through a Gaussian process does not introduce extremal dependence. The authors can refer to a large statistical literature on spatial extremes in this regard. Hence, as a spatiotemporal model for inferring spatially varying coefficients, this approach may be better suited, but may not be from a spatial-extreme perspective.*

   We thank the reviewer for this important remark.

   We agree that the proposed model belongs to the class of latent Gaussian models, in which spatial dependence is introduced through Gaussian processes defined on the transformed coefficients, while the counting processes at each location are conditionally independent given these coefficients. In this sense, we acknowledge that this structure does not induce asymptotic extremal dependence in the classical sense considered in the spatial extremes literature (e.g., max-stable or Pareto process models).

   However, the primary objective of this work is not to model joint extremal dependence in the asymptotic multivariate extreme-value framework, but rather to model the temporal frequency of local extreme events (Rnnmm-type indices) and to allow spatially structured inference and interpolation of the associated parameters. The focus of the model is therefore on temporal clustering (via the Hawkes process) and spatial smoothing of coefficients, aiming to improve predictive performance and regional interpretability of climate extreme indices.

In this context, the proposed framework is particularly well suited for inferring spatially varying coefficients and for prediction over monitoring networks, even though it does not belong to the specific class of spatial-extreme models developed to capture asymptotic extremal dependence.

To avoid potential ambiguity, we will revise the Introduction and Conclusions to clarify this methodological positioning and explicitly delineate the scope of the proposed model with respect to the broader literature on spatial extremes.

7. *The authors choose gamma priors for the variance-related hyperparameters, while an inverse-gamma prior would be conjugate. The justification for choosing a non-conjugate prior should be provided. In the algorithm, the authors mention Step 2 as GI. What does it mean? Inverse-gamma? If so, the usual notation is IG.*

   We thank the reviewer for pointing out this issue.

   First, we acknowledge that there was an inconsistency in notation in the manuscript. In Equation (11), the prior for the variance parameters σ_k^2 was incorrectly stated as Gamma, whereas the implemented and intended prior is the Inverse-Gamma distribution. This has now been corrected throughout the manuscript for consistency. Additionally, in Algorithm 1, the notation "GI" was a typographical error and has been replaced by the standard notation "IG" (Inverse-Gamma).

   In our model, for each $k \in \{W, U, M\}$, the variance parameters follow

$$\sigma_k^2 \sim \mathrm{IG}(a_{\sigma_k}, b_{\sigma_k}),$$

   while the spatial dependence parameters follow

$$\phi_k \sim \mathrm{Gamma}(a_{\phi_k}, b_{\phi_k}).$$

   The use of the Inverse-Gamma prior for σ_k^2 is motivated by its conjugacy with the Gaussian likelihood structure arising from the Gaussian process prior specification. This choice leads to closed-form full conditional distributions and facilitates efficient Gibbs sampling within the MCMC algorithm. Such a specification is standard in hierarchical spatial models (see, for example, Banerjee et al., 2014; Diggle and Ribeiro, 2007).

   For the spatial range parameters ϕ_k, conjugacy is not available under the exponential correlation function. The Gamma prior was chosen because it has support on the positive real line and provides sufficient flexibility to control prior mass over plausible correlation ranges. This specification is also commonly adopted in spatial Gaussian process modeling.

   The manuscript has been revised to ensure that all prior specifications are clearly and consistently stated.

8. *The manuscript does not include details on the MCMC diagnostics. Besides, no simulation study has been shown. While I agree that EGUsphere is an environmental sciences-focused*

journal, such details should be provided in the supplement, as the main focus of the manuscript is statistical modeling. Given that there are only 20 locations, I am highly curious about the MCMC chains for the spatial dependence parameters.

We thank the reviewer for this important and detailed comment.

In the revised manuscript, we have explicitly addressed these concerns by including a comprehensive simulation study and MCMC diagnostic analysis, now presented in Appendix A.

Specifically, the simulation study is designed to evaluate parameter recovery, convergence behavior, and predictive performance under different spatial configurations. We consider multiple sampling scenarios ($N = 25$, 36, 49, and 64 locations) over a regular spatial grid, allowing us to assess the impact of spatial sampling density on inference.

In addition, we performed a detailed MCMC diagnostic analysis. For each configuration, four independent chains were run from dispersed initial values. Convergence was assessed using the potential scale reduction factor (PSRF), and the results indicate satisfactory convergence for most parameters, with clear improvement as the number of spatial locations increases.

We also evaluated the accuracy of the estimation procedure by verifying that the 95% credible intervals contain the true parameter values used in the simulation. Furthermore, posterior distributions are presented (Figure A2), illustrating increasing concentration around the true values as the sample size grows.

These additions substantially strengthen the statistical validation of the proposed model and directly address the reviewer's concerns regarding convergence diagnostics and simulation-based evaluation.

Additional References Cited in the Responses

Banerjee, S., Carlin, B. P., and Gelfand, A. E.: *Hierarchical Modeling and Analysis for Spatial Data*, 2nd edn., Chapman and Hall/CRC, Boca Raton, FL, 2014.

Diggle, P. J. and Ribeiro, P. J.: *Model-based Geostatistics*, Springer Series in Statistics, Springer, New York, 2007.

Gelfand, A. E., Diggle, P. J., Fuentes, M., and Guttorp, P.: *Handbook of Spatial Statistics*, Chapman and Hall/CRC, Boca Raton, FL, 2010.