

Reply to reviewer comments

We sincerely thank the Editor and both Reviewers for their careful and rigorous evaluation of our manuscript. We greatly appreciate the depth and precision of the comments, which have helped us significantly improve the clarity, positioning, and robustness of the paper. The reviews raised important conceptual, methodological, and presentation-related points concerning:

- The scientific framing and novelty of the proposed formulation;
- Its operational applicability for forward-looking risk assessment;
- The implementation of recovery dynamics and state-dependent vulnerability;
- The interpretation of the Puerto Rico case study;
- Clarity, terminology, and presentation issues.

In response, we have undertaken a substantial and comprehensive revision of the manuscript. The main modifications are summarised below.

Major revisions implemented

Refined scientific positioning and revised title:

As suggested by both Reviewers, we have reformulated and better clarified the main aim of the paper and its novelty, which lies in translating previously conceptual and graphical representations of multi-hazard dynamics (e.g., de Ruiter et al., 2020, and De Angeli et al., 2022) into an “explicit and reproducible analytical structure,” implemented as modular Python code. The paper now clearly presents the proposed approach as a *multi-hazard damage formulation* integrating concurrent, consecutive, and independent hazard interactions within a unified analytical structure, rather than as a *mathematical framework*. In line with that:

- The description of the mathematical formulation has been significantly shortened.
- The abstract, introduction, discussion, and conclusion sessions have been updated to reflect this new positioning of the contribution.
- The title has been revised accordingly.

Clarified scope and operational applicability:

As encouraged by Reviewer 1, we clarified that the primary utility of the proposed analytical structure is to provide a forensic template that supports systematic exploration of multi-hazard damage interactions and facilitates structured post-disaster interpretation and learning. The Puerto Rico case study is now clearly framed as a forensic and demonstrative application of the approach, illustrating how recovery-mediated vulnerability evolution can be integrated within a consistent modelling structure.

We also explained that, beyond forensic analyses, the proposed methodology can also support anticipatory and scenario-based analyses, enabling exploration of how

cumulative multi-hazard damage may evolve under alternative assumptions, such as different recovery pathways or inter-event intervals. This potential is preliminarily examined through a model behavior analysis (now included as Section 2.3 in the revised manuscript), which provides a foundation for such forward-looking applications. However, the approach is not yet operational for predictive multi-hazard use. Its further development toward operational implementation is discussed in Section 4.2, “Future Developments”.

Strengthened explanation of recovery dynamics:

Following the suggestion of Reviewer 2, we expanded the description of the recovery function implementation and clarified how asset states are updated between consecutive events. The role of recovery trajectories in governing cumulative damage under consecutive configurations is now more explicitly described, ensuring consistency between the mathematical formulation, the Python implementation, and the case study.

Clarified the treatment of state-dependent vulnerability:

To address both Reviewers' concerns, we have reinforced that baseline fragility curves are derived from established sources (Hazus) and that the novelty lies in their dynamic adaptation within a consecutive multi-hazard structure. The mechanism of vulnerability modification due to incomplete recovery is now more clearly explained.

Improved damage-to-loss conversion explanation:

Following the suggestion of Reviewer 1, the damage-to-loss methodology has been expanded to clarify the empirical basis of predefined loss ratios and the aggregation procedure used to estimate total portfolio losses.

Clarified the role of the Python implementation:

The manuscript now clearly states that the open-source Python tool implements the generalised damage formulation in an easy-to-use, modular way, enabling customised multi-hazard damage assessment. Nevertheless, the code development is not addressed in this manuscript, clearly keeping the main scientific contribution in the analytical structure itself. These points address comments from Reviewer 2 and ensure that the contribution is consistent with the aims of the NHESS journal, in line with the Editor's recommendations.

Provided quantitative support where required:

To respond to Reviewer 1, statements regarding the non-linear escalation of cumulative loss are now supported with quantitative evidence. Units, tables, and figure references have been carefully revised for consistency and clarity.

Moreover, we integrated several comments from both reviewers, and we standardized key terminology (e.g., “slow-onset hazards”), reduced redundancy, simplified complex sentence structures, and improved figure sequencing and legend clarity throughout the manuscript.

We believe that these revisions substantially strengthen the manuscript, better align its claims with its demonstrated capabilities, and clarify its contribution to multi-hazard damage modelling. A detailed point-by-point response to each reviewer's comment is provided below. We have also **attached the revised manuscript with changes tracked**.

Line numbers referred to in our responses below are from the revised manuscript (without changes tracked).

REVIEWER 1

Comment 1: Core Contribution and Scientific Value

My primary concern concerns the manuscript's fundamental claims about its contribution and practical utility. The authors position their work as providing a "generalised mathematical framework" for multi-hazard risk assessment, but this claim requires careful examination.

Response 1

We thank the Reviewer for this important observation. In the revised manuscript, we have clarified and refined the positioning of the contribution to avoid overstatement and to better specify the scope and utility of the proposed approach. The manuscript now consistently frames the contribution as a structured quantitative methodology for multi-hazard direct physical damage assessment. It is presented as a reproducible analytical template that enables systematic exploration of multi-hazard damage interactions and supports structured post-disaster interpretation and learning, rather than as a purely generalised mathematical framework.

To better position the work, the following revisions were introduced:

Title revision:

The title has been refined to emphasise the analytical and forensic nature of the work: "A quantitative methodology for analysing physical damage and recovery dynamics from concurrent and consecutive hazards: forensic insights from Puerto Rico."

Clarification of contribution in the Introduction (Lines 55–67):

The text now explicitly states that the objective is to translate previously conceptual multi-hazard dynamics into an "explicit and reproducible analytical structure," implemented as modular Python code. The emphasis is placed on reproducibility and structured exploration of damage drivers.

Specification of what is generalised (Lines 104–112):

The formulation is described as a "generalised piecewise defined function" capable of representing concurrent, consecutive, and independent interactions through parameters Θ and μ . This clarifies that generalisation refers to interaction typologies within a unified damage function, not to universal hazard modelling.

Clarification of analytical scope (Lines 371–376):

The Discussion explicitly states that the mathematical formulation builds on existing single-hazard models and that its novelty lies in integrating multi-hazard interaction logic, recovery dynamics, and state-dependent fragility within a single computational structure.

Regarding the **practical utility**, we have expanded the Discussion section to clarify how the proposed approach can be applied in empirical contexts and how it can

also provide a basis for forward-looking applications. The presented approach, therefore, has multiple complementary uses.

First, it provides a *multi-hazard damage formulation* that translates key multi-hazard concepts, often discussed qualitatively, such as vulnerability interactions, residual damage, and recovery dynamics, into a formal and implementable modelling approach. It integrates concurrent and consecutive events within a unified structure and allows the incorporation of existing multi-hazard or state-dependent fragility models. The formulation is fully implemented in Python, ensuring reproducibility and facilitating systematic exploration of model parameters.

Second, the approach demonstrates practical utility through forensic application. As shown in the Puerto Rico case study, it allows reconstruction of sequential damage processes across concurrent and consecutive events within a real portfolio of buildings, illustrating the analytical value of modelling residual damage in consecutive hazard sequences.

Third, it enables the calibration and refinement of state-dependent vulnerability functions based on observed loss patterns, thereby improving the representation of inter-event interactions and cumulative damage mechanisms in empirical settings.

Finally, beyond retrospective analysis, the framework supports prospective and planning-oriented applications. By systematically varying recovery assumptions, inter-event time intervals, and vulnerability interactions, it enables scenario-based analyses that can inform mitigation planning and strategy assessment. This application provides a foundation for predictive analyses, although such predictions still require empirical calibration of recovery trajectories and vulnerability transitions to ensure robustness.

Comment 2: Mathematical Formalization

The mathematical framework presented is essentially a straightforward extension of existing single-hazard approaches, where total damage becomes a function of multiple hazards and recovery states. While the authors present this as novel, the mathematical formulation represents a straightforward extension. The real challenges in multi-hazard assessment lie not in the mathematical abstraction but in the empirical quantification of recovery functions, hazard dependencies, and vulnerability transitions over time - as the authors rightly state in sections 4.1 and 4.2.

Response 2

We agree that the proposed formulation builds upon established single-hazard damage representations (e.g., fragility functions and recovery curves). The objective of the manuscript is not to redefine these components individually, but to integrate them into a unified, time-dependent multi-hazard structure capable of representing

concurrent, consecutive, and independent impact dynamics within a single analytical formulation.

To clarify this point and better position the contribution, the manuscript has been revised as follows:

Clarification of the scope of contribution:

The Introduction now explicitly states that the aim is to translate conceptual multi-hazard representations into a “structured quantitative methodology” and to provide a “reproducible analytical template,” rather than to introduce new empirical vulnerability models.

Explicit positioning of the mathematical structure:

The generalised piecewise formulation is described as a compact structure whose behaviour is controlled by parameters Θ and μ (formally defined in Appendix A. The text clarifies that different interaction types (concurrent, consecutive, and independent) are represented through parameter switching within a unified function.

Integration of recovery and state dependency (Lines 154–180):

In the consecutive formulation (Eq. 4), residual damage persistence, recovery dynamics, and state-dependent vulnerability are explicitly combined. The recursive update term prevents double-counting and structurally links the residual damage state.

Clarification of structural amplification mechanism (Lines 215–226):

The manuscript now explains that cumulative amplification arises from modification of the fragility profile itself, not merely from additive residual damage, making explicit that the mechanism is embedded in the probabilistic structure of the vulnerability function.

Explicit acknowledgement of empirical challenges (Lines 408–415; 517–529):

The Discussion now clearly distinguishes between the analytical structure proposed in this work and the empirical challenges related to the calibration of recovery trajectories and state-dependent fragility transitions. It is explicitly stated that forward-looking predictive applications require empirical calibration and robust parameterisation.

Comment 3: Operational Utility Limitations

More critically, the framework suffers from a fundamental limitation that severely constrains its practical applicability. The Puerto Rico case study illustrates this problem clearly. The authors demonstrate how Hurricane Maria affected earthquake vulnerability by using post-hurricane damage data to calibrate vulnerability adjustments. However, this approach is inherently backwards-looking and requires empirical damage data that would not be available for forward-looking risk assessments.

Response 3

We thank the Reviewer for highlighting this crucial point regarding the forward-looking applicability of the proposed approach, specifically in reference to the Puerto Rico case study.

We fully acknowledge that the Puerto Rico case study represents a forensic analysis and that the availability of post-event empirical information facilitates the reconstruction of state-dependent vulnerability conditions. To further clarify this aspect, the revised manuscript makes the following points explicit:

Forensic positioning of the case study (Lines 242–248):

The Puerto Rico application is described as an evaluation of the model's explanatory capacity under empirical conditions, rather than as a forward-looking calibrated prediction.

Clarification on vulnerability modification (Lines 327–337):

The text explicitly states that, in the absence of spatially explicit residual damage data, a predefined modification range (5–30%) is explored. It is clearly specified that the objective is not parameter calibration, but evaluation of how different residual damage assumptions influence cumulative loss estimates.

Explicit statement against ex-post fitting (Lines 363–369):

The improved agreement with reported losses is described as resulting from the structural effect of incorporating incomplete recovery, and not from ex-post parameter fitting. The 15% shift is presented as one plausible scenario within a systematic exploration range.

Moreover, the manuscript does not present the approach as a calibrated predictive tool for forward-looking risk assessments. Instead, we explicitly stated that “the primary contribution of this work [...] lies in offering a reproducible analytical template that supports systematic exploration of multi-hazard damage interactions and facilitates structured post-disaster interpretation and learning.” (Lines 63-65).

Nevertheless, the forensic reconstruction allows damage accumulation mechanisms to be explicitly quantified and can, therefore, provide a pathway for extrapolating and refining multi-hazard impact models. “By reconstructing impact sequences and explicitly quantifying how vulnerability evolves between events, such approaches can enable, for example, the derivation and calibration of state-dependent vulnerability functions that capture inter-event interactions.” (Lines 405-406). These curves can then be applied for a forward-looking assessment.

Despite this capability for deriving empirical state-dependent vulnerability models, we are aware of the developments needed and limitations to overcome to have an operational predictive application.

The Discussion section (Lines 408–415) clearly states that forward-looking applications require empirically grounded parameterisation and robust calibration of vulnerability and recovery functions, and that without such calibration, results remain illustrative rather than fully predictive. A dedicated subsection (Lines

517–529) outlines the steps necessary to extend the framework toward predictive multi-hazard applications, including:

- (i) empirical calibration of state-dependent fragility transitions,
- (ii) development of transferable parameter sets, and
- (iii) integration with probabilistic hazard occurrence models.

To conclude, we agree that the present implementation does not eliminate the empirical challenges inherent in forward-looking multi-hazard risk assessment. However, the contribution of the manuscript lies in providing the analytical structure required to represent vulnerability evolution and damage accumulation explicitly. Without such a structure, even calibrated predictive applications would lack a coherent mechanism to incorporate inter-event timing and recovery-mediated fragility transitions.

In summary, the revised manuscript clarifies that:

- The Puerto Rico case study is intentionally forensic.
- The vulnerability modifications are systematically explored, not calibrated to force agreement.
- Predictive applications require additional empirical development, which is explicitly acknowledged.
- The framework provides the necessary quantitative backbone for such future developments.

Comment 4: Consider a practical scenario

If a hurricane were to strike Puerto Rico tomorrow, practitioners using this framework would need to wait for post-hurricane damage assessments before they could adjust earthquake vulnerability functions. This severely limits the framework's utility for operational risk management, emergency planning, or prospective risk assessment. The authors do not adequately address how practitioners would estimate vulnerability changes in real-time or predictive applications without extensive post-event calibration data.

Response 4

We thank the Reviewer for raising this important operational perspective. We agree that, in a real-time post-event context, detailed empirical information on residual damage may not be immediately available. The current framework does not eliminate the need for empirical input to parameterise state-dependent vulnerability transitions. However, the manuscript does not position the approach as requiring full post-event calibration prior to application.

The revised manuscript clarifies this point in several ways:

Scenario-based exploration rather than real-time calibration (Lines 327–337):

The vulnerability shifts (5–30%) are introduced as a predefined exploration range representing plausible reductions in structural capacity. The objective is explicitly

stated as evaluating how alternative residual damage assumptions influence cumulative loss estimates, rather than calibrating parameters to post-event data.

Model behaviour analysis as prospective exploration (Lines 182–192; 215–227):

Section 2.3 systematically investigates how inter-event timing, recovery trajectory, and state-dependent vulnerability modification influence cumulative loss. This structured parameter influence analysis is independent of the Puerto Rico empirical calibration and can be applied prospectively to test plausible vulnerability transition scenarios.

Explicit acknowledgement of predictive requirements (Lines 408–415):

The Discussion states that forward-looking applications require empirically grounded parameterisation of recovery trajectories and vulnerability transitions, and that without calibration, results remain illustrative but informative for scenario comparison.

Future integration with probabilistic modelling (Lines 517–529):

The manuscript outlines how coupling the damage accumulation formulation with stochastic event generation and transferable parameter sets would enable predictive, planning-oriented applications.

In practical terms, if a hurricane were to strike Puerto Rico tomorrow, the framework would not require waiting for complete post-hurricane empirical damage inventories before being used. Instead, practitioners could:

- Define a plausible range of residual vulnerability scenarios (e.g., 5–30% capacity reduction);
- Select recovery trajectory assumptions consistent with institutional or socioeconomic indicators;
- Run scenario-based simulations to quantify the sensitivity of cumulative earthquake loss to these assumptions.

The value of the framework lies in making explicit how different residual damage hypotheses translate into different cumulative risk outcomes. We therefore acknowledge that real-time operational deployment requires additional empirical and institutional inputs. However, the framework provides the analytical structure necessary to test and compare vulnerability transition assumptions.

Comment 5: Misalignment of Claimed vs. Actual Contributions

The manuscript lists several important developments as "limitations" (section 4.1) or "future research directions" (section 4.2) that would actually constitute the real scientific advances needed in multi-hazard assessment. These include empirically-derived vulnerability transition functions, standardized recovery curves based on extensive post-disaster data, and predictive models for hazard-induced vulnerability changes. Only after solving these challenges would a general mathematical framework provide meaningful operational value.

Specifically, I was excited to see the authors propose an approach to adjust vulnerability curves to account for pre-existing damage conditions (lines 394–396). Such an approach represents a potentially valuable contribution to the field.

However, the implementation still relies entirely on post-event calibration data, making it neither operational for forward-looking assessments nor generalizable across different contexts without extensive empirical datasets.

In conclusion, current framework appears most suited for retrospective analysis and systematic post-disaster impact assessment rather than the forward-looking risk management applications that the authors suggest. This represents a significant gap between the claimed contribution and the demonstrated capabilities.

Response 5

We thank the Reviewer for this careful and constructive observation. We agree that empirically derived vulnerability transition functions, standardised recovery curves based on large post-disaster datasets, and predictive models for hazard-induced vulnerability evolution represent major scientific challenges in multi-hazard assessment. These elements are indeed necessary to achieve fully operational predictive applications. However, the manuscript does not claim to have resolved these empirical challenges. Rather, it proposes a structured quantitative formulation within which such empirically derived components can be consistently embedded.

To address the perceived misalignment, the revised manuscript clarifies the following aspects:

Positioning of the contribution (Lines 55–67):

The Introduction now explicitly frames the contribution as the translation of conceptual multi-hazard dynamics into an explicit and reproducible analytical structure, implemented in modular Python code. The emphasis is on structural integration rather than empirical calibration.

Explicit statement of novelty scope (Lines 371–381):

The Discussion clarifies that the mathematical formulation builds on existing single-hazard models and that its novelty lies in integrating concurrent and consecutive interactions, recovery dynamics, and state-dependent fragility within a unified computational framework.

Explicit acknowledgement of predictive limitations (Lines 408–415; 517–529):

The manuscript now clearly states that predictive multi-hazard applications require empirical calibration of vulnerability transitions and recovery trajectories. It further identifies this as a necessary step for advancing toward operational forecasting.

We respectfully submit that the analytical framework is not positioned as a substitute for empirical development, but as a prerequisite structure. Without an explicit formulation that:

- links inter-event timing (Δt),
- recovery progression $R_{j,i}(t)$,
- and state-dependent fragility.

There is no coherent mechanism to embed empirically derived vulnerability transitions within a multi-hazard damage function.

In this sense, the framework provides:

1. A formal structure within which empirically derived transition functions can be implemented consistently;
2. A reproducible computational environment for testing and comparing alternative transition hypotheses;
3. A bridge between qualitative multi-hazard concepts and quantitative damage modelling.

We agree that, in its current form, the framework is most immediately applicable to:

- forensic reconstruction,
- systematic post-disaster impact analysis,
- and scenario-based sensitivity exploration.

This positioning is now explicitly and reinforced in the Conclusions, where forward-looking applications are described as dependent on further empirical development. We have revised the manuscript to ensure that the claimed contribution aligns strictly with the demonstrated capabilities, avoiding overextension toward predictive operational risk management. The revised text consistently presents the framework as a quantitative integration platform for multi-hazard damage dynamics, rather than as a fully operational predictive system.

Comment 6: Language and Presentation

The manuscript suffers from clarity and communication issues that hinder comprehension of the technical content. The writing style relies on unnecessarily complex sentence structures that obscure rather than illuminate key concepts. Many sentences contain multiple subordinate clauses that could be simplified without losing technical precision.

Additionally, the manuscript exhibits significant redundancy, with core concepts repeated across sections without advancing the argument or providing new information. For example, the distinction between "concurrent and consecutive hazards" is mentioned repeatedly in the abstract, introduction, and methodology sections without substantive development of how the framework addresses each case differently.

The technical exposition would benefit from more precise language and clearer logical flow. Terms like "generalised framework" are used extensively without clear definition of what makes the approach "general" compared to existing methods.

Response 6

We thank the Reviewer for this important comment. We agree that clarity and precision are essential, particularly for a manuscript introducing a quantitative formulation.

In the revised version, we have implemented the following modifications:

Simplification of sentence structure:

Several complex sentences containing multiple subordinate clauses have been rewritten to improve readability and logical flow.

Examples include:

- Introduction: The paragraph describing the contribution has been restructured to clearly separate (i) the conceptual gap, (ii) the proposed formulation, and (iii) the implementation in Python.
- Section 2: The explanation of how concurrent and consecutive dynamics are encoded through parameters Θ and μ has been simplified to avoid nested explanatory clauses.
- Section 2.3: The description of model behaviour under consecutive multi-hazard impact dynamics has been rewritten using shorter declarative sentences, clearly separating assumptions, results, and interpretation.

Reduction of redundancy:

We acknowledge that earlier versions repeated the concurrent/consecutive distinction across sections without adding analytical depth.

In the revised manuscript:

- The conceptual distinction is introduced once in the Introduction (Lines 32–45).
- It is formalised mathematically in Section 2.
- Subsequent references (e.g., Sections 3.1 and 3.2) now focus on how the framework behaves differently under each configuration, rather than restating definitions.

Redundant explanatory passages have been removed to ensure that each section advances the argument rather than repeating definitions.

Clarification of the term “generalised framework”:

We agree that the term required clearer justification. In the revised manuscript (Lines 55–67 and Lines 371–376), we clarify that “generalised” refers to:

- The ability of the same formulation to represent:
 - concurrent interactions ($\Theta = 0$),
 - consecutive interactions ($\Theta = 1, \mu = 0$),
 - independent interactions ($\Theta = 1, \mu = 1$)
- The modular integration of:
 - single-hazard vulnerability functions,
 - vector-valued fragility,
 - state-dependent fragility,
 - and alternative recovery trajectories within a unified structure.

We now avoid using “generalised” as a qualitative descriptor and instead tie it explicitly to the mathematical flexibility encoded in Appendix A.

Improved logical flow:

The structure of Section 2 has been streamlined:

Mathematical formulation → Parameter classification → Model behaviour analysis
→ Application to Puerto Rico.

Overall, the revised manuscript has been carefully edited to:

- Reduce syntactic complexity,
- Remove repetitive explanations,
- Clarify terminology,
- Align claims strictly with demonstrated capabilities.

We appreciate the Reviewer's comment, as it significantly improved the clarity and precision of the manuscript.

Comment 7: Recommendations for Improvement

Comment 7.1

The manuscript addresses an important problem in disaster risk science, but it requires substantial revision to align claims with demonstrated capabilities. The authors should consider reframing their contribution more modestly and honestly. Rather than claiming a key contribution in operational multi-hazard assessment, they could position their work as a systematic methodology for post-disaster impact assessment or as a research template for understanding multi-hazard interactions in well-documented cases.

Response 7.1

We thank the Reviewer for this constructive recommendation. We agree that the framing of the contribution must align strictly with the demonstrated analytical capabilities. In response, and in line with our previous clarifications (Response 1), the manuscript has been systematically revised to avoid overstatement toward operational multi-hazard risk assessment and to position the contribution more precisely as an analytical and forensic template.

In particular, the revised manuscript now clearly presents the main aim and novelty of the work as the translation of previously conceptual and graphical representations of multi-hazard dynamics (e.g., de Ruiter et al., 2020; De Angeli et al., 2022) into an explicit and reproducible analytical structure, fully implemented in modular Python code. The focus is placed on structural integration of multi-hazard damage dynamics, covering concurrent, consecutive, and independent interactions, rather than on generalised operational risk forecasting.

Key revisions include:

Reframing of the core contribution:

- Introduction: The contribution is now explicitly described as the translation of conceptual multi-hazard dynamics into a quantitative analytical structure and its implementation in a modular computational framework in Python. The emphasis is placed on structural integration rather than on operational risk forecasting.
- Section 2.3 (Model behaviour under consecutive multi-hazard impact dynamics) explicitly states that the objective is exploratory parameter influence analysis rather than predictive modelling.

Clarification of intended application domain:

- Puerto Rico case study (Lines 327–337; 354–369): The empirical application is clearly presented as a forensic reconstruction and systematic evaluation of vulnerability modification scenarios, rather than as a real-time operational tool.
- Discussion (Lines 408–415): The manuscript now explicitly states that forward-looking operational deployment requires empirically grounded recovery trajectories and vulnerability transition models, which are identified as future research needs.
- Conclusions (Lines 542–556): The final section emphasises that the framework provides a structured research template for analysing multi-hazard damage accumulation and supports systematic post-disaster assessment and scenario-based exploration.

Practical utility and complementary uses:

Regarding practical utility, as detailed in Response 1, the Discussion section has been expanded to clarify that the framework has multiple complementary uses, including multi-hazard damage formulation, forensic reconstruction, calibration of state-dependent vulnerability, and scenario-based prospective analyses.

Alignment between claims and demonstrated capabilities:

The revised manuscript no longer presents the framework as a ready-to-deploy operational risk management instrument. Instead, it positions the contribution as:

- A formalised multi-hazard damage accumulation structure.
- A reproducible Python-based computational platform enabling consistent integration of recovery and state-dependent vulnerability.
- A methodological basis for forensic reconstruction and structured scenario analysis.

This reframing directly addresses the Reviewer's concern regarding misalignment between claimed and demonstrated contributions. We appreciate this recommendation, as it led to a clearer and more scientifically grounded positioning of the manuscript. The revised version now presents the framework as a quantitative analytical template and structural integration tool, with predictive operational deployment identified as contingent upon further empirical development.

Comment 7.2

The framework has genuine value as a foundation for systematic vulnerability state tracking and post-disaster learning, but the authors should acknowledge its current limitations for predictive applications. Future work should focus on developing the empirical foundations needed to make such a framework operationally useful, including physics-based vulnerability transition models and standardized recovery parameters that can be estimated without extensive post-event data.

Response 7.2

We thank the Reviewer for this constructive observation. In the revised manuscript, we explicitly acknowledge the current limitations of the framework for predictive, forward-looking applications. In particular:

- The Discussion section now clearly states that predictive deployment requires empirically grounded recovery trajectories and vulnerability transition functions.
- The Conclusions emphasise that the present contribution provides a structural analytical framework and a research template for systematic vulnerability state tracking and forensic analysis, while identifying the development of empirically derived transition models and standardised recovery parameters as necessary future research directions.

The manuscript, therefore, explicitly distinguishes between the analytical structure proposed here and the empirical developments required for fully operational predictive use.

Comment 7.3

The language and presentation issues require comprehensive editing to improve clarity and eliminate redundancy.

Response 7.3

We agree with the Reviewer that clarity and conciseness are essential. The revised manuscript has undergone comprehensive editing to:

- Simplify sentence structure throughout the Introduction and Section 2;
- Remove repetitions of the concurrent/consecutive distinction;
- Clarify the definition and use of terms such as “generalised framework”;
- Improve logical flow between formulation, model behaviour analysis, and empirical application.

Redundant explanatory passages have been removed, and key contributions are now presented in a more structured and linear manner. We believe the revised version significantly improves readability and precision without reducing technical depth.

Minor comments

L30: "hazard" in brackets?

Response: The brackets around “hazard” have been removed for clarity (Line 30). The terminology is now consistent throughout the manuscript.

L 48: "underlined" seems to be a word choice error. Consider "highlighted", "identified", "outlined", "emphasized".

Response: The term “underlined” has been replaced.

Line 105: "occurs integrally at the beginning" is unclear - suggest "occurs entirely at the beginning" or "occurs instantaneously at the beginning" to clarify that all damage happens at once rather than gradually.

Response: The sentence has been revised.

L 110: To my knowledge, figures should be referenced in sequential order (Fig. 1, then Fig. 2, then Fig. 3, etc.) as they appear in the text. Fig. 3 is introduced before Fig. 1 and 2.

Response: Figures 1 and 2, as numbered in the original manuscript, have been removed following the simplification of the section related to the mathematical development of the model. In the revised manuscript, figure references have been reordered to ensure their sequential appearance in the text (Fig. 1 → Fig. 2 → Fig. 3). The numbering and corresponding in-text citations have been corrected accordingly.

L 118, 128: Avoid describing figure elements by color ("blue line", "green line"). Use descriptive labels instead for accessibility and clarity (e.g., "the horizontal line representing the response phase"). Related and for figures in general: Ensure all figure elements are clearly labeled in the legend. Figure 3 legend: "The second event in temporal order" is redundant - "second event" already implies temporal sequence. Suggest removing "in temporal order" throughout.

Response: References to figure elements by color have been replaced with descriptive labels (e.g., "the horizontal response phase segment"). Legends have been revised to ensure all elements are clearly labeled. The phrase "in temporal order" has been removed throughout, including in the Figure 3 legend.

L 179: Remove "fortunate" - scientific writing should avoid value judgments.

Response: The word "fortunate" has been removed to maintain a neutral scientific tone.

L 279: Typo in "build" back better.

Response: The typo has been corrected.

L 407, Table 2: The values in Table 2 seem to be in USD, not "thousand USD". Please double check. Also check the formatting of the values in Table 2. Table 2/Figure 7: Both show the same results. Consider moving one of the two elements to the supplementary material.

Response:

- Units have been corrected to USD (not thousand USD).
- Formatting of numerical values has been standardised.
- The detailed numerical matrix previously presented in Table 2 has been moved to the Appendix (Appendix B), and the main text now retains only the graphical representation to avoid redundancy.

L 408: Line X: The damage-to-loss conversion methodology needs brief explanation rather than just a citation. Readers should understand the key assumptions without consulting external sources.

Response: A brief methodological explanation has been added (Lines 354–361), clarifying that simulated damage state probabilities are converted into economic loss using Hazus-MH loss ratios, defined as repair cost percentages of replacement value.

L 409: "Predefined loss ratios" needs clarification - predefined by whom, based on what data, and are they appropriate for Puerto Rico conditions? Specify the source and empirical basis.

Response: The text now, at Lines 362-369, specifies that loss ratios are defined within the FEMA Hazus-MH Technical Manual, based on historical U.S. claims data and post-disaster reconstruction evidence. The empirical basis and applicability assumptions are now stated.

L 415: The claim of "clear non-linear trend" needs quantitative support. How was non-linearity assessed? Provide statistical analysis (R^2 , trend coefficients) rather than visual inspection and isolated examples.

Response: The qualitative statement has been replaced with a quantitative description.

L 421/423: "1,700 thousand" "1,742 thousand" is confusing and not scientific. Replace with scientific notation or standard units.

Response: All monetary values are now expressed in standard USD format (e.g., 1,700,000 USD). Scientific notation has been avoided for readability, and formatting has been standardised throughout.

Throughout: Use standard terminology "slow-onset hazards" rather than "long-onset hazards" to align with established disaster risk literature (e.g., UNDRR, IPCC terminology).

Response: The term "long-onset hazards" has been replaced with the standard terminology "slow-onset hazards" throughout the manuscript, in alignment with established disaster risk literature.

REVIEWER 2

Comment 1

First of all, I think that the term 'mathematical framework' feels a bit of a stretch. They are somewhat simple equations that are merely a mathematical formulation of the recovery curves as presented by De Ruiter et al. (2019) in the paper "Why we can no longer ignore consecutive disasters". As such (and also been mentioned in the abstract) I think the formulas that are presented are a generalized formulation, rather than a mathematical framework.

Response 1

We thank the Reviewer for this observation. We agree that referring to the approach as a "mathematical framework" could overstate its scope. In the revised manuscript, we have clarified this point and refined the positioning of the contribution. In particular, we now explicitly state in the text that the proposed formulation is directly based on previously published qualitative and graphical models of multi-hazard damage dynamics, such as those by de Ruiter et al. (2020) and De Angeli et al. (2022):

"This paper aims to bridge this gap by providing a structured quantitative methodology that translates conceptual representations of multi-hazard damage dynamics into an explicit and reproducible analytical structure. To this end, compound and consecutive impact dynamics described graphically and schematically by de Ruiter et al. (2020) and De Angeli et al. (2022) are reformulated into a compact mathematical expression and implemented as a modular Python code. This formulation enables explicit representation of inter-event time intervals, state-dependent vulnerability interactions, and recovery trajectories within a unified structure."

Accordingly, the manuscript now clearly presents the proposed approach as a multi-hazard damage formulation integrating concurrent, consecutive, and independent hazard interactions within a unified analytical structure, rather than as a mathematical framework. In line with this clarification, as anticipated at page 1 (Refined scientific positioning and revised title):

- The description of the mathematical formulation has been significantly shortened;
- The Abstract, Introduction, Discussion, and Conclusion sections have been updated to reflect this revised positioning of the contribution;
- The title has been revised accordingly to emphasise the analytical and forensic nature of the work.

These revisions ensure that the manuscript accurately communicates the scope and novelty of the approach as a generalized, reproducible analytical formulation, rather than implying a fully general mathematical framework.

Comment 2

And I think that also links to the lack of novelty of this study. While nicely written up, what is actually new in this formulation? Is the novelty that not too many paper wrote this up in this way? Perhaps. But then I would still expect much more clear examples with a clear time dimension included that shows how this really works. Or is the key element of this work the code behind this study? But would it perhaps not have been better to submit this to a journal like Journal for Open Source Software (JOSS), to emphasize the open source availability of the modelling framework, instead of pushing this into “a new mathematical framework”. I do also like this random event generator in the github repo. Can indeed be nice to play around with stress testing assets in a specific area in a multi-hazard concept. But I don't think I read that really in the paper?

Response 2

We thank the Reviewer for this thoughtful and constructive comment. We recognise the importance of clearly distinguishing between conceptual translation, analytical formulation, and software implementation. In the revised manuscript, we have clarified the contribution and strengthened the connection between the mathematical formulation, its time dimension, and the empirical application.

Clarification of novelty:

The novelty of the proposed approach lies in:

- Formulating a generalised piecewise damage accumulation function that:
 - explicitly integrates inter-event timing,
 - recovery progression,
 - and state-dependent fragility within a single structure;
- Encoding concurrent, consecutive, and independent configurations through parameters Θ and μ (Lines 104–130; Appendix A);
- Implementing this structure in a reproducible computational environment enables systematic parameter exploration (Lines 165–180).

This positioning has been clarified in the Introduction and in the Discussion, where we explicitly state that the contribution lies in structural integration rather than in proposing new empirical vulnerability models.

Strengthening of the time dimension:

In response to the Reviewer's suggestion for clearer temporal illustration:

- Section 2.3 “Model behaviour under consecutive multi-hazard impact dynamics” now explicitly analyses how cumulative loss evolves as a function of inter-event time interval $\Delta(t)$ under different recovery trajectories.
- Figure n.2 directly visualises the role of recovery timing in controlling damage amplification.
- The Puerto Rico case study (Section 3.2) operationalises the time dimension by explicitly modelling residual damage persistence between the 2017 hurricane and the 2020 earthquake.

We believe these additions make the temporal mechanics of the formulation more explicit and directly observable.

Role of the code and journal scope:

We appreciate the suggestion regarding JOSS. While the code is openly available and modular, the primary contribution of the manuscript is not the software architecture itself, but the formal integration of multi-hazard interaction logic within a damage accumulation formulation.

The GitHub repository provides:

- Implementation of the formulation;
- Modular recovery functions;
- State-dependent fragility modification tools;
- A random event generator for scenario exploration.

The manuscript now more clearly explains the computational structure but deliberately avoids centring the contribution on software engineering aspects. For this reason, we consider Natural Hazards and Earth System Sciences an appropriate venue, as the focus is on damage modelling methodology rather than code publication per se. We agree that the random event generator could be more explicitly mentioned. In the revised manuscript, we now clarify that the tool enables synthetic event sequence generation for stress-testing multi-hazard scenarios.

In summary, the revised manuscript clarifies that:

- The contribution lies in formalising and integrating multi-hazard damage accumulation mechanisms within a unified analytical structure.
- The time dimension is explicitly operationalised in both the model behaviour analysis and the case study.
- The code supports reproducibility and experimentation, but is not the sole or primary contribution.

Comment 3

Now, from the Puerto Rico case study example, it is really not clear how actually the recovery response is modelled. It is also highlighted in the discussion that many unknowns are still there with respect to the recovery dynamics. But how are those recovery dynamics actually incorporated in the modelling as presented here. I see it from the mathematical formulations, but not in the application? There does not seem to be a time dimension included, but just an implementation on the fragility curves in a multi-hazard setting?

Response 3

We thank the Reviewer for this important clarification. The recovery response is explicitly included in the general formulation through the recovery function, which determines the residual damage state at the onset of the second event. In the analytical formulation, recovery governs the level of structural integrity remaining at time, and this residual state directly modifies the fragility function of the subsequent hazard. In the Puerto Rico application, recovery is not reconstructed as a continuous time-evolving curve due to the absence of spatially resolved building-level recovery

data between 2017 and 2020. Instead, recovery is operationalised through its effect on residual structural capacity at the time of the earthquake.

More specifically:

- The time gap between Hurricane Maria (2017) and the seismic sequence (2020) defines the recovery window.
- Since no detailed recovery trajectory data were available, we represented the system at discrete recovery states at the time of the earthquake.
- These recovery states are implemented through systematic shifts of the median values of the structural and non-structural fragility curves (5–30%), representing different plausible levels of incomplete restoration.

In this sense, recovery is not ignored in the case study; rather, it is embedded implicitly in the residual vulnerability state at the time of the second event. The fragility curve modification is therefore not independent from recovery, but represents the structural consequence of incomplete recovery at the earthquake onset. We acknowledge that the application does not simulate a fully time-resolved recovery trajectory between 2017 and 2020. Instead, it evaluates the system at alternative residual capacity levels corresponding to different recovery scenarios. This modelling choice reflects data constraints rather than a limitation of the analytical formulation itself. We have clarified this distinction in the revised manuscript to avoid the impression that recovery is omitted in the empirical application.

Comment 4

The code that is presented behind this paper looks nice, and is cleanly written up. There I can see that (I think) in `run_framework.py` that the recovery curve indeed determines the starting state of the assets when the new hazard hits. However, this is not really clear from the Puerto Rico example.

With reference to the fragility curves, because there is no clear implementation of this recovery aspect in the application, there does not seem to be much novelty in the fragility curves. It is almost a directly implementation from HAZUS?

Response 4

We thank the Reviewer for carefully examining the code implementation and for highlighting this important clarification point. This comment led us to make the recovery–fragility linkage in the Puerto Rico example more explicit and to better distinguish between the empirical input models and the underlying structural multi-hazard formulation.

More in detail, we have made the following modifications to the manuscript:

Clarification of recovery implementation in the Puerto Rico case study:

To address the lack of clarity regarding how recovery is incorporated in the empirical application:

- In the Seismic impacts section (Lines 340–354), we have explicitly stated that the recovery function in the computational framework determines the residual damage state at the onset of the second hazard.

- We clarified that, in the Puerto Rico application, due to the absence of time-resolved building-level recovery data between 2017 and 2020, this residual state is represented through systematic fragility median modifications.
- The revised text now explicitly states that fragility modification is a proxy representation of incomplete recovery at the time of the earthquake, rather than an independent modelling adjustment.

This addition directly links the recovery logic implemented in the code (as observed in `run_framework.py`) with its operational representation in the case study.

Explicit clarification regarding Hazus fragility curves:

To avoid any ambiguity regarding novelty:

- In the same section, we now explicitly state that the baseline fragility curves are directly derived from FEMA Hazus.
- We added a clarifying sentence specifying that no new empirical fragility models are proposed in this study.

Clear distinction between empirical curves and structural contribution:

In the Discussion section we clarified that the novelty does not lie in the empirical fragility curves themselves, but in:

- their conditional adaptation within a consecutive multi-hazard configuration,
- and their integration within a unified damage accumulation formulation that explicitly accounts for recovery-mediated residual states.

This ensures that the manuscript does not present the fragility curves as novel, but rather emphasises the dynamic integration logic.

Comment 5

And to continue on this crucial point of the time component, here those non-physical asset damages are really becoming a key element. Dynamic modelling of the recovery process (which is a key element of the mathematical framework as presented here) goes really beyond the physical asset damages. And specially beyond what the example now shows with the “simple” multi-hazard or the modification of the fragility curves with different state dependencies.

I think many of my points are also summed in section 4.1 and section 4.2. For example, I would expect that a paper with this title would show mathematical formulations that move away from this simplification. This also links to the key points mentioned in the “Better understanding and modelling recovery dynamics” and “Evaluation dynamic exposure over time”. I understand that the equations as presented in this paper could perhaps provide a starting point to all of this, but I feel like some of these elements should be included already to warrant publication on a leading journal in the field, such as NHESS.

Response 5

We agree that dynamic recovery modelling extends beyond structural damage and includes broader socioeconomic and non-physical dimensions. The present study

focuses specifically on the physical damage accumulation component, while acknowledging that full recovery modelling involves additional layers of complexity.

In response to this comment, we have clarified the scope and positioning of the manuscript in the following ways.

Clear delimitation of modelling scope:

- In the Introduction we now explicitly state that the formulation addresses time-dependent physical damage accumulation, rather than modelling the full socioeconomic recovery process.
- The distinction between structural recovery representation and broader recovery dynamics has been clarified to avoid overinterpretation of the framework's scope.

Explicit limitation regarding dynamic exposure and non-physical losses:

- In the Discussion section we strengthened the wording to clarify that dynamic exposure evolution and non-physical recovery processes are not explicitly modelled in the present application.
- The sections previously titled "Better understanding and modelling recovery dynamics" and "Evaluation dynamic exposure over time" now clearly identify these aspects as future research directions rather than implied capabilities of the current implementation.

Clarification of what the formulation actually provides:

- In the Model behaviour section (Section 2.3), we emphasise that the dynamic component currently captured is the effect of recovery on residual structural capacity and subsequent vulnerability.
- In the Puerto Rico application (Lines 340–354), we clarified that the empirical example represents a residual-state assessment at the time of the second hazard, not a full dynamic reconstruction of recovery over time.

We agree that comprehensive modelling of dynamic exposure and non-physical recovery would represent a substantial advancement. However, we respectfully submit that providing a formalised and implementable structure for tracking recovery-controlled residual vulnerability within multi-hazard sequences constitutes a relevant and necessary step toward such developments.

The revised manuscript now more clearly distinguishes between the structural damage accumulation mechanisms addressed here and the broader dynamic system recovery modelling identified as future research. We appreciate the Reviewer's comment, as it has helped refine the scope and positioning of the manuscript to better align demonstrated capabilities with stated contributions.

Comment 6

So to conclude: I think the paper is published with the idea to publish the python code. I do not think that the theory presented in this paper (and the case study

results) are very exciting by themselves, and are not necessarily very novel. However, the code that is presented contains a bunch of nice elements, that I have not really seen in, for example, the DamageScanner or Delft-FIAT. There might be elements of this code in CLIMADA, but that one is sometimes a bit hard to understand what's all in there. As such, I propose to rather focus on writing an article, to a suitable journal, that mostly focuses on the publication of the code.

Response 6

We thank the Reviewer for this comment and for highlighting the practical value of the Python code. We agree that the code itself provides a flexible and modular tool for multi-hazard damage assessment.

At the same time, we respectfully highlight that the main scientific contribution of the manuscript lies in the quantitative formalization of previously conceptual multi-hazard dynamics. The Python implementation supports and operationalizes this approach, allowing systematic exploration of key parameters, but the scientific contribution remains in the analytical methodology and its ability to bridge qualitative multi-hazard concepts with a reproducible quantitative structure.

While we appreciate the suggestion to submit to a software-focused journal, we believe the manuscript's contribution aligns with the scope of NHESS because it addresses a core methodological gap in multi-hazard damage modelling, namely the explicit representation of recovery dynamics and state-dependent vulnerability within consecutive hazard sequences. The Python code supports this methodological contribution by making the formulation operational and reproducible, but the central focus of the paper remains on the damage accumulation logic and its structural implications for multi-hazard assessment. This is not excluding the possibility of publishing the code separately in a future software-focused paper, but this is not the objective of the current manuscript.