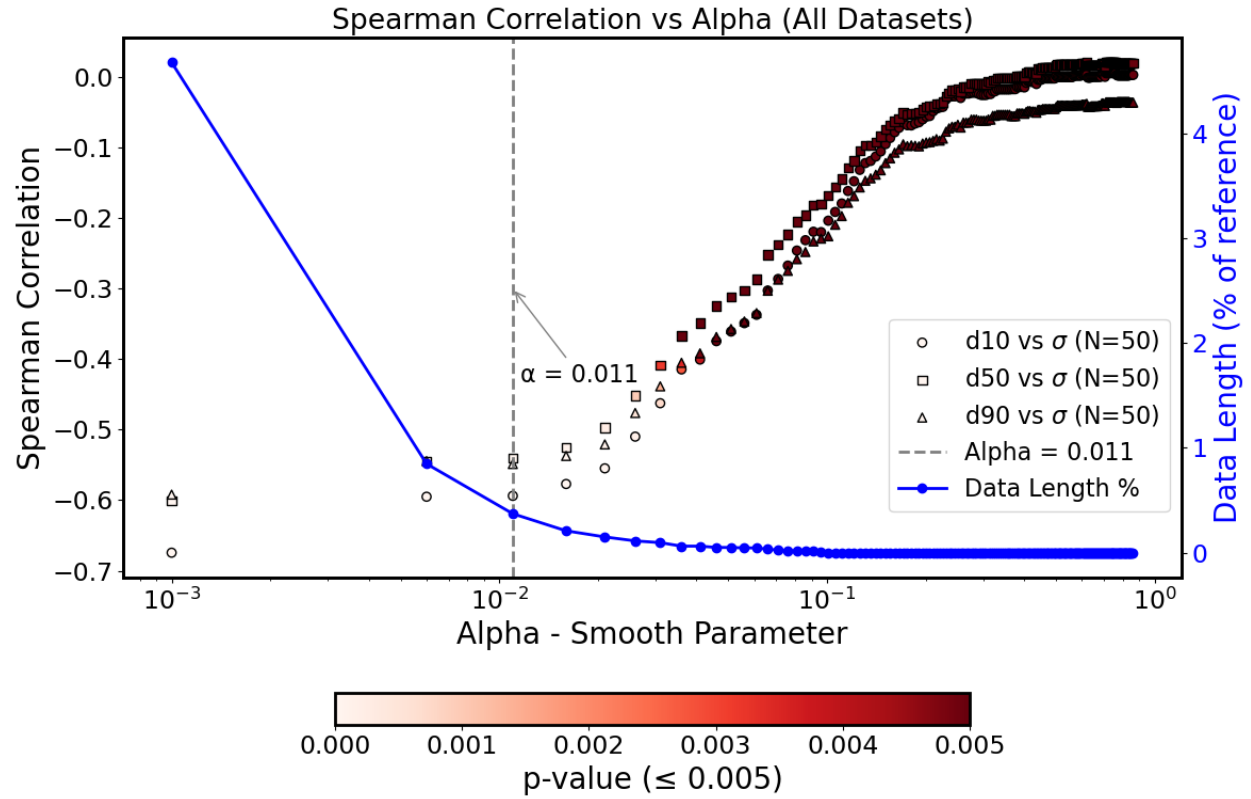


Supplemental Figure 1. This figure is the non-parametric KGE analysis conducted on the reciprocal on the percent finer grainsizes and the $\sigma_{EMI}[c]$ and $\sigma_{EMI}[nc]$. a) The Spearman's r is consistently higher for all percent finer grain size values. b) The bias and c) relative variability are closer to 1, the ideal value for each. Lastly, the KGE shows that the $\sigma_{EMI}[c]$ has a consistently higher model performance across all percent finer values relative to $\sigma_{EMI}[nc]$.



Supplemental Figure 2. The left axis is the Spearman's r between the $\sigma_{EMI}[c]$ and d10, d50, and d90. The right y-axis (blue) is the percent of negative conductivity values produced from the EMI inversion. When the smoothing parameter was set to $\alpha=0.01$, the RMSE for the data misfit for $\sigma_{EMI}[c]$ was reduced by half in most cases and also yielded higher Spearman's r values with dfiner. EMI inversions were repeated 173 times with an initial $\alpha=0.001$ and incrementing the α value by 0.005 to understand how the correlation between $\sigma_{EMI}[c]$ and grain size parameters are influenced by the smoothing. Spearman's r was computed between the inverted conductivity and six percent finer values (d5, d10, d30, d50, d90, and d95) for each value of α . Results of this analysis can be found in the supplemental material (Fig. S2 - S3). At $\alpha=0.03$, Spearman r is not statistically significant ($p\text{-value}<0.005$) between $\sigma_{EMI}[c]$ and the percent finer values. Below $\alpha=0.01$, the inversion seems to fit a larger portion of the noise and produces an increased number of nonphysical values for the inverted conductivity. Therefore, a value of $\alpha=0.01$ balanced a statistically significant relationship with grain size that provided enough smoothing to reduce fitting noise.