

REVIEWER 2

COMMENT 1

First of all, I think that the term ‘mathematical framework’ feels a bit of a stretch. They are somewhat simple equations that are merely a mathematical formulation of the recovery curves as presented by De Ruiter et al. (2019) in the paper “Why we can no longer ignore consecutive disasters”. As such (and also been mentioned in the abstract) I think the formulas that are presented are a generalized formulation, rather than a mathematical framework.

ANSWER 1

We agree that the term “mathematical framework” may overstate the level of theoretical innovation. The equations we present are indeed **intentionally simple** and **designed to generalize and integrate concepts from existing literature**, including recovery dynamics and multi-hazard vulnerability.

In the revised manuscript, we will therefore:

- reduce the emphasis on the formal mathematical component,
- clarify that the role of the theoretical formulation is to provide a transparent and implementable backbone for the Python tool, and
- replace the term “mathematical framework” with “Python-based modelling tool” or similar wording, both in the abstract and throughout the manuscript.

This change in terminology and emphasis will better reflect the actual contribution of the paper, which is to make these concepts operational and reproducible in a coherent multi-hazard modelling environment, rather than to introduce a fundamentally new mathematical theory.

COMMENT 2

And I think that also links to the lack of novelty of this study. While nicely written up, what is actually new in this formulation? Is the novelty that not too many paper wrote this up in this way? Perhaps. But then I would still expect much more clear examples with a clear time dimension included that shows how this really works. Or is the key element of this work the code behind this study? But would it perhaps not have been better to submit this to a journal like Journal for Open Source Software (JOSS), to emphasize the open source availability of the modelling framework, instead of pushing this into “a new mathematical framework”. I do also like this random event

generator in the github repo. Can indeed be nice to play around with stress testing assets in a specific area in a multi-hazard concept. But I don't think I read that really in the paper?

ANSWER 2

We acknowledge that the novelty of our work does not lie primarily in the individual equations, but in the way they are combined into a **unified, operational, and time-dependent modelling tool**. In particular, the tool explicitly links:

- multi-hazard interactions
- recovery dynamics
- multi-hazard vulnerability

within a single, modular Python implementation that supports both retrospective and scenario-based analyses.

In the revised manuscript, we will:

- streamline the presentation of the formalism and focus more on how the model behaves over time under different input conditions, explicitly showing how total damage changes as recovery, exposure, and vulnerability parameters are varied;
- include clearer examples and figures that illustrate the time dimension of the model, including the evolution of damage and residual vulnerability between consecutive events;
- present the **two main modes of application** of the tool:
 - **forensic use** (ex-post analyses based on observed damage and hazard data);
 - **anticipatory/ planning-oriented use** (scenario simulations to explore possible future outcomes).

Regarding the choice of the journal, we agree that the Python implementation and its open-source nature are central to our contribution. However, we also argue that the paper goes beyond the scope of a pure software presentation article, since it provides a methodological contribution to multi-hazard impact assessment, and a real-world case study that demonstrates how this structure can be used for both validation and scenario analysis. For this reason, we believe NHESS remains an appropriate venue.

We appreciate the reviewer's positive comment about the random event generator included in the repository. In the revised manuscript, we will more clearly describe

this component and briefly discuss its potential use for stress-testing assets in multi-hazard contexts, as suggested by the reviewer.

COMMENT 3

Now, from the Puerto Rico case study example, it is really not clear how actually the recovery response is modelled. It is also highlighted in the discussion that many unknowns are still there with respect to the recovery dynamics. But how are those recovery dynamics actually incorporated in the modelling as presented here. I see it from the mathematical formulations, but not in the application? There does not seem to be a time dimension included, but just an implementation on the fragility curves in a multi-hazard setting?

ANSWER 3

We agree that, in the current version of the manuscript, the recovery modelling and its time dynamics in the Puerto Rico application are not sufficiently explained. In the revised manuscript we will:

- provide a clearer description of **how the recovery function $R_j(t)$ is implemented in the Python tool** and how it updates the state of the asset between one event and the next;
- explain that, in principle, the tool allows for a **wide range of recovery trajectories** (linear, exponential, logistic, etc.), with different durations and shapes depending on the asset type and context;
- clarify that, for Puerto Rico, detailed empirical recovery data were not available, and therefore, instead of assuming a single, known recovery trajectory, we **explored different plausible states of residual damage** at the time of the earthquake by modifying the parameters of the fragility curves (e.g., median and dispersion) to represent varying levels of incomplete recovery.

In other words, the time dimension is incorporated through the sequence of events and the evolution of the asset state between them, which in the Puerto Rico case is represented by a set of alternative residual damage scenarios rather than by a fully data-driven recovery curve.

COMMENT 4

The code that is presented behind this paper looks nice, and is cleanly written up. There I can see that (I think) in `run_framework.py` that the recovery curve indeed

determines the starting state of the assets when the new hazard hits. However, this is not really clear from the Puerto Rico example.

With reference to the fragility curves, because there is no clear implementation of this recovery aspect in the application, there does not seem to be much novelty in the fragility curves. It is almost a directly implementation from HAZUS?

ANSWER 4

In the revised manuscript, we will:

- explicitly state that, in the computational implementation, the recovery function (or the assumed residual damage scenario) determines the initial state of the asset at the onset of the second hazard, as the reviewer correctly inferred from the code;
- add a short subsection or paragraph in the methods/application section explicitly linking the key code routines to the conceptual steps (damage from the first event → residual state → updated fragility for the second event).

Regarding the fragility curves, we confirm that the baseline fragility curves are indeed derived from HAZUS for earthquake damage, but for the Puerto Rico case, we systematically modify these curves to represent different levels of residual damage induced by the hurricane (e.g., shifting median values and, where appropriate, changing dispersion), thus **creating state-dependent fragility curves that reflect incomplete recovery**.

We will clarify that the novelty does not lie in proposing entirely new empirical fragility models, but in how standard fragility functions are dynamically adapted and integrated within a consecutive multi-hazard framework, implemented in an open and reusable way. This is precisely the type of operational integration that the tool is designed to support.

COMMENT 5

And to continue on this crucial point of the time component, here those non-physical asset damages are really becoming a key element. Dynamic modelling of the recovery process (which is a key element of the mathematical framework as presented here) goes really beyond the physical asset damages. And specially beyond what the example now shows with the “simple” multi-hazard or the modification of the fragility curves with different state dependencies.

I think many of my points are also summed in section 4.1 and section 4.2. For example, I would expect that a paper with this title would show mathematical formulations that move away from this simplification. This also links to the key points mentioned in the “Better understanding and modelling recovery dynamics” and “Evaluation dynamic exposure over time”. I understand that the equations as presented in this paper could perhaps provide a starting point to all of this, but I feel like some of these elements should be included already to warrant publication on a leading journal in the field, such as NHESS.

ANSWER 5

We fully agree that a comprehensive dynamic modelling of recovery should eventually encompass not only direct physical damage, but also non-physical dimensions such as functionality, indirect economic losses, and socio-economic recovery trajectories. We also agree that this broader scope represents an important direction for future research.

At the same time, we would like to clarify that **the present tool is explicitly focused on direct physical damage to assets**, and uses physical damage (and associated loss) as the primary metric to describe the effect of consecutive and concurrent hazards over time.

In the revised manuscript, we will:

- more clearly delimit this scope in the Introduction and Discussion, stating that indirect impacts and non-physical dimensions of recovery are outside the current implementation and are treated as future developments;
- strengthen Sections 4.1 and 4.2 by explicitly linking them to the reviewer’s remark, emphasizing that detailed, empirically based recovery models, dynamic exposure models, and functionality-based impact metrics are essential next steps to build on the foundations provided by our current tool.

We will also underline that we intend to offer a starting point, a flexible, physically grounded tool that can later be extended to non-physical damages and more complex recovery representations, as more detailed data and models become available.

COMMENT 6

So to conclude: I think the paper is published with the idea to publish the python code. I do not think that the theory presented in this paper (and the case study

results) are very exciting by themselves, and are not necessarily very novel. However, the code that is presented contains a bunch of nice elements, that I have not really seen in, for example, the DamageScanner or Delft-FIAT. There might be elements of this code in CLIMADA, but that one is sometimes a bit hard to understand what's all in there. As such, I propose to rather focus on writing an article, to a suitable journal, that mostly focuses on the publication of the code.

ANSWER 6

We sincerely appreciate the reviewer's positive assessment of the code and the recognition that it contains useful elements not readily available in other tools. We also understand the concern that the paper might be perceived as primarily motivated by the desire to publish the code.

Our intention, however, is to combine:

- a **generalized, time-dependent conceptual structure** for multi-hazard physical damage;
- a **transparent and modular Python** implementation;
- a **real-world case study** that demonstrates how the tool can be used for both forensic validation and scenario-based exploration.

In the revised manuscript, we will:

- shift some of the lower-level implementation details to supplementary material or code documentation;
- keep the main text focused on **methodological insights** arising from the simulations (e.g., underestimation of damage when residual damage is neglected, sensitivity of results to assumed residual states);
- clearly articulate how the tool advances multi-hazard impact assessment practice, rather than being solely a software release.

We agree that, in principle, a separate software-focused article (e.g., in JOSS or GMD) could complement this work. However, we believe that the combination of conceptual formulation, operational implementation, and case-study-based methodological discussion makes this manuscript suitable for NHESS, whose scope includes methodological advances in natural hazard and risk assessment supported by numerical tools.

Finally, we will explicitly highlight the **open-source and modular structure** of the code as a key asset for scientific replicability and extensibility to other hazards, asset

types, and regions. This, we hope, will underline that the code is not an accessory component, but an integral part of a broader methodological contribution.