This supplement provides additional information on the method and additional figures in support of discussion in the results section. Figures include the auto-correlation of index, horizontal circulation of IG and MRG waves at 150 and 925 hPa, total signal from the regression, the evolution of IG waves and MRG waves at 925 hPa, and the evolution of divergence associated with different waves related to the SWNP convection.

Equatorial wave filtering on the sphere

The wave filtering known as the MODES software is based on the normal-mode decomposition (NMD). It performs the projection of global circulation to the the Rossby, inertia-gravity (IG), Kelvin and mixed Rossby-gravity (MRG) eigensolutions of the linearized global primitive equations. As a complete decomposition, it enables the quantification of atmospheric variability associated with these waves, and analysis of variability in barotropic and baroclinic quasi-geostrophic Rossby waves at various zonal wavenumbers in midlatitudes and the equatorial waves for different vertical structures. The basic equations for linearized motions and their eigensolutions have been derived many times in literature and the solutions for the terrain-following (σ) coordinate by Kasahara and Puri (1981).

The NMD projects the 3D data on the eigensolutions of the linearized adiabatic equations for oscillations in the zonal wind (u'), meridional wind (v') and geopotential height (h'), superimposed on a basic state of rest with temperature T_0. Eigensolutions are derived by the method of the separation of the variables, where the vector of variables $[u',v',h']^{\mathrm{T}}$ as functions of the longitude (λ), latitude (φ), vertical level (σ) and time t is represented as the product of 2D motions (their vector denoted $[u,v,h]$ and the vertical structure functions. The set of three equations for the horizontal oscillations is identical in form with the global shallow water equations having the water depth D (Hough, 1898; Swarztrauber and Kasahara, 1985; Žagar and Tribbia, 2020).

As the non-dimensional equations are linear with respect to time, their solution $[\tilde{u}_m,\tilde{v}_m,\tilde{h}_m]^{\mathrm{T}}$ can be expressed in terms of harmonics in time as

$$\mathbf{W}_m(\lambda,\varphi,\tilde{t}) = [\tilde{u}_m,\tilde{v}_m,\tilde{h}_m]^{\mathrm{T}} = \mathbf{H}_n^k(\lambda,\varphi,m)\,e^{-iv_n^k(m)\tilde{t}}. \tag{1}$$

Here the subscript m denotes the $m-$th vertical mode after the vertical projection and the horizontal motions $[u_m,v_m,h_m]^{\mathrm{T}}$ and time t are made non-dimensional as

$$\tilde{u}_m = \frac{u_m}{\sqrt{gD_m}}, \quad \tilde{v}_m = \frac{v_m}{\sqrt{gD_m}}, \quad \tilde{h}_m = \frac{h_m}{D_m}, \quad \tilde{t} = 2\Omega t. \tag{2}$$

The horizontal structure functions $\mathbf{H}_n^k(\lambda,\varphi,m)$, known as the Hough harmonics, depend on the zonal wavenumber k and meridional mode index n for every m and the corresponding dimensionless frequency is denoted $v_n^k(m)$. The separation of the zonal and meridional dependencies and the periodic boundary conditions in the longitudinal direction lead to the following solution for $\mathbf{H}_n^k(m)$ for discrete values of k:

$$\mathbf{H}_n^k(\lambda,\varphi,m) = \Theta_n^k(\varphi,m)e^{ik\lambda}, \tag{3}$$

where the meridional dependence is described by the Hough function vector Θ, $\Theta_n^k(\varphi,m) = \left[U_n^k(\varphi,m), iV_n^k(\varphi,m), Z_n^k(\varphi,m)\right]^{\mathrm{T}}$, with U, V and Z representing the meridionally dependent profiles of the Hough functions for the zonal velocity, meridional velocity and geopotential height, respectively, all characterized by the zonal wavenumber k and meridional mode index n for each m. For every combination of the three indices k, n and m, the dimensionless frequency $v_n^k(m)$ defines three eigensolutions of the horizontal oscillations. One of the solutions is a low-frequency, westward-propagating Rossby wave, also denoted balanced mode. The other two solutions are high-frequency inertial gravity (IG) waves that propagate eastward or westward.

The derivation of solutions in the form (1) applies the following global inner product:

$$\langle \mathbf{W}_p, \mathbf{W}_r \rangle = \frac{1}{2\pi}\int_0^{2\pi}\int_{-1}^{1}\left(\tilde{u}_p^*\tilde{u}_r + \tilde{v}_p^*\tilde{v}_r + \tilde{h}_p^*\tilde{h}_r\right)\mathrm{d}\mu\mathrm{d}\lambda, \tag{4}$$

where $\mu = \sin(\varphi)$ and the asterisk $(*)$ denotes the complex conjugate. Subscript p refers to a particular mode corresponding to a zonal wavenumber k_p, a meridional index n_p and a vertical mode index m_p, while subscript r indicates another mode. The global orthogonality of the Hough functions for each m can be written as

$$\langle \mathbf{H}_n^k, \mathbf{H}_{n'}^{k'} \rangle = \frac{1}{2\pi} \int_0^{2\pi} \int_{-1}^{1} \left(\mathbf{H}_n^k\right)^* \cdot \mathbf{H}_{n'}^{k'} \, \mathrm{d}\mu \, \mathrm{d}\lambda = \delta_{nn'} \, \delta_{kk'} \, . \tag{5}$$

Similarly, the orthogonality of the vertical structure eigenfunctions is written as

$$\int_{\sigma_T}^{1} G_i(\sigma) \, G_j(\sigma) \mathrm{d}\sigma = \delta_{ij} \, , \tag{6}$$

where $\delta_{ij} = 1$ if $i = j$ and zero otherwise, and σ_T is the top level. For details of the derivation of the vertical and horizontal structure functions in the terrain following sigma coordinate presented above, as well as in the pressure coordinates, the reader is referred to various chapters in the book on normal-mode functions (Žagar and Tribbia, 2020), or original papers by Kasahara and Puri (1981), Tanaka (1985), Swarztrauber and Kasahara (1985) and follow on studies (e.g. Žagar et al., 2023).

The NMD procedure in MODES consists of two steps (Žagar et al., 2015). In the first step, the input data vector $\mathbf{X}(\lambda, \varphi, \sigma_j)$ at time step t on j-th σ level is decomposed using a series of M orthogonal vertical structure functions $G_m(j)$ as

$$\mathbf{X}(\lambda, \varphi, \sigma_j) = \sum_{m=1}^{M} G_m(j) \, \mathbf{S}_m \, \mathbf{X}_m(\lambda, \varphi) \quad , \tag{7}$$

where $\mathbf{S}_m$ is a 3×3 diagonal matrix with elements $\sqrt{gD_m}$, $\sqrt{gD_m}$, and D_m for the purpose of normalization (2). The vertical mode index m varies from the external (barotropic) mode, $m = 1$, to the total number of vertical modes M, with $M \leq J$. The number of σ levels with data is J. The result of (7) is the vector $\mathbf{X}_m(\lambda, \varphi) = [\tilde{u}_m, \tilde{v}_m, \tilde{h}_m]^{\mathrm{T}}$ defined above.

In the second step, the dimensionless horizontal motions for each m are represented by a series of the Hough harmonics $\mathbf{H}_n^k$ using the complex Hough expansion coefficients $\chi_n^k(m)$ as

$$\mathbf{X}_m(\lambda, \varphi) = \sum_{n=1}^{R} \sum_{k=-K}^{K} \chi_n^k(m) \, \mathbf{H}_n^k(\lambda, \varphi; m) \, . \tag{8}$$

In (8), the parameter K denotes the maximal number of zonal waves whereas the total number of meridional modes, denoted R, combines all of the Rossby modes, the eastward-propagating IG modes and the westward-propagating IG modes. For each of the three types of eigensolutions, n varies between $n = 1$ and $n = R/3$ i.e. the meridional mode $n - k$ varies from 0 to $R/3 - 1$. The three indices k, n, and m constitute the 3-component modal index $\nu = (k, n, m)$ of the Hough expansion coefficients, χ_ν.

In practice, the computation of $\mathbf{X}_m$ is performed first as

$$\mathbf{X}_m(\lambda, \varphi) = \mathbf{S}_m^{-1} \sum_{j=1}^{J} G_m(j) \, \mathbf{X}(\lambda, \varphi, \sigma_j) \quad , \tag{9}$$

where we use (7) and the orthogonality (6) for discretized vertical structure functions. This is followed by the computation of the Hough expansion coefficients χ_ν as

$$\chi_\nu = \chi_n^k(m) = \frac{1}{2\pi} \int_0^{2\pi} \int_{-1}^{1} \left(\mathbf{H}_n^k\right)^* \cdot \mathbf{X}_m \, d\mu \, d\lambda \, . \tag{10}$$

The zonal expansion is performed by using the fast Fourier transform while the Gaussian quadrature approximates the integration in the meridional direction. Equations (7) and (9) are the vertical transform pair whereas equations (8) and (10) are the horizontal transform pair.

The result of the expansion (9-10) is the time series of the non-dimensional coefficients $\chi_\nu(t)$, such as used in this study. Note that the study used the pressure system version of MODES, which is described in Žagar et al. (2023).

Additional figures

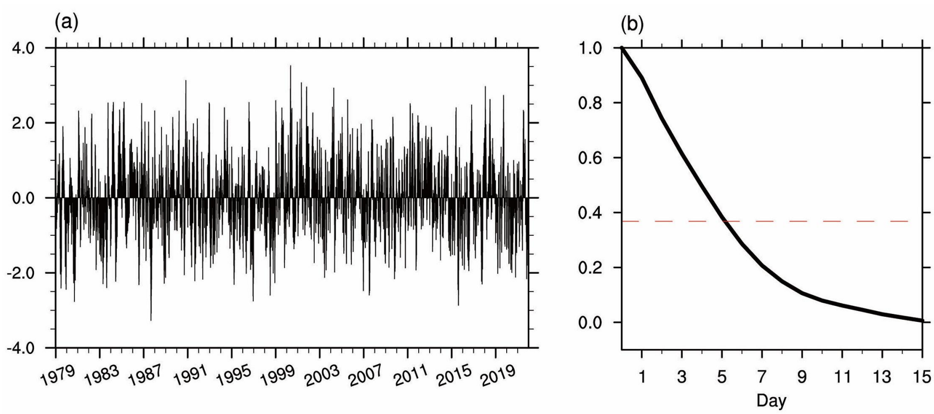

Figure S 1. (a) Time series of the standarized daily SWNPI from June to August during 1979-2021 and (b) the auto-correlations of SWNPI. The red dash line denotes the value of e^{-1}.

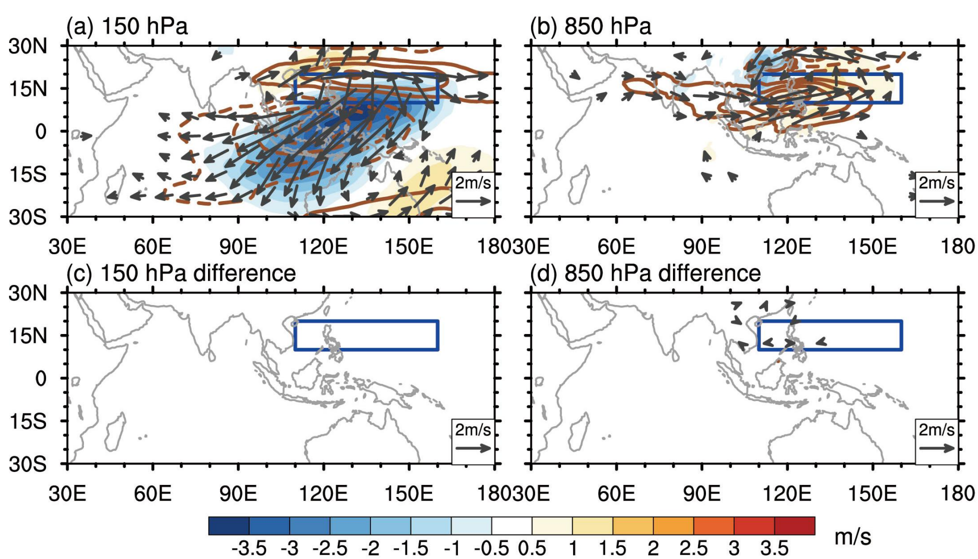

Figure S 2. As in Figure 2 but for (a-b) all modes related to the SWNPI and (c-d) differences between Figures 3a-b and Figures 4a-b. Black vectors indicate the horizontal wind anomalies whose magnitude is larger than 0.5 m/s for all modes and 0.1m/s for differences at 150 hPa and 850 hPa.

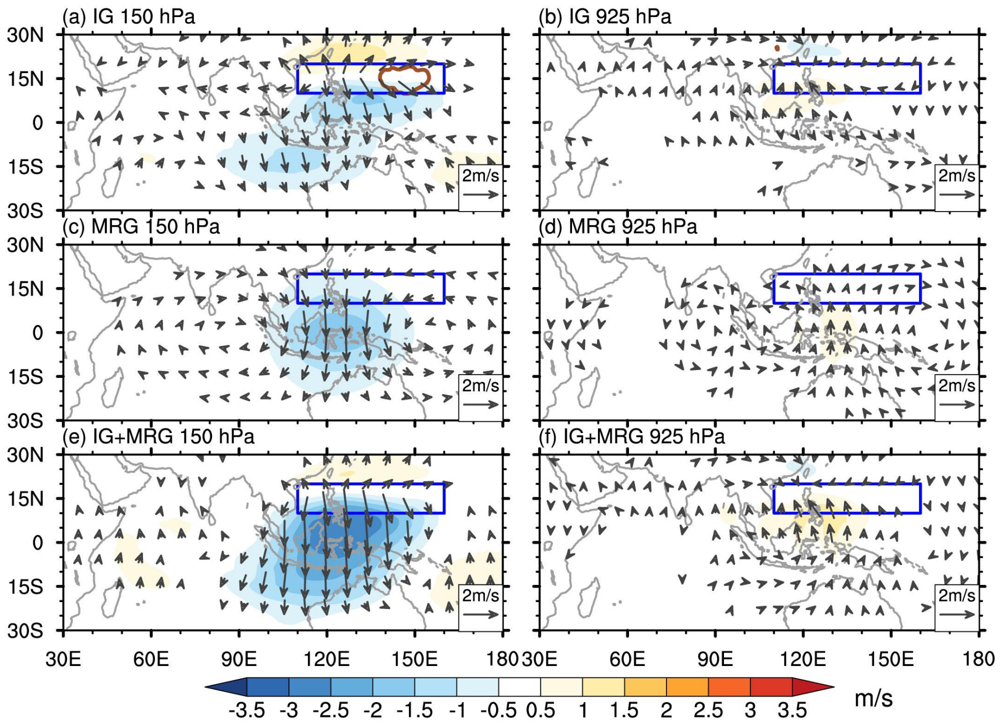

Figure S 3. As in Figure 5, but at 150 hPa and 925 hPa

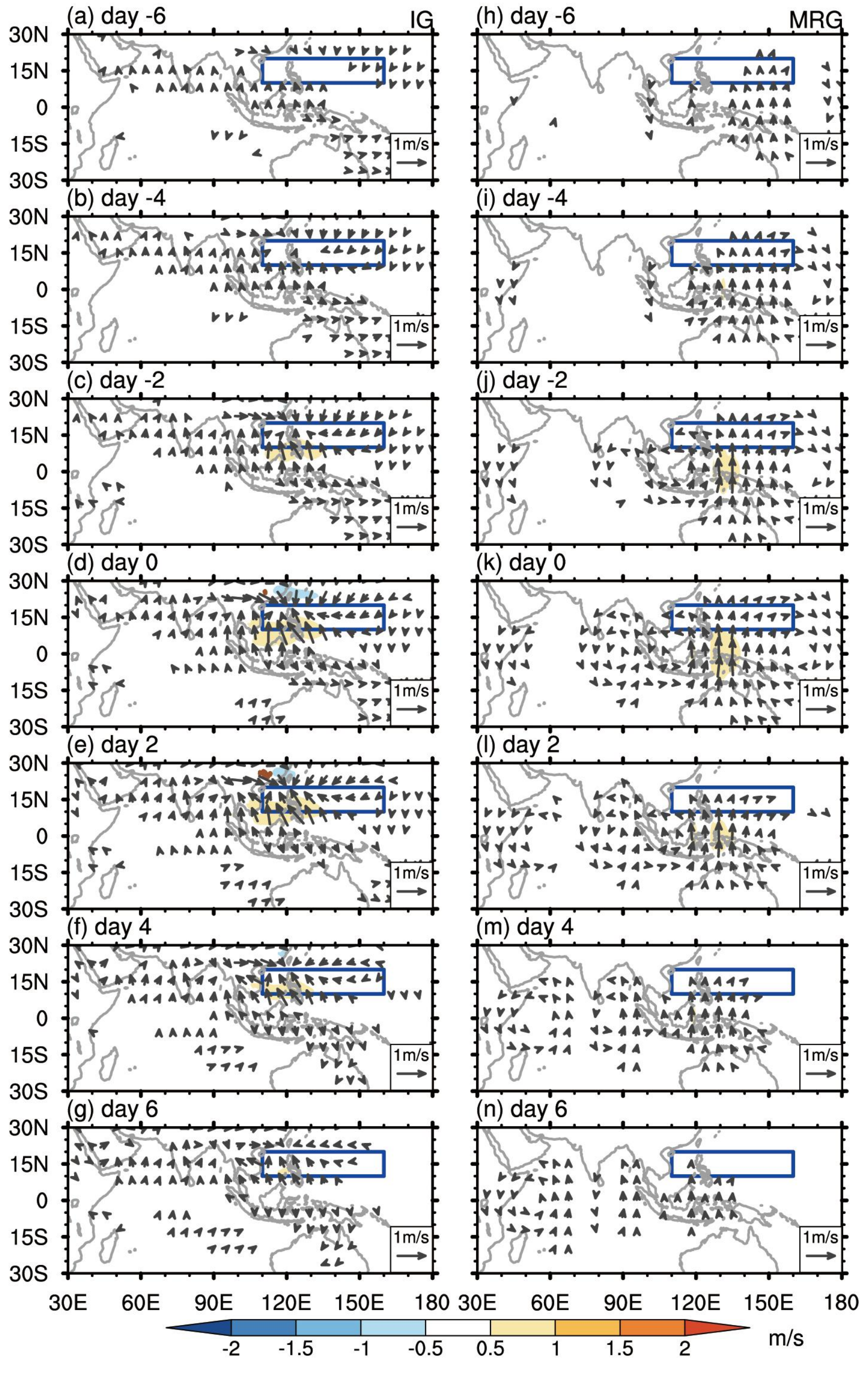

Figure S 4. As in Figure.9 but at 925 hPa. Black vectors indicate the horizontal wind anomalies whose magnitude is larger than 0.1 m/s at 150 hPa and 925 hPa.

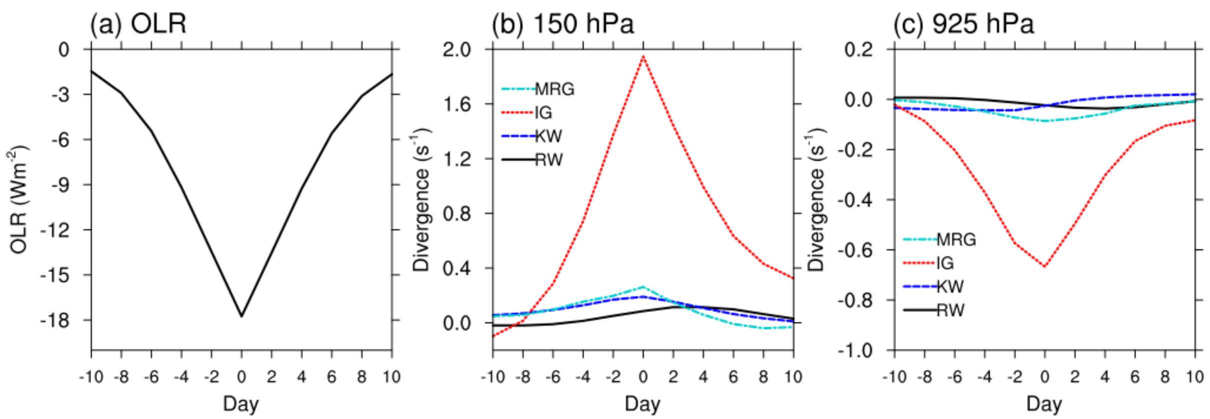

Figure S 5. (a) OLR anomalies regressed onto the SWNPI from day -10 to day 10 averaged over the SWNP region. Divergence (1×10^{-6}) of different waves related to the SWNPI from day -10 to day 10 at (b) 150 hPa and (c) 925 hPa averaged over the SWNP region.

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