

REVIEWER 1

Major Concerns

Core Contribution and Scientific Value

My primary concern relates to the fundamental claims about the manuscript's contribution and its practical utility. The authors position their work as providing a "generalised mathematical framework" for multi-hazard risk assessment, but this claim requires careful examination.

Mathematical Formalization: The mathematical framework presented is essentially a straightforward extension of existing single-hazard approaches, where total damage becomes a function of multiple hazards and recovery states. While the authors present this as novel, the mathematical formulation represents a straightforward extension. The real challenges in multi-hazard assessment lie not in the mathematical abstraction but in the empirical quantification of recovery functions, hazard dependencies, and vulnerability transitions over time - as the authors rightly state in sections 4.1 and 4.2.

ANSWER

We acknowledge that the mathematical framework represents an extension of existing single-hazard formulations, and we agree that its novelty lies less in the mathematical abstraction itself and much more in its **capacity to integrate multiple elements usually treated separately** (multi-hazard interactions, recovery dynamics, and multi-hazard vulnerability), and its **practical application, through the development of a dedicated Python code**.

In the revised manuscript, we will therefore reduce the emphasis on the formal mathematical aspects and clarify that the theoretical structure primarily serves as the foundation for a computational tool implemented in Python. To better reflect this focus, we will replace the term "*mathematical framework*" with "*Python-based modelling tool*" (or a similar term, as appropriate).

By strengthening these aspects, the revised manuscript will more accurately convey the originality of our contribution. It will position our work not as a purely theoretical and mathematical development, but as a practically

oriented, Python-based tool that operationalizes the relationships among multi-hazard overlaps, vulnerability dynamics, and recovery processes.

Operational Utility Limitations: More critically, the framework suffers from a fundamental limitation that severely constrains its practical applicability. The Puerto Rico case study illustrates this problem clearly. The authors demonstrate how Hurricane Maria affected earthquake vulnerability by using post-hurricane damage data to calibrate vulnerability adjustments. However, this approach is inherently backward-looking and requires empirical damage data that would not be available for forward-looking risk assessments.

ANSWER

We agree with the reviewer that the Puerto Rico case study mainly demonstrates a retrospective (ex-post) application based on post-event damage data. Nevertheless, we think that the proposed model can also be applied for anticipatory/ planning-oriented analyses.

In the revised version of the manuscript, we will explicitly introduce two modes of application within the same Python-based tool:

- **Forensic use:** ex-post analysis of consecutive events, where empirical data on damage and vulnerability are available. The Puerto Rico case study falls under this category and serves as a real-world example where we can compare two sets of damage curves and economic loss estimates, validating how earthquake fragility evolves after hurricane-induced damage. This demonstrates that the tool can reproduce physically observed effects without relying on the invention of new fragility curves.
- **Anticipatory/ planning-oriented use:** we will streamline the description of the mathematical framework and focus more on demonstrating how the model “behaves” under varying input parameters, specifically illustrating how total damage evolves as recovery, exposure, and vulnerability parameters change. This forward-looking approach enables sensitivity analyses that are valuable for preventive planning and the assessment of mitigation strategies, thereby better supporting anticipatory applications of the model.

To improve clarity, we will add a schematic figure summarizing these two operational pathways (forensic and anticipatory) within the same framework, showing how users can transition from retrospective validation to forward-looking assessments.

Consider a practical scenario: if a hurricane were to strike Puerto Rico tomorrow, practitioners using this framework would need to wait for post-hurricane damage assessments before they could adjust earthquake vulnerability functions. This severely limits the framework's utility for operational risk management, emergency planning, or prospective risk assessment. The authors do not adequately address how practitioners would estimate vulnerability changes in real-time or predictive applications without extensive post-event calibration data.

ANSWER

We would like to clarify that the objective of the Puerto Rico case was not to calibrate recovery or vulnerability functions for “predictive” purposes, but to use a real-world dataset where observed damages were available to validate the internal behaviour of the model. This forensic validation demonstrates that the tool can reproduce physically consistent variations in fragility following sequential hazards, without introducing arbitrary new curves.

In the revised manuscript, we will more clearly present the Puerto Rico case as a validation of the model's forensic capabilities. We will also clarify how, beyond this validation, the same framework can support forward-looking (i.e., anticipatory/ planning-oriented) analyses by simulating future scenarios through parameterized adjustments of recovery, exposure, and vulnerability functions. This will help illustrate the tool's relevance both for retrospective studies of past events and for forward-looking applications in disaster risk management.

Misalignment of Claimed vs. Actual Contributions:

The manuscript lists several important developments as "limitations" (section 4.1) or "future research directions" (section 4.2) that would actually constitute the real scientific advances needed in multi-hazard assessment. These include empirically-derived vulnerability transition functions, standardized recovery curves based on extensive post-disaster data, and predictive models for hazard-induced vulnerability changes. Only after solving these challenges would a general mathematical framework provide meaningful operational value.

Specifically, I was excited to see the authors propose an approach to adjust vulnerability curves to account for pre-existing damage conditions (lines 394-396). Such an approach represents a potentially valuable contribution to the field. However, the implementation still relies entirely on post-event calibration data, making it neither operational for forward-looking assessments nor generalizable across different contexts without extensive empirical datasets.

In conclusion, current framework appears most suited for retrospective analysis and systematic post-disaster impact assessment rather than the forward-looking risk management applications that the authors suggest. This represents a significant gap between the claimed contribution and the demonstrated capabilities.

ANSWER

We fully agree that empirically-derived vulnerability transition functions, standardized recovery curves, and predictive models for hazard-induced vulnerability changes are critical for achieving fully operational, forward-looking applications. These represent important avenues for future research and development, and we have clearly marked them as limitations or future directions in Sections 4.1 and 4.2.

We would like to clarify that the primary objective of the current work is not to deliver a fully operational anticipatory/ planning-oriented tool, but rather to provide a flexible, Python-based framework capable of integrating multi-hazard, recovery, and dynamic vulnerability components in a consistent, modular way.

In the revised manuscript, we will:

1. Emphasize that the framework is intended primarily for retrospective validation and sensitivity analyses rather than real-time operational deployment.
2. Highlight the novelty and added value of the tool in integrating components often treated separately in the literature (multi-hazard, recovery, dynamic vulnerability) and providing a Python-based tool that can support both forensic validation and anticipatory scenario exploration.

This clarification will help align the presentation of the framework with its actual demonstrated capabilities, while still pointing to its potential for future improvements.

Language and Presentation

The manuscript suffers from clarity and communication issues that hinder comprehension of the technical content. The writing style relies on unnecessarily complex sentence structures that obscure rather than illuminate key concepts. Many sentences contain multiple subordinate clauses that could be simplified without losing technical precision.

ANSWER

We will revise the whole manuscript and simplify the sentence structure, avoiding as much as possible the use of subordinate clauses.

Additionally, the manuscript exhibits significant redundancy, with core concepts repeated across sections without advancing the argument or providing new information. For example, the distinction between "concurrent and consecutive hazards" is mentioned repeatedly in the abstract, introduction, and methodology sections without substantive development of how the framework addresses each case differently.

ANSWER

We will significantly shorten the part devoted to the presentation of the mathematical framework, to dedicate more space to presenting how the model "behaves". This will help in reducing redundancy and concept repetition.

The technical exposition would benefit from more precise language and clearer logical flow. Terms like "generalised framework" are used extensively without clear definition of what makes the approach "general" compared to existing methods.

ANSWER

We will revise the language and ensure coherence and precision of the applied terminology. Moreover, as anticipated, we will avoid using the term "mathematical framework" and rather refer to a "Python-based tool" or similar.

Recommendations for Improvement

The manuscript addresses an important problem in disaster risk science, but it requires substantial revision to align claims with demonstrated capabilities. The authors should consider reframing their contribution more modestly and honestly. Rather than claiming a key contribution in operational multi-hazard assessment, they could position their work as a systematic methodology for post-disaster impact assessment or as a research template for understanding multi-hazard interactions in well-documented cases.

ANSWER

We thank the reviewer for this suggestion and agree that the manuscript should better align its claims with demonstrated capabilities. In the revised version, we will place **less emphasis on the mathematical formalization** and instead **highlight the Python-based framework** and its two complementary modes of application: (i) forensic and (ii) anticipatory/ planning-oriented.

This revised framing clarifies that the framework's **operational value lies in structured forensic analyses and scenario-based explorations**, while not implying real-time predictive applicability, and highlights the practical benefits of its Python-based implementation.

The framework has genuine value as a foundation for systematic vulnerability state tracking and post-disaster learning, but the authors should acknowledge its current limitations for predictive applications. Future work should focus on developing the empirical foundations needed to make such a framework operationally useful, including physics-based vulnerability transition models and standardized recovery parameters that can be estimated without extensive post-event data.

ANSWER

We fully agree that the framework is not applicable for real-time predictions. In the revised manuscript, we will explicitly acknowledge these limitations and clarify that the framework is intended primarily for forensic analyses and scenario-based forward-looking assessments. This reframing will ensure an honest representation of the tool's current capabilities while highlighting avenues for future advancement, including the development of a stronger empirical foundation for operational use, such as physics-based vulnerability

transition models and standardized recovery parameters that can be applied without relying on extensive post-event datasets. These aspects are already discussed in the “Future Developments” section of the manuscript.

The language and presentation issues require comprehensive editing to improve clarity and eliminate redundancy.

ANSWER

As anticipated, we will review the whole manuscript, simplifying the sentences, improving the coherence and precision of the adopted terminology, and avoiding unnecessary repetitions.

Minor comments

L30: "hazard" in brackets?

L 48: "underlined" seems to be a word choice error. Consider "highlighted", "identified", "outlined", "emphasized".

Line 105: "occurs integrally at the beginning" is unclear - suggest "occurs entirely at the beginning" or "occurs instantaneously at the beginning" to clarify that all damage happens at once rather than gradually.

L 110: To my knowledge, figures should be referenced in sequential order (Fig. 1, then Fig. 2, then Fig. 3, etc.) as they appear in the text. Fig. 3 is introduced before Fig. 1 and 2.

L 118, 128: Avoid describing figure elements by color ("blue line", "green line"). Use descriptive labels instead for accessibility and clarity (e.g., "the horizontal line representing the response phase"). Related and for figures in general: Ensure all figure elements are clearly labeled in the legend. Figure 3 legend: "The second event in temporal order" is redundant - "second event" already implies temporal sequence. Suggest removing "in temporal order" throughout.

L 179: Remove "fortunate" - scientific writing should avoid value judgments.

L 279: Typo in "build" back better

L 407, Table 2: The values in Table 2 seem to be in USD, not "thousand USD". Please double check. Also check the formatting of the values in Table 2. Table 2/Figure 7: Both show the same results. Consider moving one of the two elements to the supplementary material. L 408: Line X: The damage-to-loss conversion methodology needs brief explanation rather than just a citation. Readers should understand the key assumptions without consulting external sources.

L 409: "Predefined loss ratios" needs clarification - predefined by whom, based on what data, and are they appropriate for Puerto Rico conditions? Specify the source and empirical basis.

L 415: The claim of "clear non-linear trend" needs quantitative support. How was non-linearity assessed? Provide statistical analysis (R^2 , trend coefficients) rather than visual inspection and isolated examples.

L 421/423: "1,700 thousand" "1,742 thousand" is confusing and not scientific. Replace with scientific notation or standard units.

Throughout: Use standard terminology "slow-onset hazards" rather than "long-onset hazards" to align with established disaster risk literature (e.g., UNDRR, IPCC terminology).

ANSWER

We will carefully revise all these minor comments and integrate them into the revised version of the manuscript.