

A ~~mathematical framework~~ quantitative methodology for quantifying analysing physical damage over time and recovery dynamics from concurrent and consecutive hazards: forensic insights from Puerto Rico

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Abstract. Space and time play a crucial role in multi-hazard impact assessment. When two or more natural hazards occur at the same location simultaneously or ~~within a short time frame~~ in sequence, the physical integrity of assets and infrastructures can be compromised, and the resulting damage can be greater than ~~the one that~~ generated by individual hazards occurring in isolation. ~~The current literature highlights the lack of quantitative standardised frameworks for~~ Despite widespread conceptual
5 recognition of these interactions, the literature lacks quantitative, standardised methods for systematically analysing multi-hazard ~~impact assessment. This research presents a generalised mathematical framework for quantitatively assessing impacts.~~
This study presents a quantitative methodology for evaluating multi-hazard physical damage ~~on exposed assets, such as~~
~~buildings or critical infrastructures, over time. The proposed framework to the built environment, translating qualitative impact~~
dynamics into a transparent and reproducible analytical framework, implemented as modular Python code. The approach
10 covers both concurrent and consecutive hazards ,by modelling: (i) the increased damage resulting from the combined impact of two or more concurrent hazards that overlap in space and time, and (ii) the effects of cumulative damage on asset vulnerability and the recovery dynamics in case of consecutive hazards that overlap in space. ~~The framework~~ Using the
Python implementation, a model behaviour analysis is conducted to systematically explore how variations in inter-event time
intervals, vulnerability interactions, and recovery trajectories affect cumulative physical damage. The methodology is ap-
15 plied to a ~~real-world case study~~ real multi-hazard sequence in Puerto Rico, including the concurrent wind and flood impacts ~~generated by the passage of Hurricane Maria~~ ,as well as the consecutive impacts caused by the subsequent seismic sequence of 2019-2020. Based on simulations performed on a building portfolio, we found that neglecting residual damages caused by the

hurricane when assessing the impacts of the subsequent earthquake would lead to a significant underestimation of the overall damage experienced by the assets and the consecutive seismic impacts of the 2019–2020 earthquake sequence. The forensic reconstruction highlights that ignoring residual hurricane damage would significantly underestimate the subsequent earthquake losses, and that damage accumulation is path-dependent, strongly influenced by event timing and recovery processes. By providing a generalised formalisation to perform quantitative, transparent, and reproducible quantitative structure, this study offers a tool for forensic analysis of past multi-hazard impact assessment, able to account for amplification phenomena and recovery dynamics, the framework can offer scientists and decision-makers a comprehensive and deeper understanding of the impacts caused by compound and consecutive events, events, systematic exploration of damage drivers, and scenario-based assessment of alternative hazard and recovery conditions, supporting both post-disaster learning and planning-oriented applications.

1 Introduction

In recent years, increasing attention from the scientific community and international frameworks has been drawn to multi- (hazard)-risk assessment and management (Ward et al., 2022). There has been a growing acknowledgement that natural hazard events may occur simultaneously, in cascade, or cumulatively over time, and can interplay with societal factors such as exposure and vulnerability, to generate complex multi-risk disaster scenarios (de Ruiter et al., 2020; de Ruiter and van Loon, 2022). These compound events, like such as heavy rainfall and wind extremes extreme wind from the same convective storm (e.g. ,Tilloy et al. (2022)), cascading events like such as earthquakes triggering tsunamis (e.g. ,Mimura et al. (2011)), or even consecutive independent events (e.g. ,NASA (2021))NASA (2021)), can generate an impact that is different from that of the individual hazards occurring in isolation (Gill and Malamud, 2014). Moreover, in an increasingly interconnected world, natural hazards' impacts cascade across geographical and sectoral boundaries, leading to considerable challenges for disaster risk managers, including emergency management agencies, asset managers, and operators of critical infrastructures and lifelines (Hochrainer-Stigler et al., 2023; Pasino et al., 2021). Although more complex, approaches that account for multiple hazards and their interconnections better capture the real risk many areas of the world are exposed to and can support the definition of effective disaster risk reduction (Kappes et al., 2012).

In contrast to single-hazard risks, the multi-risk assessment (hazard) poses a series of challenges at each step of the risk or impact assessment, from the hazard modelling to the vulnerability characterisation, until the final risk or impact assessment (Kappes et al., 2012; De Angeli et al., 2022). Although significant improvements have been made in developing approaches that can identify and quantify interrelationships between hazards (Tilloy et al., 2019), as well as understanding their spatial and temporal overlap dynamics (Claassen et al., 2023), the quantification of impacts arising from multiple hazards and the development of generalised models for assessing multi-hazard physical damage remain a partially unexplored domain (Gentile et al., 2022). When modelling physical impacts caused by multiple concurrent or consecutive hazards, several aspects can influence the magnitude and nature of the final damage, including the order in which the hazards affect the exposed assets, the time window between the hazards (if any), and the recovery rate of the impacted asset, among others (De Angeli et al., 2022).

50 This paper presents a generalised mathematical framework for quantitatively assessing Several conceptual frameworks offer qualitative descriptions of multi-hazard ~~direct physical damage on exposed assets, such as buildings or critical infrastructures, over time. Current quantitative multi-hazard damage models developed to assess direct physical damages to assets or infrastructures are available only for specific combinations of asset types (e.g., buildings or bridges, that are the most investigated) and hazards. A comprehensive review of these models is reported by Gentile et al. (2022). From this systematic review, it emerges~~

55 that impact interactions, as well as schematic representations of their temporal dynamics in the majority of quantitative multi (dual)-hazard physical impact models available are seismic-related, i.e. are models where at least one of the two considered hazards is an earthquake. To overcome this specificity, the framework presented in this manuscript is general enough to be applied to a wide range of natural hazards, including geological, hydrological, and climatological ones, and to be integrated with existing multi-hazard damage models.

60 The framework aims at formalising and quantifying the full range of aspects underlined by De Angeli et al. (2022) as crucial for multi-hazard damage assessment, offering a comprehensive time-dependent mathematical description of: (i) *concurrent (or compound) impacts*, where the impacts result from two hazard events that either completely or partially overlap in space and time, with the second event occurring before the response phase of the first one; (ii) *consecutive impacts*, wherein the impacts are generated by spatially overlapping hazards which occur close enough in time that the assets do not have sufficient time to

65 completely recover before the onset of the second hazard; (iii) *independent impacts*, which are generated by consecutive hazards that occur so distantly in time that the assets have adequate time to restore their original conditions before the second hazard ~~occurs~~ form of graphs and conceptual schemes, such as those proposed by de Ruiter et al. (2020), De Angeli et al. (2022), and Wenzel et al. (2026). These frameworks also identify key factors that can influence the magnitude of impacts, such as spatial overlap, the time window between successive events, and the recovery dynamics of the impacted elements, among others.

70 Specifically, in the case of consecutive events, the impacts of an earlier event are likely to change the vulnerability at the time of the next event (de Ruiter and van Loon, 2022). In addition, recovery of vulnerability to pre-disaster conditions after the event ended plays a key role. Societies and systems that recover quickly from disasters become less vulnerable to the next event than societies that follow a slower recovery path (Di Baldassarre et al., 2018). ~~Despite its importance in a deeper understanding of complex risk dynamics, the~~

75 Nevertheless, there remains a lack of studies that attempt to translate these considerations into a quantitative formulation and move towards their quantification. The majority of available multi-hazard physical impact quantitative models are seismic-related models, i.e., models where at least one of the two considered hazards is an earthquake, and focus on specific critical infrastructures, such as bridges (Gentile et al., 2022). Furthermore, modelling of the recovery process remains a significant challenge even from a single-hazard perspective (Mohammadi et al., 2024). Most of the damage models currently used do not address the temporal

80 dimension of post-disaster loss and recovery or treat it in a simplistic fashion (Miles and Chang, 2006; Sarker and Lester, 2019). Very little research has been conducted on how recovery proceeds over time (e.g., Cimellaro et al. (2010); Loos et al. (2023)), or on the social and economic factors ~~affecting that affect~~ the recovery process (e.g., Miles et al. (2019); Koliou et al. (2020); Hariri-Ardebili et al. (2022)). ~~Without aiming to solve this complex issue, the generalised mathematical framework presented~~

in this manuscript has been conceptualised to incorporate recovery dynamics into the quantitative assessment of damage over time. This approach provides a comprehensive understanding of post-disaster loss dynamics.

To encourage widespread practical application, the framework has been implemented as ~~Consequently, a deeper understanding of how these factors affect cumulative multi-hazard impacts is still lacking.~~

This paper aims to bridge this gap by providing a structured quantitative methodology that translates conceptual representations of multi-hazard damage dynamics into an explicit and reproducible analytical structure. To this end, compound and consecutive impact dynamics described graphically and schematically by de Ruiter et al. (2020) and De Angeli et al. (2022) are reformulated into a compact mathematical expression and implemented as a modular Python code. This formulation enables explicit representation of inter-event time intervals, state-dependent vulnerability interactions, ~~allowing users to customise multi-hazard parameters, vulnerability functions, and recovery dynamics, and perform multi-hazard damage assessments.~~ trajectories within a unified structure. Using this implementation, a model behaviour analysis is conducted to systematically investigate how these key parameters influence compound and cumulative physical damage in a multi-hazard context. Although such mechanisms are widely acknowledged qualitatively in the literature, they are rarely modelled in quantitative terms within a single coherent framework. The primary contribution of this work, therefore, lies in offering a reproducible analytical template that supports systematic exploration of multi-hazard damage interactions and facilitates structured post-disaster interpretation and learning. Beyond reconstructing past events, the proposed methodology can also support anticipatory and scenario-based analyses, enabling exploration of how cumulative multi-hazard damage may evolve under alternative assumptions, such as different recovery pathways or inter-event intervals.

This paper is organised as follows: first, we present the conceptualisation of our mathematical framework (Sect. ??), starting with a single-hazard setting (Sect. ??) and then progressing to multi-hazards (Sect. ??). We focus on the quantification of concurrent (Sect. 2.1), consecutive (Sect. 2.2) and independent (Sect. ??) impacts. Next, we illustrate the application of this ~~The proposed approach is then applied to a real multi-hazard quantitative impact framework in sequence in Puerto Rico from a forensic perspective. In this application, the multi-hazard damage model is used to reconstruct and interpret the evolution of direct physical damage following Hurricane Maria and the subsequent seismic sequence, illustrating how residual vulnerability and incomplete recovery shape cumulative impacts. The case study, therefore, serves as a demonstration of how the proposed quantitative structure can support post-disaster analysis and learning in complex multi-hazard contexts.~~

This paper is organised as follows. Section 2 presents the structured quantitative methodology for analysing multi-hazard damage dynamics, beginning with the mathematical formulation of the damage model and followed by the analysis of model behaviour. Section 3 illustrates the forensic application of the approach to a real-world case study in Puerto Rico, ~~covering addressing~~ the concurrent wind and flood impacts ~~from of~~ Hurricane Maria and the consecutive impacts ~~from the 2019-2020 seismic sequence (Sect. 3). Finally, we discuss of the 2019-2020 seismic sequence. Finally, Section 4 discusses~~ limitations and future developments ~~(Sect. 4) and present, and Section 5 summarises~~ the main conclusions ~~(Sect. 5).~~

2 Mathematical Framework for Damage Estimation From qualitative frameworks to quantitative multi-hazard dynamics

We conceptualise a mathematical framework to quantitatively estimate the impact over time t on an asset or infrastructure j caused by one or more natural hazards. More specifically, we focus on the quantification of the direct physical damage $d_j(t)$, which can be also seen as an indirect measure of the physical integrity $y_j(t)$. Based on the works by de Ruiter et al. (2020) and De Angeli et al. (2022), two main multi-hazard impact dynamics can be identified: (i) concurrent impacts, generated by hazards which either completely or partially overlap in space and time; (ii) consecutive impacts, generated when spatially overlapping hazards occur close enough in time that the assets do not have sufficient time to completely recover before the onset of the second hazard. These two dynamics are qualitatively depicted in Fig. 1, using the physical integrity as an impact variable. The physical integrity $y_j(t)$ of the asset, existing an asset j is related to its physical damage $d_j(t)$ by the relationship:

$$y_j(t) = 1 - d_j(t), \quad y_j \in [0, 1] \quad (1)$$

$y_j(t) = 1$ indicates no impact on the element's physical integrity and $y_j(t) = 0$ represents complete destruction. The concept of physical integrity was introduced by Minciardi et al. (2006) as a proxy to quantify the asset's functionality. However, our framework does not cover the assessment of asset functionality, as discussed in previous studies (e.g. Disse et al. (2020); Disse et al. (2019)), or the evaluation of systemic impacts and system resilience (e.g., Cimellaro et al. (2008); Cimellaro et al. (2009)). Quantifying asset functionality, which refers to the inherent ability of an asset to perform its designated task (Ellingwood et al., 2016), requires evaluating various factors such as social, physical, economic and accessibility aspects. It also requires estimating indirect impacts resulting from the loss of functionality in other elements of the territorial system to which the asset belongs.

Accordingly, $d_j(t)$ measures the relative (or percentage) damage of the asset. The absolute damage $D_j(t)$ can be obtained by multiplying the relative damage $d_j(t)$ times the exposure value of the asset $E_j(t)$, as outlined in Eq. (??):

$$D_j(t) = d_j(t) \cdot E_j(t)$$

The exposure value has to be considered as variable over time due to various factors unrelated to hazards, such as restructuring, maintenance, or improvement interventions, as well as due to lack of maintenance or change of use. Moreover, when considering the effects of a hazard, exposure may diminish due to the damages incurred by the hazardous events, it can dynamically change during the whole recovery phase, and can even reach a higher level following the completion of the recovery process. Therefore, it is crucial to emphasise that to perform a dynamic damage assessment, the focus must also be on evaluating how exposure can dynamically change over time.

2.1 Single-Hazard Scenario Modelling

To facilitate the understanding of some of the key elements of the proposed framework, we start with the analysis of a single-hazard scenario. Figure ?? illustrates the variation in physical integrity over time $y_j(t)$ for a general asset j that is impacted by a single natural hazard event, for example, a flood, an earthquake or a hurricane.

We start observing the asset at $t = t_0$. Physical integrity is initially assumed to be set to one, that is, $y_j(t_0) = 1$. At a specific time t_S , the asset starts to be impacted by the hazard, resulting in a loss of physical integrity Δy_j dependent on the intensity of the hazard event, as well as the vulnerability of the asset. The hazard event ends at t_F , with a total duration $\Delta t_{EV} = t_F - t_S$.

For simplicity and to facilitate the graphical representation, it is assumed that the loss of physical integrity occurs integrally at the beginning of the event. However, in many real cases the maximum loss is reached only when the hazard reaches its peak (that is, the maximum of its intensity) and sometimes, as in the case of earthquakes, physical damage can manifest even after the end of the event (e.g., the collapse of buildings hours or days after the main shock) (Raghunandan et al., 2015). The temporal evolution of $y_j(t)$ during the event phase could be relevant only in the case of concurrent events that have a non-null interarrival shorter than Δt_{EV} . In the scheme adopted for concurrent events (see Fig.1), such interarrival is always considered negligible. Furthermore, the duration of the event is assumed to be comparable to or shorter than the response phase. This implies that we primarily focus on so-called sudden-onset hazards, for which the event phase typically lasts no longer than a few days. In contrast, in the case of events such as drought or subsidence, the event phase can last for years, and the distinction between the event, response, and recovery phases is not always easily distinguished (Terzi et al., 2022).

Variation of the physical integrity over time for a general asset j which is impacted by a single natural hazard event.

When the event ends, the response phase starts. It consists of actions that are aimed at saving lives, containing losses, and restoring essential services. Response actions may include activating the emergency operations centre, evacuating populations, setting up temporary shelters, providing medical care, and searching and rescuing. The response phase persists until time t_{RES1} and is represented in Fig.?? by a horizontal blue line. For reasons of simplification, it is assumed that the value of physical integrity $y_j(t)$ remains constant throughout the response phase. Although it is conceivable that in real-world scenarios slight fluctuations might occur, these variations can be deemed negligible compared to the significant changes observed once the asset begins reconstruction during the subsequent recovery phase.

Following the response, the recovery phase begins with the aim of restoring the pre-event level of physical integrity. It is widely acknowledged in the scientific community that the recovery phase begins after the threat to human life has subsided. Nevertheless, the boundaries between the response and recovery phases are blurred, and making a clear distinction is challenging in the context of real disaster development (Mohammadi et al., 2024). The goal of the recovery phase is to bring the affected area back to some degree of normalcy, including restoring basic services and repairing physical, social, and economic damages. During the recovery phase, the reconstruction activities for the asset j begin and continue until time t_{REC} . Estimating the duration and dynamics of recovery processes is critical. In Fig. ??, the recovery pattern is represented by a tilted green line. According to Cimellaro et al. (2010), this assumption is valid in contexts where the organisational and response capacity of the system is unknown. However, the recovery function of an asset, $R_j(t)$, can follow various patterns, such as exponential, trigonometric, or linear, with different rates depending on technological, logistic, and economic factors, among others. A trigonometric curve is more suitable for contexts where the initial recovery phase is hampered by a lack of materials and resources needed for reconstruction. If appropriate political measures are swiftly implemented and updated civil protection plans are in place, the response phase will be shorter. This will lead to a faster start to the long-term recovery phase, which

will initially progress rapidly before slowing down as conditions improve. This scenario is best represented by an exponential recovery function Cimellaro et al. (2010).

Key concepts and their definitions as used in this paper

185 ~~Element-Description Physical integrity $y_j(t)$ (Fig.1) Complementary to the relative damage $d_j(t)$, it measures how much~~
~~an asset remains integer in its physical components. It can vary continuously from 0 to 1, with 1 indicating no impact on the~~
~~element's physical integrity and 0 representing destruction. Event Phase Δt_{EV} (Fig.1) Time window between the beginning~~
~~and the end of a hazardous event. For sudden events, such as earthquakes or flash floods, it can span from seconds to minutes.~~
~~For long-onset events, such as riverine floods or droughts, it can span from days to years. Response Phase Δt_{RES} (Fig.1) Phase~~
~~that immediately follows the conclusion of the event. It consists of actions that are aimed at saving lives, containing losses, and~~
190 ~~restoring essential services. Response actions may include activating the emergency operations centre, evacuating populations,~~
~~setting up temporary shelters, providing medical care, and searching and rescuing. Recovery Phase Δt_{REC} (Fig.1) Phase that~~
~~follows the response, starting when the threat to human life has subsided. It aims to bring the affected area or asset back to~~
~~some degree of normalcy, including the restoration of basic services and the repair of physical, social, and economic damages.~~
~~Concurrent Impacts (Fig.2, panel a) Impacts resulting from two hazard events that either completely or partially overlap in~~
195 ~~space and time. The second event occurs simultaneously with the first one or during its response phase. The Concurrent~~
~~impacts result from two hazard events that overlap in space and time, either completely or partially. The second event occurs~~
~~simultaneously with the first or during its response phase, so that recovery has not yet begun when the second event occurs.~~
~~Real-world examples include the 2018 events in Indonesia, where a 7.5 magnitude earthquake in Sulawesi triggered a tsunami,~~
~~causing extensive coastal damage (Sabah and Sil, 2023; Naik et al., 2023; Opabola et al., 2023). Another example, which does~~
200 ~~not involve causal relationships between events, is the 2013 earthquake in the Philippines, followed by a typhoon before~~
~~recovery from the earthquake had begun (Lagmay and Eco, 2014; Pormon et al., 2023; Lagmay et al., 2015). In such scenarios,~~
~~the impacts of both events combine and affect the asset. The primary challenge is to estimate quantitatively assess the overall~~
~~amplification of physical damage due to caused by the superimposition of the two loads on the same assets. Consecutive Impacts~~
~~(Fig.2, panel b) Impacts generated when spatially overlapping hazards multiple loads.~~
205 ~~Consecutive impacts occur when hazard events that spatially overlap occur close enough in time that the assets do not~~
~~have sufficient time to completely recover before the onset of the second hazard. The main challenge Real-world examples~~
~~include the sequence of earthquakes and floods in Nepal in 2015. An earthquake with a magnitude of 8.0 struck Nepal in~~
~~April 2015, causing significant structural and non-structural damage. The region was then hit by heavy rains and floods~~
~~in 2017 (Gautam and Dong, 2018). Recovery and reconstruction documents (He et al., 2018; Liu et al., 2021) show that the~~
210 ~~floods affected structures and areas still recovering from the previous earthquake. Another example is Hurricane Irma and~~
~~Maria, which devastated Puerto Rico in 2017, followed by an earthquake in the same area two years later. This case is analyzed~~
~~in detail in Sect. 3. The main challenge with consecutive impacts is to estimate residual damage according to the recovery~~
~~dynamic of the asset and understand how this residual damage can influence the based on the asset's recovery dynamics and to~~
~~understand how the residual damage influences vulnerability to the second hazard. Independent Impacts (Fig.2, panel c) The~~

215 two impacts are independent of each other since they were generated by hazards that occurred so distantly in time that the assets had adequate time to restore their physical integrity before the second hazard occurred. subsequent hazard.

Moreover, at the end of the reconstruction, the assets may not be restored to the same conditions as before the event. Indeed, the goal is usually to rebuild them in a way that enhances their resilience and reduces vulnerability, as emphasised by the Build-Back-Better theory (see, e.g., Kennedy et al. (2008); Saya et al. (2017); Hallegatte et al. (2018)). This improvement can

220 be reflected in a change in both the exposure, in terms of the total economic value of the assets, and the asset's vulnerability. To enable the quantitative investigation of compound and consecutive impacts, the dynamics graphically described in Fig. 1 are translated into a compact and easily implementable mathematical formulation, as presented in Appendix A. This generalised piecewise defined function allows the estimation of relative damage over time to an asset or infrastructure j exposed to n natural hazards. The behaviour of the function is controlled by the parameters Θ and μ , defined by Eqs. (A5) and (A6) in

225 Appendix A. Depending on the values assumed by the parameters, the formulation represents different multi-hazard impact dynamics:

To summarise, given a generic hazard event able to generate a loss of physical integrity on a generic asset j , the following three main phases can be identified:-

- the event phase $\Delta t_{EV} = t_F - t_S$
- 230 - the response phase $\Delta t_{RES} = t_{RES} - t_F$
- the recovery phase $\Delta t_{REC} = t_{REC} - t_{RES}$

A concise definition of each of these three phases, as well as of physical integrity, is provided in Table ??.

2.1 Multi-Hazard Scenario Modelling

Moving from a single to a multi-hazard scenario and building upon the impact mechanisms classification suggested by

235 De Angeli et al. (2022), the framework enables the analysis of three distinct types of impact interactions, as given in Fig. ??:- (i) If $\Theta = 0$, the function models concurrent impacts, generated by hazards which either completely or partially i.e., damage caused by hazards that overlap in space and time; (ii), either fully or partially.

- If $\Theta = 1$ and $\mu = 0$, it models consecutive impacts, generated when where spatially overlapping hazards occur close enough in time that the assets do not have sufficient time to completely asset has insufficient time to fully recover before
- 240 the onset of the second hazard; (iii) independent impacts, generated by hazards that occur so distantly in time that the assets have adequate time to restore their physical integrity before the second hazard occurs. second hazard occurs.

Figure ?? is a flowchart in which ovals represent the initial and final states, and diamonds represent conditions that require a Boolean decision (that is, 'Yes' or 'No'). The process begins with a specific temporal evolution scenario involving two hazard events, labelled 1 and 2, affecting one or more assets of interest (depicted in the grey oval at the top of Fig. ??).

245 Depending on whether certain temporal overlap conditions are verified, If $\Theta = 1$ and $\mu = 1$, the three different cases of impact

interactions shown in the ovals can be identified. We will now detail the two Boolean decision points (diamonds) and discuss the results that result from confirming ('Yes') or rejecting ('No') the statements associated with each of these decision points. The two Boolean decision points are labelled A and B. Impacts are treated as independent, allowing the function to perform a series of single-hazard damage assessments without incorporating any multi-hazard interactions. The detailed formulation of the two multi-hazard impact dynamics, concurrent and consecutive, is presented in Sect. 2.1 and Sect. 2.2 respectively. The formulation for independent impacts is provided in Appendix A1.

Identification of multi-hazard impact interaction typologies resulting from the temporal evolution of two different hazard events. Diamonds A ($t_{S_2} \leq t_{RES_1}$) and B ($t_{S_2} < t_{REC_1}$) represent Boolean decisions ("Yes" or "No").

The first condition labelled A checks whether the hazard event 2 begins before the end of the event response phase 1', that is, if $t_{S_2} \leq t_{RES_1}$. This condition is mathematically represented by the Heaviside step function $\mathcal{H}$, as shown in Eq. (A5).

$$\Theta(t_{S_2}, t_{RES_1}) = \mathcal{H}(t_{S_2} - t_{RES_1})$$

If A = "Yes", i.e., $\Theta = 0$, hazard event 2 begins before the end of event 1's response phase, and The generalised formulation is implemented in Python to enable reproducible and modular multi-hazard damage assessments. The implementation follows directly the mathematical structure described in Appendix A, where event classification (concurrent, consecutive, independent) is determined dynamically through the parameters Θ and μ , and damage evolution is computed at the asset level across discrete time steps. Within the tool, recovery functions $R_{j,i}(t)$ are explicitly defined as time-dependent functions (linear, exponential, logistic, or user-defined), allowing different durations and restoration trajectories depending on asset type and contextual assumptions. After each hazard event, the state of the asset is updated by storing residual damage, which subsequently modifies the vulnerability function applied to the impacts are considered *concurrent*. In this scenario, a series of impacts simultaneously affect the considered asset, regardless of the relationship between the hazards that generated them. This first type of impact interaction is thoroughly described in Sect. 2.1 and visually represented by Fig. 1, panel (a).

If A = "No", i.e., $\Theta = 1$, hazard event 2 begins after the end of event 1's response phase. In this case, two different scenarios can occur depending on the outcome of the second Boolean decision point, labelled B.

The second condition B additionally checks whether hazard event 2 begins before the end of event 1's recovery phase, i.e., if $t_{S_2} \leq t_{REC_1}$. This condition is mathematically represented again by the Heaviside step functions, as shown in Eq. (A6).

$$\mu(\Theta, t_{S_2}, t_{REC_1}) = \mathcal{H}(t_{S_2} - t_{REC_1}) - \text{if } \Theta = 1$$

If A = "No", i.e., $\Theta = 1$, and B = "Yes", i.e., $\mu = 0$, hazard event 2 begins after the end of event 1's response phase but before the end of event 1's recovery phase. In this case, the impacts are considered *consecutive*. In this scenario, the asset has not had sufficient time to complete the recovery process from the first event, and when the second event occurs, the asset still possesses a level of integrity lower than one, rendering it more vulnerable to the impacts of the subsequent event. This second type of impact interaction is investigated in detail in Sect. 2.2 and visually represented by Fig. 1, panel (b).

If $A = \text{"No"}$, i.e., $\Theta = 1$, and $B = \text{"No"}$, i.e., $\mu = 1$, hazard event 2 begins after the end of event 1's recovery phase and the impacts are considered *independent*. This last type of impact interaction refers to the fortunate scenario in which events occur so far apart in time or the recovery has been so fast that, by the time the second event starts, the asset has had sufficient time to fully recover from the previous damage. In this context, the second impact causes damage when the physical integrity of the asset is fully restored from the first event in temporal order, next event in the case of consecutive impacts. This ensures consistency between recovery progression and state-dependent fragility adaptation. The framework is modular in structure: hazard inputs, exposure data, fragility curves, and its current value is again equal to $y_j(t_0)$. The scenario is detailed recovery assumptions are treated as interchangeable components. This design allows users to test alternative recovery trajectories, introduce modified fragility parameters, or simulate different event sequences without altering the core computational logic. While the present study applies the implementation for model behaviour analysis and forensic validation in Puerto Rico, the structure supports broader scenario-based applications and integration with external hazard workflows. Full implementation details and data access are provided in Sect.?? and visually represented by Fig. 1, panel (e). Code and data availability.

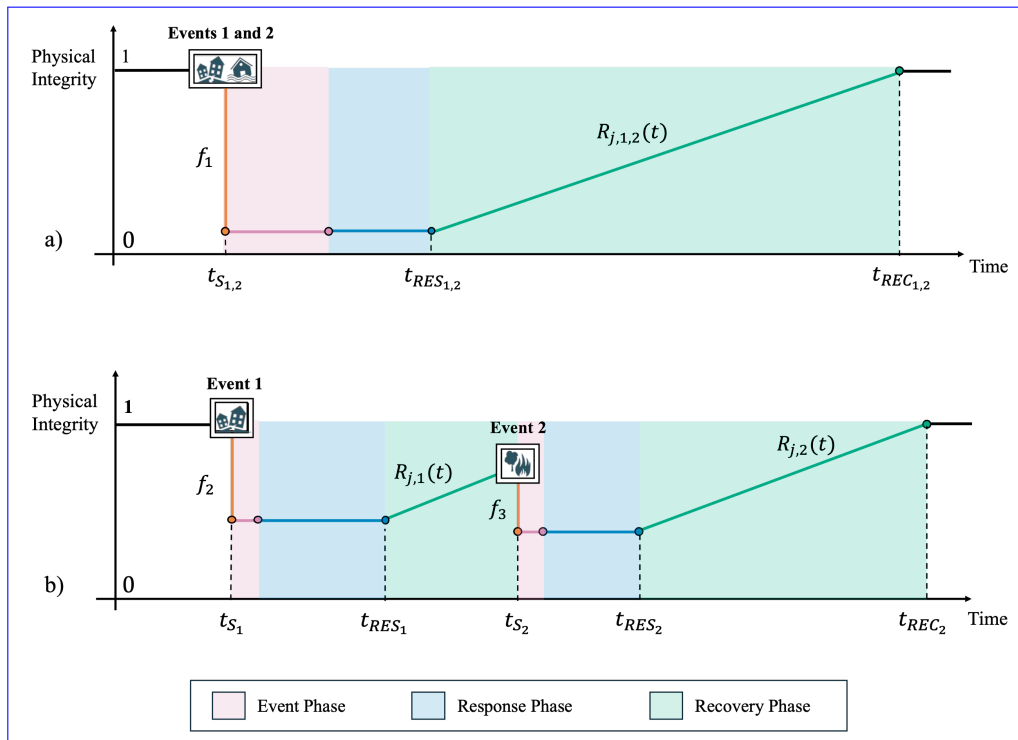


Figure 1. Variation of physical integrity over time for ~~the different interactions between impacts considered by the general framework~~concurrent and consecutive multi-hazard impact dynamics. **Panel a:** The impacts are concurrent. The second event in temporal order affects the asset during the response phase of the previous one. The second impact and the preceding one are considered simultaneous. The loss of physical integrity is the amplified combination of the two stressors. Details in Sect. 2.1. **Panel b:** The impacts are consecutive. The second event in temporal order affects the asset during the long-term recovery phase of the previous one. Details in Sect. 2.2. ~~**Panel c:** The impacts are independent of each other. The second event in temporal order affects the asset when it has already completed the recovery phase of the previous one. Details in Sect. ??.~~

2.0.1 Concurrent Impacts

290 2.1 Concurrent impact dynamics

Concurrent impacts result from two hazard events that overlap in space and time either completely or partially. The second event occurs simultaneously with the first or during its response phase. Real-world examples include the 2018 events in Indonesia, where a 7.5 magnitude earthquake in Sulawesi triggered a tsunami, causing extensive coastal damage (Sabah and Sil, 2023; Sabah and Sil, 2023). Another example, which does not involve causal relationships between events, is the 2013 earthquake in the Philippines followed by a typhoon before recovery from the earthquake had begun (Lagmay and Eco, 2014; Lagmay and Eco, 2023; Lagmay and Eco, 2023). In such scenarios, the impacts of both events combine and affect the asset. The primary challenge is to quantitatively assess the overall amplification of physical damage caused by the superimposition of multiple loads.

Direct physical damage over time t to an asset or infrastructure j caused by ~~concurrent impacts is two concurrent impacts,~~
as graphically represented in Fig. 1, panel (a), can be calculated according to Eq. (2).

$$d_j(t)[t_{S_1}, t_{REC_{1,2}}] = \begin{cases} f_1 & \text{if } t_{S_1} \leq t < t_{RES_{1,2}} \\ f_1 \cdot R_{j,1,2}(t) & \text{if } t_{RES_{1,2}} \leq t < t_{REC_{1,2}} \end{cases} \quad (2)$$

with

$$f_1 = f_{v_{j,1,2}}(h_{1,\max}, h_{2,\max}) \quad (3)$$

According to Eq. (2), at any time t between the start of the first hazard event t_{S_1} and the end of the response phase of the multi-hazard event $t_{RES_{1,2}}$, asset j experiences a level of damage equal to f_1 . ~~Similarly to the single-hazard scenario (Sect. ??), for simplicity and to facilitate the graphical representation, it is assumed that the damage (and therefore the loss of physical integrity) occurs entirely at the onset of the first event.~~ After the response phase of the multi-hazard event concludes (i.e., for $t \geq t_{RES_{1,2}}$), the asset begins to recover from the concurrent impacts according to the recovery function $R_{j,1,2}(t)$, until $t = t_{REC_{1,2}}$. ~~This evolution of damage in time, as described in Eq. (2), is graphically represented in Fig. 1, panel (a).~~

Equation (A1) represents the damage model used to quantify the effects of two concurrent hazards, referred to as the "con-
 310 current damage model." This model assesses the combined impact of both hazards on asset performance. In the literature, concurrent damages are typically evaluated using bivariate vulnerability or fragility functions. ~~Therefore, we express this model.~~ Accordingly, the model is expressed as $f_{v_{j,1,2}}$, where the resulting damage depends on the maximum intensity of each hazard event, $h_{1,\max}$ and $h_{2,\max}$. Gentile et al. (2022) classifies this approach as a "vector-valued fragility model." Bivariate vulnerability functions have been applied in single-hazard scenarios, such as assessing flood damage considering both flow
 315 velocity and duration (Elmer et al., 2010; Nofal and van de Lindt, 2020; Ming et al., 2015), and in multi-hazard contexts, such as hurricanes, where structural impacts are evaluated by combining the effects of coastal surges and wind forces (Do et al., 2020; Nofal et al., 2021).

However, there are instances where concurrent damages are calculated using "state-dependent fragility models" (Gentile et al., 2022), where the fragility of an asset to a secondary hazard is conditional on its damage state after the primary hazard.
 320 This approach is applied, for example, to assess the impacts of an earthquake-tsunami combination, where the fragility curve for the post-shock tsunami depends on the earthquake-induced damage (Xu et al., 2021; Gómez Zapata et al., 2023).

2.1.1 ~~Consecutive Impacts~~

2.2 Consecutive impact dynamics

~~Consecutive impacts occur when hazard events that spatially overlap occur close enough in time that assets do not have
 325 sufficient time to completely recover before the onset of the second hazard. Real-world examples include the sequence of earthquakes and floods in Nepal in 2015. An earthquake with a magnitude of 8.0 struck Nepal in April 2015, causing significant~~

structural and non-structural damage. The region was then hit by heavy rains and floods in 2017 (Gautam and Dong, 2018). Recovery and reconstruction documents (He et al., 2018; Liu et al., 2021) show that the floods affected structures and areas still recovering from the previous earthquake. Another example is Hurricane Irma and Maria, which devastated Puerto Rico in 2017, followed by an earthquake in the same area two years later. This case will be discussed in detail in Sect. 3. The main challenge with consecutive impacts is to estimate residual damage based on the asset's recovery dynamics and to understand how the residual damage influences vulnerability to the subsequent hazard.

Direct physical damage over time t to an asset or infrastructure j caused by two consecutive impacts is calculated according to Eq. (4).

$$d_j(t)[t_{S_1}, t_{REC_2}] = \begin{cases} f_2 & \text{if } t_{S_1} \leq t < t_{RES_1} \\ f_2 \cdot R_{j,1}(t) & \text{if } t_{RES_1} \leq t < t_{S_2} \\ f_3 \cdot [1 - f_2 \cdot R_{j,1}(t_{S_2})] + f_2 \cdot R_{j,1}(t_{S_2}) & \text{if } t_{S_2} \leq t < t_{RES_2} \\ \{f_3 \cdot [1 - f_2 \cdot R_{j,1}(t_{S_2})] + f_2 \cdot R_{j,1}(t_{S_2})\} \cdot R_{j,2}(t) & \text{if } t_{RES_2} \leq t < t_{REC_2} \end{cases} \quad (4)$$

with

$$f_2 = f_{v_{j,1}}(h_{1,\max}) \quad (5)$$

$$f_3 = f_{v_{j,2|d(t_{S_2})}}(h_{2,\max}) \quad (6)$$

According to Eq. (4), at time t_{S_1} , due to the impact of the first hazard event, the asset j experiences a level of damage equal to f_2 . Equation (A2) represents a single hazard damage model (e.g., a depth-damage curve for floods or a fragility curve for earthquakes), as a function of the maximum intensity of the hazard events $h_{1,\max}$. ~~For the single-hazard scenario (Sect. ??), it is assumed that the damage (and therefore the loss of physical integrity) occurs entirely at the onset of the event.~~ After the response phase of the first event is complete (that is, for $t \geq t_{RES_1}$), the asset begins to recover from the impacts of a single danger according to the recovery function $R_{j,1}(t)$.

At time t_{S_2} , the asset is still recovering from the previous hazard event when it is impacted by a second event. The damage caused by this consecutive event is expressed as $f_3 \cdot [1 - f_2 \cdot R_{j,1}(t_{S_2})] + f_2 \cdot R_{j,1}(t_{S_2})$. This expression accounts for the sum of the residual damage at time t_{S_2} , denoted by $R_{j,1}(t_{S_2})$, and the additional damage from the second event, which is calculated as the percentage damage caused by the second event, represented by f_3 , applied to the remaining undamaged portion of the asset, i.e., $[1 - f_2 \cdot R_{j,1}(t_{S_2})]$. This approach prevents the double-counting of damage.

After the response phase of the second event concludes (i.e., for $t \geq t_{RES_2}$) the asset begins to recover from the consecutive impacts according to the recovery function $R_{j,2}(t)$, until $t = t_{REC_2}$. This evolution of damage in time, as described in Eq. (4), is graphically represented in Fig. 1, panel (b).

Equation (A3) represents the damage model used to quantify the effects of two consecutive impacts, known as the "consecutive damage model." This model is based on a "state-dependent fragility model" (Gentile et al., 2022), denoted as $f_{v_{j,2|d(t_{S_2})}}$. It defines the fragility or vulnerability of an asset to a secondary hazard, conditional on the asset's residual damage state due to

incomplete recovery. The resulting damage is a function of the maximum intensity of the second hazard $h_{2,\max}$ and the residual damage to the asset calculated at t_{S_2} according to the recovery function of the asset. State-dependent fragility models are relatively scarce in the literature and are mainly developed within the seismic field to assess the effects of a primary event followed by aftershocks on an already weakened asset (Li et al., 2014; Aljawhari et al., 2021). Another application in the seismic domain involves evaluating the worsening condition of assets, often bridges, due to factors like time and corrosion. In this scenario, the primary seismic event impacts an asset in a deteriorated state, resulting in increased damage (Otárola et al., 2022).

2.2.1 Independent Impacts

~~In the case of consecutive hazards where the second hazard event occurs after the end of recovery of the first one,~~

2.3 Model behaviour under consecutive multi-hazard impact dynamics

To investigate the internal mechanics of the ~~impacts can be considered independent~~. In such a case, the evolution of the damage over time can be seen as a series of single-hazard ~~proposed formulation under consecutive multi-hazard dynamics, similar to those introduced in Sect.??~~. a model behaviour analysis is conducted via a structured parameter influence assessment. The objective is to examine how the mechanisms embedded in Eq. 4 generate additional cumulative damage relative to the independent baseline configuration. The analysis, therefore, isolates the endogenous amplification effects arising from the interaction between residual damage, recovery progression, and state-dependent vulnerability. The investigation focuses on consecutive interactions because their cumulative effects emerge over time as each hazard occurs sequentially. In consecutive events, residual damage and evolving fragility modify the response to subsequent loading, producing additional loss amplification through the recursive update term in Eq. 4–A3. In contrast, concurrent impacts are captured directly by the bivariate damage function in Eq. 2, which defines the combined effect of simultaneous hazards at a single time step. Since concurrent effects are already structurally integrated in the formulation, the analysis concentrates on consecutive interactions to isolate and quantify the unique mechanisms driving cumulative damage across sequential events.

~~Direct physical damage over time t to an asset or infrastructure j caused by concurrent impacts can therefore be simply calculated according to Eq.(A7).~~ Three parameters are examined, consistent with the conceptual drivers identified by de Ruiter et al. (2020) and De Angeli et al. (2022):

$$d_j(t)[t_{S_1}, t_{REC_2}] = \begin{cases} f_2 & \text{if } t_{S_1} \leq t < t_{RES_1} \\ f_2 \cdot R_{j,1}(t) & \text{if } t_{RES_1} \leq t < t_{REC_1} \\ 0 & \text{if } t_{REC_1} \leq t < t_{S_2} \\ f_4 & \text{if } t_{S_2} \leq t < t_{RES_2} \\ f_4 \cdot R_{j,2}(t) & \text{if } t_{RES_2} \leq t < t_{REC_2} \end{cases}$$

with-

$$f_2 = f_{v_{j,1}}(h_{1,\max})$$

$$f_4 = f_{v_{j,2}}(h_{2,\max})$$

(i) the inter-event time interval (Δt);

385 (ii) the recovery trajectory $R_{j,1}(t)$;

(iii) state-dependent modifications of the second-event vulnerability function.

According to Eq. (A7), at time t_{S1} , the first hazard event causes the asset j to sustain damage quantified as f_2 . Once the response phase of the first event concludes (i.e., for $t \geq t_{RES1}$), the asset begins to recover from the damage using the recovery function $R_{j,1}(t)$. By $t = t_{REC1}$, the asset has fully recovered, resulting in a residual damage level of zero, which remains unchanged until a second hazard occurs at t_{S2} . Figure 2 reports additional cumulative loss as a function of Δt under three alternative recovery trajectories: exponential, linear, and logistic. These functional forms, commonly adopted in resilience modelling (Cimellaro et al., 2010; Loos et al., 2023), represent distinct temporal distributions of restoration capacity and therefore different persistence patterns of residual damage.

At t_{S2} , the second hazard affects the asset, causing damage quantified as f_4 . Similarly, once the response phase of this second event is completed (i.e., for $t \geq t_{RES2}$), the asset begins to recover according to the recovery function $R_{j,2}(t)$. By $t = t_{REC2}$, the asset again reaches a residual damage level of zero. Equations (A8) and (A9) describe single-hazard damage models as functions of the maximum intensities of hazard events, $h_{1,\max}$ and $h_{2,\max}$, respectively. Consistent with the single-hazard scenario (Sect. ??), it is assumed that the damages from both events occur entirely at the onset of each event. This evolution of damage in time, as described in Eq. (A7), is graphically represented in Fig. 1, panel (c).

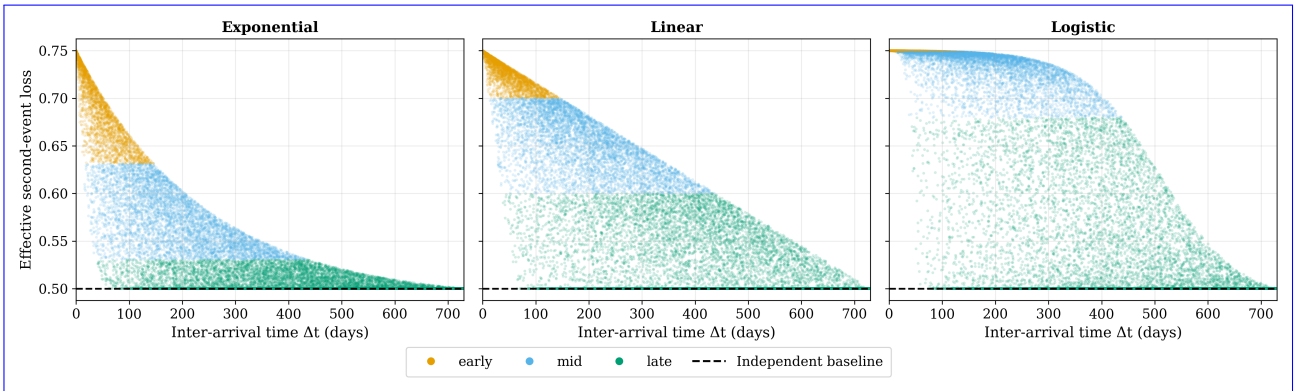


Figure 2. Additional cumulative loss as a function of inter-arrival time (Δt) under exponential, linear, and logistic recovery assumptions. The dashed line represents the independent baseline.

400 Although it may seem that the impacts in this scenario could be assessed by treating the two events as independent single hazards, the situation is more complex. In an ideal analytical scenario, the system's vulnerability may have either decreased or

increased after reconstruction following the first event. Some studies suggest that both structural and non-structural attributes can improve in response to previous events, as seen in the "built back better" theory (Kennedy et al., 2008; Neeraj et al., 2021). Consequently, a second event with similar characteristics could cause more or less damage, depending on how the recovery and reconstruction process unfolded after the initial response phase. Given this scenario, it becomes crucial for risk assessment and management to update the system's vulnerability and exposure after the recovery process to accurately calculate the risks posed by consecutive hazards. When the second event occurs during the early recovery phase, cumulative damage substantially exceeds the independent baseline. As Δt increases, additional cumulative loss decreases and asymptotically converges toward independent behaviour. However, both the rate and magnitude of convergence are governed by the functional form of $R_{j,1}(t)$. Under exponential recovery, rapid early restoration reduces residual damage quickly, limiting amplification. Linear recovery produces intermediate behaviour. Logistic recovery, characterised by delayed early restoration and accelerated later recovery, prolongs vulnerability persistence and sustains elevated amplification over a wider Δt range. These results show that recovery trajectory assumptions are structurally influential within Eq. 4. The assumed recovery trajectory directly determines how long residual damage persists and, therefore, how likely the second event is to interact with a weakened system. Assuming instantaneous or implicitly complete recovery collapses this mechanism and systematically biases cumulative loss toward additive behaviour. The second test isolates the contribution of state-dependent vulnerability (Eq. A3). The vulnerability function of the second event is progressively shifted (+10%, +20%, +30%) to represent increasing fragility due to incomplete recovery. Although state-dependent fragility formulations have been explored primarily in seismic contexts (Gentile et al., 2022; Gentile et al., 2023), they are rarely embedded within a general multi-hazard quantitative structure.

2.4 Generalised Damage Framework

The three distinct types of impact interactions, as defined by Eqs. (2), (4), and (A7), are unified into a single formulation, as presented in Table A1. This generalised piecewise-defined function allows the estimation of relative damage over time to an asset or infrastructure j exposed to n natural hazards. The parameters Θ and μ , as defined in Eqs. (A5) and (A6), govern the classification of impact interactions:-

- $\Theta = 0$: Impacts are classified as *concurrent*, and formulation, presented in Table A1 simplifies to Eq. (2).
- $\Theta = 1, \mu = 0$: Impacts are classified as *consecutive*, and formulation, presented in Table A1 simplifies to Eq. (4).
- $\Theta = 1, \mu = 1$: Impacts are classified as *independent*, and formulation, presented in Table A1 simplifies to Eq. (A7).

The framework has been implemented in Python to facilitate its practical application. The Python code provides a modular structure, enabling users to easily customise-

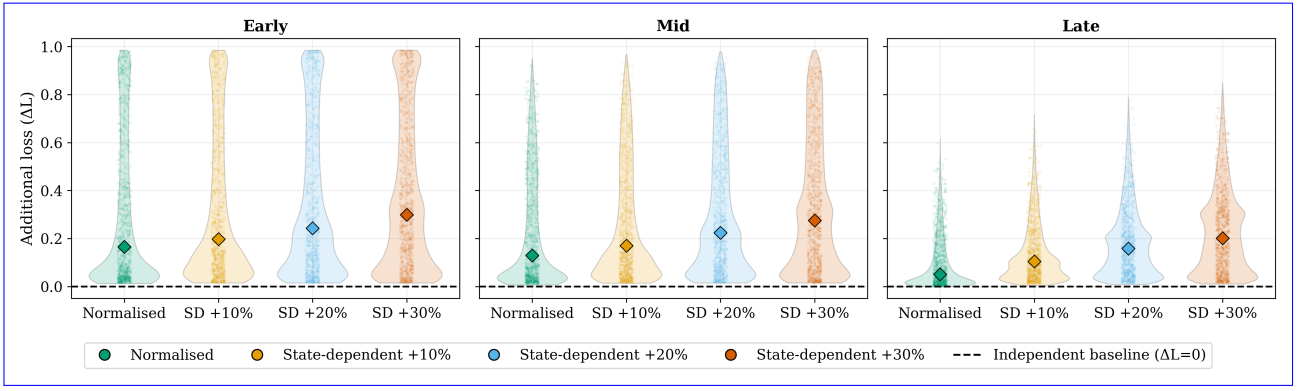


Figure 3. Distribution of additional cumulative loss (ΔL) under increasing state-dependent vulnerability modification. The dashed line indicates the independent baseline ($\Delta L = 0$).

Figure 3 shows that state-dependent vulnerability modification generates convex, non-linear amplification of cumulative damage. Even moderate fragility shifts (+10%) produce measurable increases in additional loss, while larger shifts (+30%) substantially increase both median outcomes and upper-tail behaviour. Crucially, amplification does not arise solely from residual damage magnitude, but from the alteration of the fragility profile itself. Through Eq. A3, incomplete recovery modifies the system's response function, changing the probability structure of damage exceedance. In this sense, consecutive multi-hazard parameters (e.g., intensity, duration, and temporal overlap of the events), vulnerability functions, interaction is not merely additive accumulation but a structural reconfiguration of vulnerability between events. The recovery trajectory should not be interpreted as an abstract modelling parameter. Empirical evidence shows that recovery dynamics reflect governance effectiveness, institutional capacity, infrastructure robustness, and recovery dynamics. The modularity of the Python implementation enhances flexibility by allowing straightforward integration with existing hazard analysis workflows, scalability across multiple scenarios and case studies, and adaptability to user-defined data formats and external modelling tools. For further details on the code, see Sect. Code and data availability at the end of the manuscript. socioeconomic conditions (Miles et al., 2019; Miles et al., 2020; Miles et al., 2021). At the macro scale, such dimensions can be synthesised through composite recovery capacity indicators, such as the Recovery Gap Index (RGI) (Borre et al., 2025). Lower recovery capacity implies prolonged vulnerability persistence, increasing the likelihood that subsequent hazards interact with residual damage and trigger multiplicative effects.

The core of the framework is the recovery function $R_{j,i}(t)$, which models the temporal dynamics of response and recovery for each hazard. Recovery rarely follows a uniform trajectory, as it is shaped by socioeconomic, infrastructural, and political factors. Linear recovery, for instance, assumes a constant rate of progress over time, suitable for regions with predictable and stable resource availability. Exponential recovery captures scenarios where initial efforts are rapid but diminish as resources are exhausted, often observed in regions with strong emergency response but limited sustained capacity.

More complex recovery dynamics can be modelled using logistic functions, particularly useful to describe real-world conditions in which initial delays are followed by a period of rapid progress once external support or efficiency gains come into play. Generically, the framework supports any custom recovery functions, enabling users to tailor their models precisely

~~to specific contexts~~ Overall, the parameter influence analysis shows that cumulative loss in consecutive multi-hazard scenarios depends primarily on three factors: the time interval between events, the speed and shape of recovery, and possible modifications of vulnerability due to residual damage. When the second event occurs after substantial recovery has taken place, total damage remains close to the sum of the two independent events. In contrast, when recovery is incomplete and vulnerability is increased, the second event acts on a weakened system, producing higher cumulative losses than those obtained under independent modelling assumptions. The proposed formulation makes these mechanisms explicit by linking inter-event timing, recovery progression, and vulnerability modification within a single damage function. Rather than assuming that assets fully recover between events, the model allows vulnerability to evolve over time and directly influence subsequent damage estimates. In Sect. 3, this modelling framework is applied to the Puerto Rico disaster sequence, where incomplete recovery and evolving fragility conditions significantly influenced cumulative impacts.

3 ~~Forensic multi-hazard damage analysis in~~ Puerto Rico ~~Case Study and Multi-Hazard Damage Assessment~~

~~This section aims to demonstrate the applicability and added value of the proposed generalised damage framework in capturing the complex, cumulative effects of temporally compounding hazards.~~ Puerto Rico offers a highly relevant and well-documented case of consecutive multi-hazard interaction, having experienced a sequence of major disasters within a relatively short time window. The ~~consecutive~~ impacts of Hurricanes Irma and Maria in 2017 ~~, were~~ followed by the ~~2019-2020 earthquake sequence, underscore the need for~~ 2019–2020 seismic sequence, creating a configuration in which the second hazard occurred during an incomplete recovery phase from the first.

~~The objective of this section is to apply the proposed quantitative multi-hazard damage assessment approaches that go beyond traditional single-hazard models. This case study offers a real-world example to underline how multi-hazard interactions, recovery trajectories, and vulnerability evolution can meaningfully influence the outcomes of a disaster sequence.~~

formulation to a real disaster sequence to assess its explanatory capacity under empirical conditions. Adopting a forensic perspective, the analysis evaluates whether explicitly modelling residual damage and recovery trajectory, as formalised in Sect. 2.2 and analysed in Sect. 2.3, improves consistency between simulated and officially reported losses. In contrast to ~~single-hazard risk assessments, independent hazard modelling approaches, the proposed multi-hazard damage modelling poses specific challenges at each stage of the analysis, from hazard interaction modelling and exposure characterization to the representation of dynamic vulnerability and the quantification of cumulative impacts. The spatial and temporal interdependencies among hazards may significantly alter the nature and magnitude of damage. The Puerto Rico case study provides a compelling example of how neglecting these interactions can lead to an underestimation of systemic risk, overlooking how prior damage and interrupted recovery processes influence vulnerability to subsequent events~~ model allows vulnerability to evolve dynamically between events through the recovery trajectory $R_{j,1}(t)$ and state-dependent vulnerability modification. The Puerto Rico sequence provides an empirical setting in which these mechanisms can be evaluated against observed damage outcomes.

In September 2017, Puerto Rico was struck by ~~two Category-5 hurricanes, Hurricanes~~ Irma and Maria, ~~within just a few weeks. These events caused catastrophic damage to housing, lifelines and public infrastructure causing extensive structural~~

damage and infrastructure disruption (Amir et al., 2020; Morales-Velez et al., 2021; Kwasinski et al., 2019; Cangialosi et al., 2018; Pasch et al., 2023; Fischbach et al., 2020). Hurricane Maria ~~-, in particular, caused nearly 3,000 deaths and an estimated alone resulted in nearly~~ \$90 billion in damage ~~-, ranking as the third most expensive hurricane in US history (Kishore et al., 2018; Kishore et al., 2018a). Recovery efforts were slow and uneven; of the \$42 billion allocated by Congress,~~
490 ~~only \$14 billion had been disbursed by (FEMA, 2018a). Reconstruction progressed unevenly, and by~~ the end of 2019 ~~(FEMA, 2019a; FEMA~~
~~-. As a result, large portions of critical infrastructure remained vulnerable when-, significant portions of the building stock had~~
not yet been fully restored (FEMA, 2019a; FEMA, 2020). In January 2020, a magnitude 6.4 earthquake struck ~~on 7 January~~
2020. This earthquake, the island as part of a wider seismic sequence that began in late 2019, exacerbated the vulnerability of an
already weakened territory (Melendez-Colon et al., 2021; Melendez-Colon et al., 2019). More than 300 schools were reported
495 ~~to have suffered structural damage, and more than 8~~ broader seismic sequence (Melendez-Colon et al., 2021; Melendez-Colon et al., 2019)
~~, 000 residents were displaced (Rodriguez et al., 2021). The disaster further destabilised the power grid and impeded access~~
~~to essential services, especially in southern municipalities. The healthcare sector, already strained, faced growing operational~~
~~challenges (Peters et al., 2021), while the tourism and agricultural sectors registered significant economic losses (Puerto Rico Chamber of C~~
~~-~~
500 affecting regions that were still recovering from hurricane-induced damage. This temporal configuration makes Puerto Rico
a particularly suitable case for examining the role of residual damage and incomplete recovery in shaping cumulative loss.
Figure 4 illustrates the sequence and overlap of the two major types of hazards that affected Puerto Rico during the 2017-2020
period.

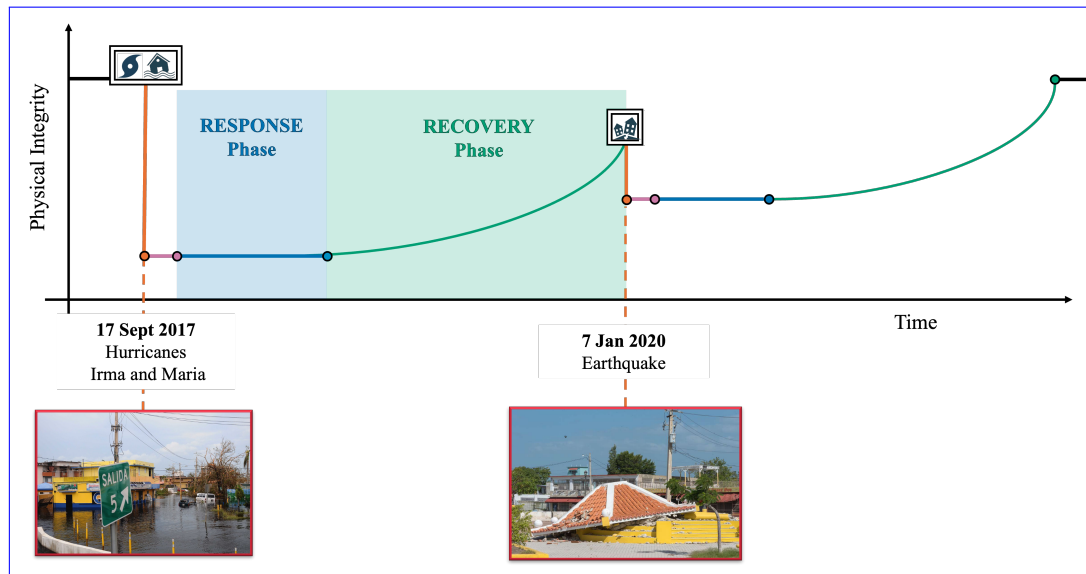


Figure 4. Representation of Puerto Rico's 2017-2020 disaster sequence. [Images, from left to right: \(i\) Flooded area in Carolina, Puerto Rico, following Hurricane Maria's impact on the island, 29 September 2017. Photo by Sgt. Jose Ahiram Diaz-Ramos. Online image, Flickr. \(ii\) Severe earthquake damage to a gazebo in a public park in Guánica, Puerto Rico, 11 February 2020. Photo by Liz Roll/FEMA. Online image, NARA & DVIDS Public Domain Archive.](#)

Puerto Rico is a densely inhabited island, characterised by a higher concentration of buildings in the San Juan metropolitan area, adjacent northern municipalities, and in the southern region around Ponce. These areas not only host a large share of the residential, commercial, and public infrastructure ,but also represent key nodes in the island's economic and logistical networks. The multi-hazard damage assessment performed applying our framework focuses on damages to the built-up area, using as input a portfolio of buildings provided by FEMA (2020). This portfolio was compiled as part of a recent data inventory conducted on the island after Hurricane Maria, and includes the following categories, along with their corresponding estimated replacement values FEMA (2020):

- **Residential:** $\sim 1.5 \times 10^6$ buildings; estimated replacement value ~ 300 billion USD.
- **Commercial:** $\sim 6 \times 10^3$ buildings; estimated value ~ 12 billion USD.
- **Industrial:** $\sim 1 \times 10^3$ buildings; estimated value ~ 2 billion USD.
- **Educational:** $\sim 1.2 \times 10^3$ buildings; estimated value ~ 3 billion USD.

The analysis is carried out in two phases. In the first, ~~we model~~ the impacts of Hurricane Maria ~~using a Compound Damage Model~~ [are modelled by implementing a compound damage model](#) (Sect. 2.1) ~~,integrating that integrates~~ wind and flood hazards. ~~The impacts.~~ [The central objective of the case study is to assess the damage dynamics associated with the subsequent seismic sequence and its interaction with a partially recovered built environment. Accordingly, the compound](#)

520 damage assessment from the hurricane impact simulation primarily serves to define the post-event damage and recovery status that constitute the input for the consecutive damage model applied to the earthquake phase. The fragility curves used for this phase are those provided by FEMA for the specific context of Puerto Rico (Federal Emergency Management Agency, 2020). In the second phase, the ~~Consecutive Damage Model~~ consecutive damage model (Sect. 2.2) is applied to simulate the seismic impacts on structures already weakened by hurricane damage. Here, ~~we started from the~~ standard fragility curves provided by FEMA ~~, and we modified them~~ are modified ad-hoc to capture the changed vulnerability of assets previously affected by the hurricane ~~, and still exhibit~~ and still exhibiting residual damage.

Input data includes:

- **Structural parameters:** Material, type, number of stories, code compliance.
- **Hazard inputs:** Wind speed, flood depth, spectral acceleration.
- **Vulnerability functions:** Hazus-based curves, modified to reflect state dependency.
- 530 – **Economic indicators:** Repair cost per unit area, downtime, and replacement costs.

The results are expressed in terms of physical loss (reduction in structural integrity) and economic cost.

3.1 Compound ~~Hurricane Impacts~~ hurricane impacts: Wind-wind and Flood Interaction flood interaction

~~Within the context of Puerto Rico's~~ The 2017 disaster sequence, the compound impacts landfall of Hurricane Maria ~~represent a particularly critical component of the~~ provides a real configuration of concurrent multi-hazard ~~damage landscape. As highlighted~~ in the preceding sections, the spatial distribution of exposure and the intensity of the event created conditions for widespread disruption. This subsection focuses on the estimation of the damages caused by the interaction between two cooccurring hazards: ~~extreme winds and flooding~~ interaction, in which wind and flood hazards overlapped spatially and temporally across Puerto Rico. In this setting, damage amplification arises from the simultaneous action of multiple stressors acting on the same asset during the emergency phase, as formalised in Eq. 2.

540 ~~According to our framework, these damages are assessed using a Concurrent Damage Model (Sect. 2.1). The vulnerability functions applied are those provided by~~ Within the proposed damage model, compound hurricane damage is represented through a multi-parameter vulnerability formulation $f_{v,j,1,2}(h_{1,max}, h_{2,max})$ that accounts for the joint influence of peak wind intensity and flood depth under overlapping conditions. In practice, the concurrent formulation integrates standard Hazus wind fragility curves and flood depth-damage functions within the unified structure of Eq. 2. Hazus does not provide an explicit bivariate fragility surface for combined wind-flood loading. Instead, wind- and flood-induced damages are evaluated consistently using the respective Hazus vulnerability components, while their interaction is represented through the concurrent damage operator, which accounts for their temporal and spatial superposition. This approach ensures methodological consistency with the standard Hazus framework while preserving the multi-hazard interaction logic embedded in the proposed formulation.

550 Vulnerability functions are derived from the FEMA Hazus-MH methodology FEMA (2020). Hazus-based fragility curves
for wind damage are defined (FEMA, 2020). Wind fragility curves are parametrized according to structural characteristics
such as primary typology, construction material, roof anchorage, and building typology. Similarly, flood damage is modelled
using regionally calibrated number of storeys, and code compliance level. Flood damage is represented through depth-damage
functions ; which express the probability of structural damage based on the depth of the inundation. In the case of Hurricane
555 Maria, the synergistic nature of these hazards became particularly evident: high wind speeds contributed to roof detachment
and building envelope failures, while subsequent or simultaneous flooding infiltrated already compromised structures. FEMA
after-action assessments reported that buildings affected by overlapping hazards showed repair costs up to 30–40calibrated
for different occupancy classes. From a physical standpoint, wind and flood mechanisms are structurally interdependent. High
wind speeds induce roof uplift, envelope breach, and cladding failure, increasing structural susceptibility to water ingress.
560 Subsequent or simultaneous inundation accelerates the deterioration of load-bearing and non-structural components. Post-event
assessments indicate that buildings exposed to overlapping wind and flood loading experienced repair costs 30–40% higher than
those affected by wind or flood alone FEMA (2018b) buildings exposed to a single hazard (FEMA, 2018b). Field observations
highlighted further show that flood depths exceeding 1.5metres resulted in near-complete m resulted in near-complete failure of
unreinforced masonry buildingsstructures, particularly where prior wind damage had already compromised structural elements
565 Rodriguez et al. (2021) reduced lateral resistance (Rodriguez et al., 2021).

The simulation results indicate that the combined impacts of wind and flood during Hurricane Maria resulted in modelled
compound loss for Hurricane Maria is approximately \$90billion in damage in Puerto Rico. This estimate is billion, con-
sistent with official reports, which attribute nearly \$90 billion in total damage to the effects of the hurricane on the island
National Oceanic and Atmospheric Administration (2018). economic assessments (National Oceanic and Atmospheric Administration, 20
570 . The agreement with reported losses does not result from post-event parameter fitting, but from the direct application of
observed hazard intensities and standardised Hazus vulnerability functions within the concurrent formulation. In this phase
of the Puerto Rico sequence, vulnerability parameters are not modified between hazards; amplification derives from hazard
superposition under overlapping conditions rather than from recovery-mediated fragility evolution. The subsequent seismic
sequence, in contrast, introduces a fundamentally different mechanism, where damage persistence and state-dependent vulnerability
575 shifts govern cumulative impacts.

3.2 The ~~Recovery Process~~ recovery process and ~~Subsequent Seismic Impacts~~ subsequent seismic impacts

The earthquake sequence that began in late 2019 and culminated in the magnitude 6.4 event on 7 January 2020 struck Puerto
Rico at a time when many regions were still in the process of recovering from Hurricane Maria. This temporal overlap be-
tween the recovery phase and the occurrence of a new hazard event represents a critical scenario within multi-hazard damage
580 assessment, where pre-existing damage and delayed reconstruction substantially influence the vulnerability landscape.

To spatially contextualise this interaction, Fig. 5 integrates three key spatial data sets: the track of Hurricane Maria (Septem-
ber 2017), the distribution of buildings still damaged as of November 2018 (based on Hazus post-event assessments), and the

contours of the peak ground acceleration (PGA) from the January 2020 earthquake. The post-hurricane assessment identified nearly 138,000 buildings in Puerto Rico that had not yet been fully repaired more than a year after the hurricane.

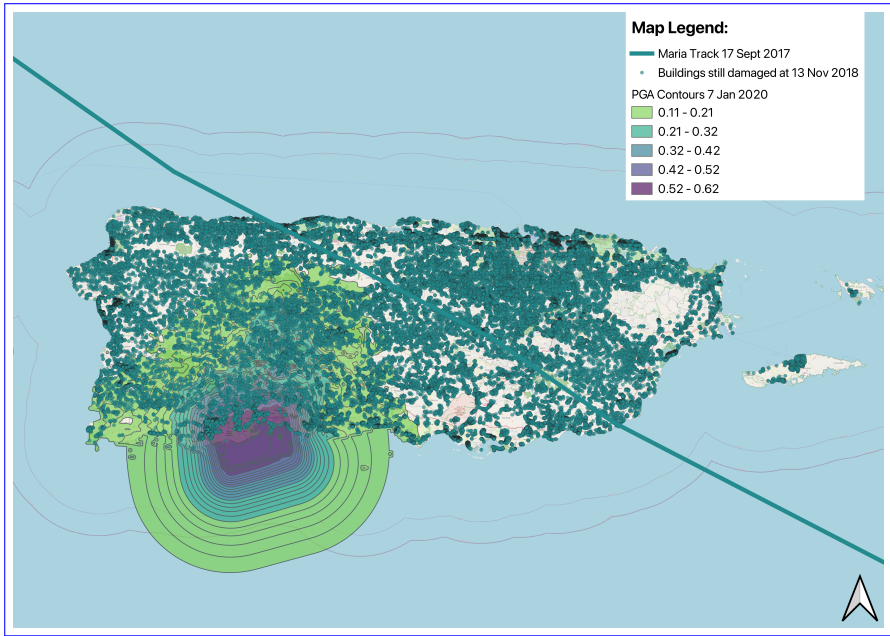


Figure 5. Map showing the track of Hurricane Maria (Sept. 2017), the spatial distribution of buildings with remaining hurricane damage (as of Nov. 2018), and PGA [g] contours from the January 2020 earthquake.

As illustrated in Fig. 5, many of the damaged buildings were concentrated in coastal and southern municipalities, including regions such as Ponce and Guayanilla, which later experienced the highest levels of ground shaking during the earthquake ($PGA > 0.5g$). These areas faced dual exposure, first to high winds and flooding from Hurricane Maria and later to seismic loading from the 2020 event. Residual structural weaknesses, whether due to incomplete reconstruction or substandard repairs, likely increased the risk of damage and collapse.

To model the seismic impacts on the buildings still damaged by the hurricane, ~~we applied a Consecutive Damage Model a~~ consecutive damage model (Sect. 2.2) ~~;-We started from the standard is applied. Standard~~ fragility curves provided by FEMA (Federal Emergency Management Agency, 2012) ~~;-and we transformed them into are transformed into~~ state-dependent curves ~~;-to properly account for residual damage. Our~~ The approach introduces parametric adjustments to account for pre-existing damage conditions. Specifically, the median values of both structural and non-structural fragility curves are shifted according to hypothetical levels of residual damage.

Given the absence of detailed ~~spatialised~~ spatialized building-level recovery data, ~~we assumed~~ a homogeneous distribution of residual damage is assumed throughout the building portfolio. ~~Furthermore, since no specific residual damage values were available, we simulated different degrees of recovery to retrospectively identify those that best captured the final estimated losses.~~

600 To explore these variations, we implemented a sensitivity analysis in which structural and non-structural fragility curves are adjusted by increments of In the absence of spatially explicit residual damage data, a predefined range of vulnerability modifications (5% to 30%) is explored, representing plausible reductions in structural capacity. The objective is not parameter calibration, but to evaluate how different residual damage assumptions influence cumulative loss estimates and their consistency with reported values. This approach allowed us to estimate how the results of the damage could vary under different assumptions about the state of the building stock at the time of the earthquake. Although the modifications were applied uniformly, their impact was is particularly pronounced in areas with high PGA values, where even small increases in vulnerability result in disproportionately large increases in expected losses.

605 Figure 6 illustrates the conceptual effect of residual damage on fragility curves. As residual damage increases, the curves change to the left, reflecting a greater susceptibility to damage at lower levels of seismic demand.

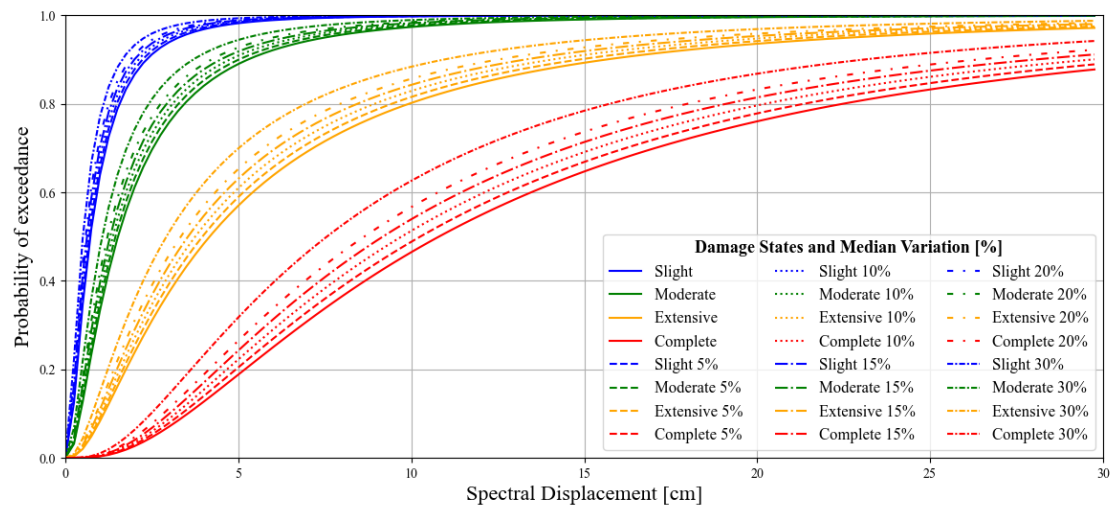


Figure 6. State-dependent fragility curves reflecting changes in structural vulnerability due to residual damage.

610 The results of this analysis are presented in Table ?? The matrix shows total estimated damages (in thousands of USD) for combinations of structural and non-structural degradation. We obtained these values by translating the simulated physical damage into economic losses, following the damage-to-loss conversion methodology described in the Simulated damage probabilities are translated into direct economic losses using the Hazus-MH Technical Manual damage-to-loss methodology (FEMA, 2020). For each type of building building class and damage state, predefined loss ratios were assigned, representing the expected repair cost Hazus defines empirically derived loss ratios representing expected repair costs as a percentage of the replacement value of the asset total replacement value. These ratios differ for are based on historical U.S. claims data and post-disaster reconstruction evidence, and differentiate between structural and non-structural components and increase with the severity of damage. The replacement cost values used in this analysis correspond to the exposure $E_j(t)$ provided in the FEMA

asset-inventory. This approach ensures a consistent and standardised translation from simulated damage to direct economic
620 ~~loss across the entire building portfolio.~~ In this study, replacement values are taken directly from the FEMA building inventory
for Puerto Rico (FEMA, 2020), ensuring internal consistency between exposure, vulnerability, and economic conversion
assumptions. Total portfolio loss is obtained by multiplying damage state exceedance probabilities by the corresponding loss
ratios and aggregating across building typologies.

A clear non-linear trend emerges from the resulting loss estimates: as both forms of residual damage increase, the total losses
625 ~~escalate sharply.~~ For instance, at 10% structural degradation and 15% ~~To explore these variations, structural and non-structural~~
degradation, losses rise by over 40% compared to the baseline (undamaged) scenario.

Total damage estimates (in thousands of \$) for different combinations of fragility curve modifications.

To better visualise this increase in losses, Fig. 7 presents a three-dimensional surface plot. The curvature of the surface
underscores the accelerating cost of residual damage, especially in scenarios where ~~fragility medians were progressively shifted~~
630 ~~between 5% and 30%, representing increasing levels of residual vulnerability.~~ For each modification level, seismic damage was
recomputed and converted into economic loss, allowing systematic evaluation of how cumulative impacts vary under different
recovery assumptions. The results show that total loss increases more than proportionally as fragility medians are shifted
toward lower capacity. Even modest modifications produce noticeable increases in simulated loss, while larger shifts lead to
disproportionately higher damage estimates. The complete numerical matrix of results is reported in Appendix B, while Fig. 7
635 ~~illustrates how total loss varies across structural and non-structural elements remain damaged~~ modification levels.

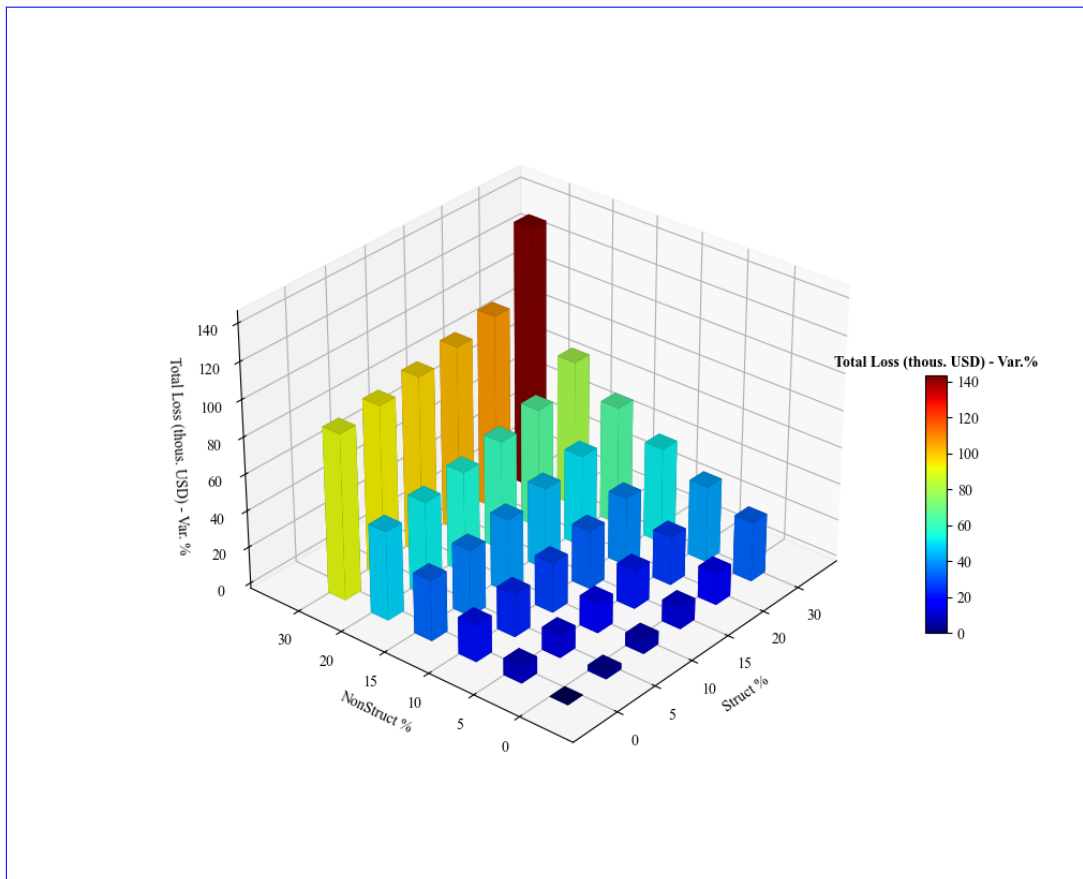


Figure 7. Three-dimensional representation of total losses Relative (in thousands of \$%) change in total simulated losses compared to the single-hazard case for different combinations of increasing structural and non-structural fragility curve vulnerability modifications.

The official damage estimates provided by the Government of Puerto Rico (2020) amount to Official damage estimates for the 2019–2020 seismic sequence amount to approximately 1,700thousand\$. By introducing ,000 USD (Government of Puerto Rico, 2020) . When the earthquake is modelled as an independent single-hazard event—thus assuming full recovery of building vulnerability after Hurricane Maria—the simulated loss equals 1,205,525 USD. This configuration underestimates the official figure by approximately 29%. The discrepancy indicates that assuming complete restoration of structural capacity before the earthquake is not consistent with the observed loss pattern. When state-dependent fragility curves with a vulnerability is introduced, by shifting the median of both structural and non-structural fragility curves by 15%adjustment for structural and non-structural damage components, we obtained a total simulated loss of , the simulated total loss increases to 1,742thousand\$,296 USD, closely matching the official estimate. In contrast, applying standard fragility curves, without accounting for residual damage, yields a simulated loss approximately This modification represents a plausible level of residual vulnerability at the time of the seismic sequence and falls within the systematic range explored in Sect. 2.3 (5–30% lower than the official figure. A comparison of the official estimates and those obtained with these two modelling configurations is presented in Table 1.%).

Table 1. ~~Total damage in thousands of \$ caused by the seismic sequence of 2019-202 in Puerto Rico: comparison~~ Comparison between official ~~estimates and modelling outcomes~~ modelled seismic losses (USD).

Source	Total damage [Thous. \$ <u>USD</u>]
Official Portal of the Government of Puerto Rico (Government of Puerto Rico, 2020)	1.700 <u>1,700,000</u>
Modelled damage using with state-dependent fragility curves (consecutive hazards damage model) <u>vulnerability modification (15%)</u>	1.742 <u>1,742,296</u>
Modelled damage using standard fragility curves (single hazard damage model) <u>with independent fragility curves</u>	1.206 <u>1,205,525</u>

From a technical standpoint, this methodology offers a pragmatic solution for capturing the effects of incomplete recovery in multi-hazard settings. By adjusting the standard fragility functions originally developed in the Hazus framework to reflect state-dependent vulnerability, The difference between the two modelling configurations directly reflects the mechanism described in Eq. 4. In the independent configuration, the model bridges the gap between sequential hazards and evolving exposure conditions. Although the analysis is limited by spatial resolution and the assumption of uniform adjustment, it provides valuable information on how delayed recovery can dramatically amplify future losses recovery function implies full restoration before the second event, and the earthquake acts on an undamaged system. In the state-dependent configuration, residual damage from the hurricane shifts the fragility curves toward lower capacity, increasing damage probabilities under the observed PGA levels. The improved agreement with reported losses does not result from ex-post parameter fitting, but from the structural effect of incorporating incomplete recovery within the vulnerability formulation. The 15% modification lies within the predefined exploration range and represents one plausible residual capacity scenario rather than a calibrated value. The Puerto Rico case confirms that neglecting residual damage persistence leads to systematic underestimation of cumulative impacts in consecutive multi-hazard sequences.

4 Discussion

This ~~research introduces a generalised framework for quantitatively assessing~~ study translates key concepts frequently discussed qualitatively in the multi-hazard ~~direct physical damage to exposed assets, such as buildings or critical infrastructures, over time. It provides scientists and decision-makers with a comprehensive and deeper understanding of the impacts caused by compound and consecutive events. The framework is operationalised as risk literature by translating them into a structured quantitative methodology, implemented as an open-source Python code~~ (please refer to Sect. Code and data availability at the end of the manuscript), allowing the easy application of various users in a wide range of case studies and applications. It comprehensively covers damage mechanisms resulting-. The results demonstrate that these concepts are not only theoretically relevant but can quantitatively affect damage estimates when explicitly represented. While the mathematical formulation

670 builds on existing single-hazard models, its novelty lies in its capacity to integrate multiple elements that, when addressed quantitatively, are usually treated separately (such as multi-hazard vulnerability models and recovery dynamics), and in its practical and easy-to-use application through the development of a dedicated Python code.

The methodology accounts for damage mechanisms arising from both concurrent and consecutive ~~hazard events by modelling the increased damage resulting from the combined impact of two or more concurrent hazards, and the effect of cumulative~~ hazards, including the combined effects of simultaneous hazards and the influence of residual damage and recovery ~~dynamics in case of consecutive hazards. By offering in successive events. By adopting~~ a hazard-independent formulation, the ~~framework model~~ can be applied to any combination of sudden-onset hazard typologies. Furthermore, it allows for the seamless integration of ~~available existing~~ multi-hazard damage models, ~~such as including~~ vector-valued or state-dependent fragility models (Gentile et al., 2022).

680 The ~~efficacy of the framework is demonstrated through a real-world case study in model behaviour analysis (Sect. 2.3)~~ identifies inter-event time intervals, vulnerability interactions, and recovery trajectories (shape and duration) as key drivers of multi-hazard direct physical damage. Additional cumulative loss is highest when subsequent events occur during early recovery, and its magnitude strongly depends on the assumed recovery trajectory. Slower or prolonged recovery phases sustain vulnerability and increase compound losses. Even moderate vulnerability modification levels generate non-linear increases
685 in cumulative damage, indicating that incomplete recovery alters the fragility profile of assets rather than simply adding residual damage. Overall, the results confirm that multi-hazard impacts are path-dependent and may transition from additive to multiplicative behaviour depending on the interaction between event timing and evolving system conditions.

The forensic application to Puerto Rico (Sect. 3) ~~, where both damage mechanisms have been observed: the concurrent impacts of wind and flood generated by the passage of Hurricane Maria, as well as the consecutive impacts caused by the~~ subsequent seismic sequence of 2019–2020. Following the computation of ~~provides a real-world setting in which both concurrent~~ (wind–flood interactions during Hurricane Maria) and consecutive (hurricane–earthquake sequence of 2019–2020) damage mechanisms were observed. After estimating hurricane-induced losses, the ~~framework is used to model the damages incurred by the consecutive effects of the hurricane and earthquake model~~ simulates the additional impact of the subsequent seismic sequence on a portfolio of buildings. ~~To achieve this, a consecutive damage model is applied, involving~~ Central to this
695 application is the development of ~~ad hoc~~ seismic state-dependent fragility curves. ~~This model adjusts fragility function, systematically explored within a predefined modification range to reflect the residual damage conditions caused by the hurricane. Rather than applying standard fragility functions, the approach adjusts fragility parameters for both structural and non-structural components, accounting for the conditional damage state of assets due to incomplete recovery from hurricane-induced damage according to the conditional post-event damage state, thereby embedding incomplete recovery directly into the vulnerability~~
700 representation. This forensic reconstruction allows damage accumulation mechanisms to be explicitly quantified. Multi-hazard damage ~~is impacts are~~ evaluated in both physical (e.g., the percentage loss of physical integrity) and economic terms (e.g., monetary loss), enabling comparison with official earthquake loss estimates (Table 1). The ~~application of a consecutive damage model better estimates real losses~~ closer agreement between modelled and reported losses demonstrates that incorporating temporally dynamic vulnerability substantially improves cumulative impact estimation. The results confirm that damage is not

705 ~~just additive in concurrent and consecutive events, but is path-dependent and strongly conditioned by event order, timing, and recovery dynamics. By introducing-~~

~~Notably, quantitative forensic analyses of multi-hazard events do more than retrospectively explain observed losses, as they can provide a pathway for extrapolating and refining multi-hazard impact models. By reconstructing impact sequences and explicitly quantifying how vulnerability evolves between events, such approaches can enable, for example, the derivation and~~
710 ~~calibration of state-dependent fragility curves, the total damage for the entire portfolio of buildings better captures the order of magnitude of the official total economic damage estimates for the earthquake, as depicted in Table 1 vulnerability functions that capture inter-event interactions, as those derived for the Puerto Rico building portfolio.~~

~~These results provide user evidence of the need for more complex damage models that can account for interaction mechanisms between hazards, vulnerabilities, and impacts. From a policy perspective, they underscore the importance of prioritising rapid and equitable recovery, particularly in high-exposure zones, while also highlighting the necessity of multi-hazard-informed planning frameworks that account for the temporal dynamics of vulnerability~~
715 ~~Beyond retrospective investigation, the quantitative methodology implemented here has potential for anticipatory and planning-oriented applications. By systematically varying key parameters, such as recovery trajectories, inter-event time intervals, or vulnerability interactions, the model allows exploration of how cumulative damage may develop under alternative scenarios. Such scenario-based and “what-if” analyses can inform~~
720 ~~preventive planning and mitigation strategy assessment.~~

~~To enhance such assessments, future efforts should focus on coupling this modelling approach with finer-resolution recovery data, to enable more targeted and locally nuanced assessments. However, this study represents a concrete step forward in operationalising the concepts of cascading risk and temporally dynamic vulnerability in the context of compound hazard events~~
725 ~~the reliability of these forward-looking applications depends on the availability of empirically grounded parameterisations and robust calibration of vulnerability and recovery functions. Without calibration, results remain illustrative rather than fully predictive, but they still provide a comparison of alternative scenarios, structured information on how total damage responds to variations in key parameters, supporting post-disaster analysis and scenario comparison.~~

4.1 Assumptions and limitations

~~The development and application of this framework in the context of the Puerto Rico case study~~
730 ~~quantitative methodology presented for multi-hazard damage dynamics and its forensic application in Puerto Rico are subject to various assumptions and limitations that inevitably introduce uncertainty but also define the scope and applicability of the study. These assumptions and limitations, previously mentioned in various sections of this manuscript, are further detailed below to provide clarity and transparency regarding the boundaries of this analysis~~
~~of validity of the results.~~

Focus on sudden-onset hazards. ~~The framework~~
735 ~~multi-hazard damage formulation proposed in this study~~ primarily targets sudden-onset hazards, characterised by short-duration events such as hurricanes and earthquakes, where the event phase is brief compared to the subsequent response and recovery phases. This assumption is particularly relevant for the Puerto Rico case study, where Hurricane Maria and the 2019-2020 earthquake sequence are treated as discrete events with well-defined temporal boundaries. However, this focus excludes ~~long-onset~~
~~slow-onset~~ hazards such as droughts, sea-level rise, or subsidence, which

740 evolve over extended periods and may blur the distinctions between the event, response, and recovery phases (Terzi et al., 2022). Although this limitation does not diminish the applicability of the framework to sudden-onset events, it underscores the need for ~~separate~~ad-hoc modelling approaches for ~~prolonged~~long-duration slow-onset hazards.

~~**Simplification of hazard interactions.** The framework models hazard interactions through state-dependent fragility curves that adjust building vulnerability based on residual damage from previous events. While this approach provides a robust means of capturing cumulative damage, it simplifies certain complexities inherent in real-world interactions. For example,~~
745 ~~the overlapped impacts of flooding and wind during Hurricane Maria were modelled as independent contributions to total damage. However, field observations suggest that wind-induced roof damage often exacerbated flooding by allowing water infiltration into buildings, creating a compounding effect (FEMA, 2020). These nuances are not fully represented in the current implementation.~~

~~**Direct physical impacts only.** The framework is designed to estimate direct physical damages to assets, such as the percentage of structural integrity lost, using vulnerability or fragility functions as represented by Eqs. (A1), (A3), (A8), and (A9). While these functions are effective for quantifying physical impacts, they do~~quantitative implementation focuses exclusively on direct physical damage to the built environment and does
750 ~~not capture indirect effects such as business interruptions, supply chain disruptions, or social impacts. In Puerto Rico, the prolonged loss of electricity and water after Hurricane Maria led to widespread socioeconomic challenges, including health crises, migration, and economic decline in key sectors such as agricul-~~
755 ~~ture and tourism (Fischbach et al., 2020; Peters et al., 2021). These indirect effects, while significant, fall outside the scope of the current framework~~presented quantitative investigation.

~~**Assumptions about the recovery process.** The~~In the Python code, and as a consequence, in the model behaviour analysis presented, the
760 ~~recovery process is modelled as a series of linear or exponential functions representing slow, medium, or fast recovery speeds. This approximation facilitates analysis but oversimplifies the inherently non-linear nature of recovery, which is influenced by a multitude of factors. In Puerto Rico, for example, bureaucratic inefficiencies, delayed federal funding, and inequitable distribution of resources significantly delayed recovery efforts in certain regions (Group, 2020; OCHA, 2020). Moreover, the~~framework-proposed multi-hazard damage formulation
765 ~~assumes uniform recovery rates across structural and non-structural components, which may not reflect reality. Non-structural elements, such as utilities and interior finishes, often recover faster than structural systems, leading to mismatches in the actual recovery~~paee-trajectory (FEMA, 2019b).

~~**Simplification of temporal dynamics.** In the~~framework-proposed multi-hazard damage formulation,
770 ~~it is assumed that physical integrity is lost immediately at the onset of the event, and no further degradation occurs during the response phase. While this assumption is practical for mathematical modelling, it does not capture cases where damage continues to accumulate after the hazard peaks, such as when flooding persists or secondary hazards occur. In addition, physical integrity is assumed to remain constant during the response phase, despite minor fluctuations that may occur due to emergency repairs or further deterioration. Although these simplifications do not introduce significant bias, they limit the ability to model more complex temporal dynamics observed in real-world scenarios (Porter et al., 2020).~~

~~**Generalized spatial resolution.** The~~framework-forensic analysis in Puerto Rico adopts a portfolio-based approach, grouping buildings into categories based on general characteristics such as structural type, construction material, and occupancy use.

While this approach is suitable for regional-scale assessments, it does not capture localised variations in vulnerability or hazard exposure. In Puerto Rico, differences in topography, proximity to fault lines, and floodplain dynamics played a significant role in shaping the spatial distribution of damage (Baker et al., 2021; FEMA, 2020). These localised effects were approximated rather than explicitly modelled, potentially reducing the precision of damage estimates.

Economic impact modelling limitations. The economic losses in the Puerto Rico case study were estimated using standard unit repair costs, downtime estimates, and sector-specific multipliers. Although these methods provide a useful approximation, they do not account for broader macroeconomic effects, such as long-term population decline, loss of workforce productivity, or disruptions in international trade. For example, the loss of agricultural infrastructure during Hurricane Maria had cascading effects on food security and export revenues, which are not explicitly captured in ~~the framework~~ this study (Fischbach et al., 2020; Group, 2020).

Uncertainty in state-dependent fragility curves. Adjustment of fragility curves to reflect residual damage is based on proportional modifications to the median curve values. Although this method effectively accounts for cumulative damage, it assumes uniform vulnerability changes across all buildings within a category. In reality, building-specific factors such as construction quality, maintenance history, and localised damage patterns may lead to significant variability in vulnerability (Nofal and van de Lindt, 2020; Porter et al., 2020). This limitation highlights the need for more granular data to refine the state-dependent fragility model.

In conclusion, while the ~~framework provides a robust and adaptable tool for modelling~~ proposed approach provides a reproducible analytical template that supports systematic exploration of multi-hazard ~~impacts~~ damage interactions and facilitates forensic analyses, its assumptions and limitations must be carefully considered, specifically when interpreting the results of the Puerto Rico case study. Addressing these limitations in future ~~iterations~~ developments, such as incorporating ~~dynamic~~ more sophisticated recovery models, indirect impacts, and higher spatial resolution, will improve the accuracy and applicability of the ~~framework~~ approach, allowing more comprehensive assessments of compound and consecutive multi-hazard impact scenarios.

4.2 Future developments

~~The proposed framework, while comprehensively addressing the assessment of direct physical damages caused by multiple hazards, presents opportunities for significant improvements. These advancements~~ Building on the current limitations of the proposed methodology, several promising avenues for improvement can be identified. These advances would not only improve the accuracy and applicability of the ~~framework~~ approach but also broaden its potential for integration into various fields of disaster risk reduction and management. ~~In the following, we highlight several key~~ Key directions for future developments are discussed below.

Better understanding and modelling recovery dynamics. The dynamics of the recovery process are central to the assessment of residual damage and the interactions between consecutive hazard impacts. Specifically, the timing of the response phase (t_{RES}) determines whether the hazard impacts are classified as concurrent or consecutive, while the timing and shape of the recovery phase (t_{REC} and $R_j(t)$) significantly influence the residual damage for subsequent events. The response phase, while critical for civil protection and disaster preparedness, has received limited attention in the scientific literature, often being

conflated with the recovery phase. However, its proper evaluation is essential to improve disaster safety plans and understand its cascading effects on recovery (UNISDR, 2015; Consiglio dei Ministri, 2018; IFRC, 2020; Toyoda et al., 2021). More attention
810 has been paid to recovery dynamics, particularly in the context of seismic resilience. Recovery dynamics are highly variable, often following linear, exponential, or logistic patterns depending on socio-economic, political, and governance factors. The pioneering work of Miles and Chang introduced one of the first models of recovery behaviour after an earthquake, proposing recovery trajectories of the community (Miles and Chang, 2006). Based on this, Cimellaro et al. (2010) developed an advanced method for the quantification of resilience by integrating recovery and preparedness metrics. Recent studies have highlighted
815 the importance of recovery models in post-disaster decision-making (Marasco et al., 2022; Fountain and Cradock-Henry, 2020; Loos et al., 2023), underscoring the need for reliable recovery functions. However, challenges remain in quantifying recovery due to the complexity of the influencing factors, including governance, technology, economic conditions, and social cohesion. Emerging methodologies, such as Agent-Based Models (ABMs), offer promising avenues to simulate recovery processes by integrating interactions between individuals, institutions, and resources (Xu and Chopra, 2022). These models complement
820 traditional approaches like community surveys and questionnaires, which are invaluable for understanding local recovery dynamics and identifying barriers to resilience (Jones and Ballon, 2020; Opabola et al., 2023).

Providing more reliable multi-hazard damage models. Improving multi-hazard damage models is essential for capturing the cumulative effects of consecutive hazards. Future research should focus on integrating damage-state-dependent functions that dynamically modify vulnerability and fragility curves based on residual damage from prior events. For example, scenarios involving hurricanes followed by earthquakes, as demonstrated in the Puerto Rico case study, require models capable of
825 adapting to changes in structural and non-structural vulnerabilities caused by sequential impacts (Porter et al., 2020; Nofal and van de Lindt, 2020).

Evaluating dynamic exposure over time. ~~Exposure is a dynamic parameter that evolves as a result of~~ In the current implementation, exposure is assumed to remain constant and is not explicitly modelled within the framework. However, exposure is inherently
830 dynamic, evolving over time due to maintenance, retrofitting, and functional changes in assets. Hazard impacts can reduce exposure through physical damage, but exposure may increase following recovery processes or new construction. Future developments should aim to model these variations over time, capturing the cyclical nature of exposure changes in response to hazards and human interventions (Gomez-Cunya et al., 2022; Loos et al., 2023).

Incorporating indirect impacts through functionality modelling. The ~~framework presented approach~~ currently focuses on
835 direct physical damage but does not explicitly address indirect impacts, such as loss of functionality, economic disruption, or cascading failures in interconnected systems. Future iterations could incorporate functionality as a key metric, using physical integrity as a proxy to estimate functionality (Miles et al., 2019; Opabola et al., 2023). This approach would enable assessments of broader systemic impacts, including business interruptions and supply chain disruptions.

Adapting the framework for long-onset hazards Adapting the formulation to slow-onset hazard dynamics. The ~~current~~
840 ~~framework is optimised for multi-hazard damage assessment~~ formulation is tailored to sudden-onset hazards ~~hazard dynamics,~~ where event durations are short relative to the response and recovery phases. However, ~~long-onset~~ slow-onset hazards such as

droughts, coastal erosion, and subsidence require a fundamentally different modelling approach. Extending the framework to account for these dynamics would improve its applicability to a wider range of hazard scenarios (Terzi et al., 2022).

~~By addressing these areas, the framework can evolve into a more comprehensive tool for~~ Advancing the model toward
845 predictive multi-hazard applications. While the present approach demonstrates its value in forensic reconstruction and model
behaviour analysis, its extension toward predictive multi-hazard risk assessment and management. These advances would
~~support decision-makers in developing more effective mitigation strategies, improving community resilience, and optimising~~
~~resource allocation during the recovery process.~~ represents a crucial direction for future research. At present, vulnerability
transitions and recovery trajectories are parameterised using stylised assumptions that are suitable for exploratory analysis but
850 may not be sufficiently robust for forward-looking risk forecasting. To enable predictive applications, future developments
should focus on: (i) empirical calibration of state-dependent fragility transitions across hazard combinations, (ii) development
of transferable parameter sets that can be applied in data-scarce regions, and (iii) integration with probabilistic hazard occurrence
models to simulate realistic multi-hazard event sequences. In particular, coupling the current damage accumulation formulation
with stochastic event generation models would allow simulation of long-term compound risk under different climate or seismic
855 scenarios. Such integration would transform the model from a post-event analytical tool into a prospective decision-support
instrument capable of evaluating mitigation strategies and recovery policy alternatives under uncertainty. However, achieving
this transition requires a stronger empirical foundation. Without these developments, predictive outputs may remain highly
sensitive to modelling assumptions, limiting their reliability for operational risk management.

5 Conclusions

860 This paper ~~presents a generalised mathematical framework~~ provides a structured quantitative methodology for assessing direct physical damage to exposed assets subjected to multiple natural hazards that can overlap or occur sequentially over time. Specifically, it translates previously conceptual representations of multi-hazard impact dynamics into an explicit and reproducible analytical structure, implemented as a modular Python code. Unlike existing models that focus on specific asset-hazard combinations, ~~our framework~~ the proposed formulation is designed to be broadly applicable to multiple hazards, capturing the complexities of concurrent, consecutive, and independent hazard interactions. A key contribution of this work is the incorporation of recovery dynamics and state-dependent vulnerability interactions into the damage assessment process. ~~By formalising the changes in vulnerability between consecutive hazards, our approach allows for capturing post-disaster loss dynamics.~~ The modular Python implementation ~~of the framework~~ ensures flexibility, enabling users to customise hazard parameters, vulnerability functions, and recovery processes and apply the ~~framework~~ methodology to their specific case study.
870 It can support structured forensic reconstruction of past multi-hazard events, such as the Puerto Rico sequence, as well as forward-looking scenario-based analyses under alternative assumptions of hazard timing, vulnerability evolution, and recovery pathways.

~~Its effectiveness is demonstrated in a Puerto Rico case study, where it was applied to model damage from~~ A structured model behaviour analysis was conducted through the Python code to systematically investigate how inter-event timing, vulnerability

875 interactions, and recovery trajectories influence compound and cumulative physical damage. While such mechanisms are
widely acknowledged qualitatively in the literature, they are rarely examined quantitatively within a single coherent and
reproducible analytical structure. The results highlight that neglecting recovery dynamics or state-dependent vulnerability
adjustments can lead to substantial underestimation of multi-hazard damage. The model behaviour analysis also illustrates the
880 potential of the approach for scenario-based planning, as it enables systematic exploration of alternative assumptions on hazard
timing, recovery pathways, and vulnerability evolution, supporting anticipatory multi-hazard risk assessment.

The proposed approach is then applied to a real multi-hazard sequence in Puerto Rico from a forensic perspective. In this
application, the Python-based quantitative structure is used to reconstruct and interpret the evolution of direct physical damage
following Hurricane Maria and the subsequent 2019-2020 seismic sequence. By incorporating state-dependent fragility curves
to adjust asset vulnerability based on residual damage, the framework produced damage estimates that closely align with
885 official economic loss assessments. This highlights model reproduces damage patterns consistent with reported loss estimates,
highlighting the importance of explicitly accounting for hazard interactions and recovery dynamics to better capture multi-
hazard losses -better. Importantly, the forensic application also enabled the derivation of state-dependent damage curves from
observed cumulative impacts, demonstrating that post-disaster analysis can inform vulnerability models that explicitly account
for residual damage and incomplete recovery.

890 Despite its contributions, the framework methodology has limitations, particularly in introducing some simplifications in
the hazard interactions modelling, assuming uniform recovery rates , and focusing solely on direct physical impacts while
neglecting broader indirect socio-economic consequences. Future developments will address these challenges by refining re-
covery modelling, incorporating dynamic exposure changes, and expanding the applicability of the framework to long-onset
slow-onset hazards. In addition, integrating functionally-based assessments and indirect economic losses will enhance its com-
895 prehensiveness.

Code and data availability. The Python implementation of the framework, along with detailed documentation, is freely available on GitHub
at the following link: <https://github.com/AlessandroBorre/multi-hazard-impact-framework.git>

Appendix A: ~~Generalised Damage Formula~~ Multi-hazard damage formulation

900 The generalised damage formula is summarised continuous piecewise-defined function presented in Table A1 , which shows
how relative damage provides the relative damage over time $d_j(t)$ evolves over time depending on the type of hazard interaction
(concurrent, consecutive, or independent), governed by the parameters Θ and μ to an asset or infrastructure j caused by n natural
hazard events. The events $i \in [1, n]$ are ordered according to their starting time $t_{S,i}$, and considered two by two.

Table A1. Generalised damage formulation across time intervals for asset j

Start Time	End Time	Damage Function $d_j(t)$
t_{S_i}	$\Theta \cdot t_{RES_i} + (1 - \Theta) \cdot t_{S_{i+1}}$	$\Theta \cdot f_2 + (1 - \Theta) \cdot f_1$
t_{RES_i}	$t_{S_{i+1}}$ (only if $\Theta = 1$)	$\Theta \cdot f_2 \cdot R_{j,i}(t)$
$t_{S_{i+1}}$	$\Theta \cdot t_{RES_{i+1}} + (1 - \Theta) \cdot t_{RES_{i,i+1}}$	$\Theta \cdot (\mu \cdot f_4 + (1 - \mu) \cdot (f_3 + [f_2 \cdot R_{j,i}(t_{S_{i+1}})] \cdot (1 - f_3))) + (1 - \Theta) \cdot f_1$
$\Theta \cdot t_{RES_{i+1}} + (1 - \Theta) \cdot t_{RES_{i,i+1}}$	$\Theta \cdot t_{REC_{i+1}} + (1 - \Theta) \cdot t_{REC_{i,i+1}}$	$\Theta \cdot (\mu \cdot f_4 + (1 - \mu) \cdot (f_3 + [f_2 \cdot R_{j,i}(t_{S_{i+1}})] \cdot (1 - f_3))) \cdot R_{j,i+1}(t) + (1 - \Theta) \cdot f_1 \cdot R_{j,i,i+1}(t)$

The generalised damage formulation presented with

$$905 \quad \underline{f_1 = f_{v_{j,i,i+1}}(h_{i,\max}, h_{i+1,\max})} \quad (\text{A1})$$

$$\underline{f_2 = f_{v_{j,i}}(h_{i,\max})} \quad (\text{A2})$$

$$\underline{f_3 = f_{v_{j,i+1|d(t_{S_{i+1}})}}(h_{i+1,\max})} \quad (\text{A3})$$

$$\underline{f_4 = f_{v_{j,i+1}}(h_{i+1,\max})} \quad (\text{A4})$$

and the parameters reported in Table A2.

Table A2. Description of symbols used in the multi-hazard damage formulation.

<u>Symbol</u>	<u>Description</u>
t_{S_i}	<u>Starting time of hazard event i</u>
t_{E_i}	<u>Ending time of hazard event i</u>
t_{RES_i}	<u>End of the response phase from hazard event i</u>
$t_{RES_{i+i+1}}$	<u>End of the response phase from the overlapping events i and $i+1$</u>
t_{REC_i}	<u>End of the recovery phase from hazard event i</u>
$t_{REC_{i+i+1}}$	<u>End of the recovery phase from overlapping events i and $i+1$</u>
f_{vji}	<u>Single-impact damage model for asset j under hazard event i</u>
$f_{vji+i+1}$	<u>Concurrent impact damage model: vulnerability function quantifying damage caused by compound impacts from overlapping events i and $i+1$</u>
f_{vji+d}	<u>Consecutive impact damage model: vulnerability function quantifying damage caused by event i, conditional on residual damage d</u>
$h_{i,max}$	<u>Maximum magnitude reached by hazard event i</u>
R_{ji}	<u>Recovery rate of asset j from the damage caused by hazard event i</u>
$R_{j,i+i+1}$	<u>Recovery rate of asset j from the damage caused by overlapping events i and $i+1$</u>

910 The type of impact interaction is governed by the parameters Θ and μ , which allow the model to represent different dynamics (concurrent, consecutive, or independent) within a single, consistent framework. These parameters are formally defined using Heaviside step functions as follows:

$$\Theta(t_{S_{i+1}}, t_{RES_i}) = \mathcal{H}(t_{S_{i+1}} - t_{RES_i}), \quad (\text{A5})$$

$$\mu(\Theta, t_{S_{i+1}}, t_{REC_i}) = \mathcal{H}(t_{S_{i+1}} - t_{REC_i}), \quad \text{if } \Theta = 1. \quad (\text{A6})$$

915 The parameters classify the type of multi-hazard impact as follows:

- $\Theta = 0$: Impacts are classified as *concurrent*. The formulation in Table A1 provides a flexible framework for modelling how asset damage evolves over time in response to multiple hazard events. By adjusting the parameters Θ and μ , the model captures different types of interactions, concurrent, consecutive, simplifies to Eq. (2) in the main text.
- $\Theta = 1, \mu = 0$: Impacts are classified as *consecutive*. The formulation in Table A1 simplifies to Eq. (4) in the main text.
- 920 – $\Theta = 1, \mu = 1$: Impacts are classified as *independent*. The formulation simplifies to Eq. (A7) reported in this appendix.

A1 Independent impacts

In the case of two consecutive hazards where the second hazard event occurs after the end of recovery of the first one, the impacts can be considered independent. In such a case, the evolution of the damage over time can be seen as a series of single-hazard dynamics.

925 Direct physical damage over time t to an asset or infrastructure j caused by concurrent impacts can therefore be simply calculated according to Eq. (A7).

$$d_j(t)[t_{S_1}, t_{REC_2}] = \begin{cases} f_2 & \text{if } t_{S_1} \leq t < t_{RES_1} \\ f_2 \cdot R_{j,1}(t) & \text{if } t_{RES_1} \leq t < t_{REC_1} \\ 0 & \text{if } t_{REC_1} \leq t < t_{S_2} \\ f_4 & \text{if } t_{S_2} \leq t < t_{RES_2} \\ f_4 \cdot R_{j,2}(t) & \text{if } t_{RES_2} \leq t < t_{REC_2} \end{cases} \quad (\text{A7})$$

with

$$f_2 = f_{v_{j,1}}(h_{1,\max}) \quad (\text{A8})$$

$$930 \quad f_4 = f_{v_{j,2}}(h_{2,\max}) \quad (\text{A9})$$

According to Eq. (A7), at time t_{S_1} , the first hazard event causes the asset j to sustain damage quantified as f_2 . Once the response phase of the first event concludes (i.e., for $t > t_{RES_1}$), the asset begins to recover from the damage using the recovery function $R_{j,1}(t)$. By $t = t_{REC_1}$, the asset has fully recovered, resulting in a residual damage level of zero, which remains unchanged until a second hazard occurs at t_{S_2} .

935 At t_{S_2} , the second hazard affects the asset, causing damage quantified as f_4 . Similarly, once the response phase of this second event is completed (i.e., for $t \geq t_{RES_2}$), the asset begins to recover according to the recovery function $R_{j,2}(t)$. By $t = t_{REC_2}$, the asset again reaches a residual damage level of zero. Equations (A8) and (A9) describe single-hazard damage models as functions of the maximum intensities of hazard events, $h_{1,max}$ and ~~independent, allowing for a consistent representation of complex multi-hazard sequences:~~ $h_{2,max}$, respectively. It is assumed here that the damages from both events occur entirely at the onset of each event. This evolution of damage in time, as described in Eq. (A7), is graphically represented in Fig. A1.

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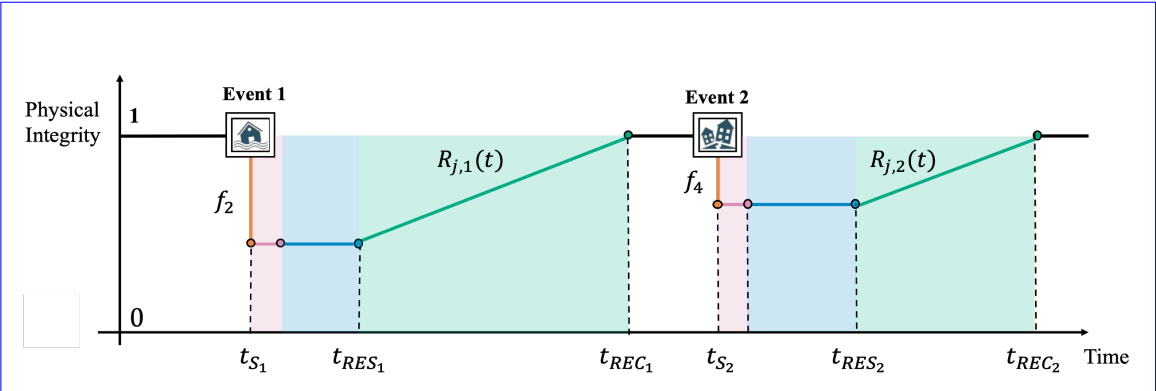


Figure A1. Variation of physical integrity over time in the case of two independent impacts. The second event in temporal order affects the asset when it has already completed the recovery phase of the previous one.

Although it may seem that the impacts in this scenario could be assessed by treating the two events as independent single hazards, the situation is more complex. In an ideal analytical scenario, the system's vulnerability may have either decreased or increased after reconstruction following the first event. Some studies suggest that both structural and non-structural attributes can improve in response to previous events, as seen in the "build back better" theory (Kennedy et al., 2008; Neeraj et al., 2021). Consequently, a second event with similar characteristics could cause more or less damage, depending on how the recovery and reconstruction process unfolded after the initial response phase. Given this scenario, it becomes crucial for risk assessment and management to update the system's vulnerability and exposure after the recovery process to accurately calculate the risks posed by consecutive hazards.

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Appendix B: Numerical results for state-dependent vulnerability modification

950 This appendix reports the complete numerical matrix of simulated seismic losses for different combinations of structural and non-structural vulnerability modification levels applied in the Puerto Rico case study (Sect. 3.2).

The baseline fragility curves are derived from FEMA Hazus (Federal Emergency Management Agency, 2012). State-dependent vulnerability modification is introduced by progressively reducing the median parameters of the fragility functions between 0% and 30% for both structural and non-structural components. For each combination, seismic damage probabilities are computed and converted into economic loss using the Hazus damage-to-loss ratios described in Sect. 3.2.

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Loss values are expressed in USD and represent aggregated portfolio losses across all building classes included in the FEMA inventory for Puerto Rico.

Table B1. Total simulated seismic losses (USD) under different combinations of structural and non-structural vulnerability modification levels.

	NonStruct 0%	NonStruct 5%	NonStruct 10%	NonStruct 15%	NonStruct 20%	NonStruct 30%
Struct 0%	1,205,525	1,316,141	1,446,902	1,602,471	1,789,239	2,294,623
Struct 5%	1,245,367	1,355,773	1,486,192	1,641,565	1,828,054	2,331,851
Struct 10%	1,292,172	1,402,442	1,532,614	1,687,487	1,873,685	2,376,321
Struct 15%	1,347,889	1,457,732	1,587,716	1,742,296	1,927,840	2,429,457
Struct 20%	1,414,843	1,524,206	1,653,575	1,807,901	1,933,072	2,493,172
Struct 30%	1,595,392	1,703,806	1,832,623	1,985,067	2,168,596	2,934,148

The matrix shows that loss increases more than proportionally when both structural and non-structural vulnerability modifications are combined.

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