Review of "Sea ice albedo bounded data assimilation and its impact on modelling: A regional approach"

By Joseph F. Rotondo et al.

Version 1

Date of review: 10 September 2025

Summary

This manuscript presents an interesting study on the potential usefulness of assimilating seaice albedo observations in sea-ice modelling and predictions. Although the study is restricted to a single-column sea-ice model, its promising results merit future investigations on this topic in the context of a full, three-dimensional model. The reporting of findings from the present study is therefore highly relevant to the scientific community. Hence, noting that the material in the manuscript is within the remit of *The Cryosphere*, I do recommend the manuscript's acceptance in the journal, though subject to some minor corrections as detailed below.

Main Comment

In Section 4, you analyse the results from a number of experiments assimilating different types of observations or different combinations of observation types. These include SIC-only, SIT-only, SIAL-only, and all variables (SIC, SIT and SIAL) combined. As the paper is aimed at demonstrating the added value of assimilating SIAL observations, it would be good to run an SIC-and-SIT-only experiment and compare it with the all-variable experiment to single out the impact of additionally assimilating SIAL observations. The findings from this comparison can complement the discussion around the comparison between an SIAL-only experiment and a free run (which also demonstrates the added value of assimilating SIAL observations, but without the SIC and SIT constraints).

On a related note, in Section 5.1 you say that assimilating SIAL may "complement or replace" the assimilation of SIT (line 374). These two observation types provide complementary information about sea ice, so I can't see why one might want the assimilation of SIT to be replaced – as opposed to being complemented – by the assimilation of SIAL. To me, the focus should be on the added value of SIAL observations, rather than having only one or the other observation type (but not both).

Other Comments

- 1. Lines 8 9: The use of "three-quarters of the Arctic regions studied" and "across all regions" could be potentially misleading, as you are only looking into 4 discrete points in the Arctic Ocean.
- 2. Line 13: Is this a typo? "improved" → "improving"

- 3. Lines 150 151: Does it mean there is no guarantee that the synthetic observations will be within the hard physical bounds?
- 4. Lines 154 155: The phrase "as aggregates of modelled quantities categorized by thickness... over these thickness categories" sounds a bit clumsy and could be a bit confusing. Is there a simpler way of expressing it?
- 5. Line 165: Please elaborate on what is meant by "direct and indirect shortwave radiation are treated equivalently".
- 6. Line 185: The acronym ICESat-2 is undefined.
- 7. Table 1:
 - a. In the heading of the right-most column, what is the significance of the multiplier 2 in front of σ ? Why do you show uncertainties in the form of 2σ instead of σ ?
 - b. I presume $aice_n$ and $vice_n$ are normalised quantities ("per unit area", i.e. normalised by the area of the grid cell); otherwise, the equations for the forward operators don't make sense. This should be mentioned somewhere, ideally when $aice_n$ and $vice_n$ are first defined.
 - c. How often do you get SIC values of exactly 0 or 1? In those cases, the observational uncertainty for SIC would be exactly 0. Would this be theoretically problematic for the QCEFF? Would you consider imposing a minimum value for the observational uncertainty?
 - d. In the expressions for the observational uncertainty of the different types of SIAL, the α should come with the appropriate subscripts. It would be even better if you make it clear, ideally through different notations, that the observational uncertainties (across all observation types) are parametrised based on the **background** values of these model variables, as opposed to the (yet-to-be-known) analysis values.
- 8. Lines 198 199: Does it mean that the assimilation window is 1 day long?
- 9. Equation 1 and line 225: Is the summation in the equation taken over the data points of many time-instances? How many data points are there? Please clarify.
- 10. Line 227: "separately for" may be a better choice of words than "across".
- 11. Line 233: Do you mean you compare the SIAL experiments against SIC-only and SIT-only experiments? (See also the Main Comment above.)
- 12. Equation 2: The subscript "SIC, SIT" could be misleading. As things stand, the subscript refers to the reference experiment you compare against, but it is easy to misinterpret the subscript as the quantity over which the RMSE is computed (which is not indicated in the notations of Equation 2). As you demonstrate in the rest of the article, these two aspects should be separate.
- 13. Lines 244 245: In Table 1 you show that SIAL is defined after normalisation by SIC. With that in mind, what drives the finding here that SIAL is well-correlated with SIC?
- 14. Lines 272 273: You say that thick perennial ice in the Central Arctic results in "high SIT RMSE that are not easily reduced by DA due to our prescribed SIT uncertainty". I am not sure how they are related (high observational uncertainty does not necessarily mean high RMSE). What about in experiments that don't assimilate SIT observations?

- 15. Figures 6 and 7: Please move the figures up so that they could be close to the text that discusses about them.
- 16. Line 297: The phrase "the added value of SIAL assimilation over SIC or SIT assimilation" is not clear. Do you mean the comparison between the SIAL-only and SIC-only / SIT-only experiments, or do you mean the impact of additionally assimilating SIAL observations? (See also the Main Comment above.)
- 17. Line 319: The term "category-wise DA" isn't clear; you may consider using "assimilating category-specific observations" instead.
- 18. Equation 4 and lines 335-340: What are "the width of the representative albedo interval" (Δa_n) and the notation $a_{n,right}$? Also, you say that the formulation in Equation 4 "concentrates uncertainty in the low-albedo, thin-ice regime", but I don't find it easy to infer it from the equation. The motivation behind the equation needs to be more clearly explained.
- 19. Lines 350 351: That small n refers to thin ice needs to be more explicitly mentioned somewhere.
- 20. Lines 352 353: "helping explain why aggregate SIAL assimilation generally performs well or comparably in other areas" how?
- 21. Line 388: What does "inflation" mean in this context?
- 22. Lines 466 467: Is there an appendix missing?