

Modelling diffusion, decay and ingrowth of U–Pb isotopes in zircon

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Abstract. Understanding the thermal evolution of geological terranes provides essential insights into tectonic processes, crustal evolution, and mineral resource formation. Zircon U–Pb geochronology is widely used to date geological events, yet these dates are altered by a wide-range of processes, including diffusion of radiogenic isotopes at high (>800°C) temperatures. This study utilises the *underworld3* numerical code to couple diffusion processes with radioactive decay and ingrowth in two-
5 dimensions. We assess the numerical solutions against a series of benchmarks to test the implementation, and apply the models to examine lead-loss due to thermal events and complexities that arise from multiple zircon growth episodes. Our approach bridges analytical U–Pb isotope measurements with a diffusion-decay-ingrowth numerical model, providing insights into how the thermal evolution of a region alters zircon U–Pb isotope ratios. We apply the methodology to the Trivandrum block in southern India, a region characterised by a prolonged high-temperature event, comparing multiple temperature–time paths with
10 analytical U–Pb isotope data to provide constraints on the thermal evolution of the region. We provide a modelling framework, through the package UWDiffusion, that can be easily modified to investigate diffusion-decay-ingrowth across various minerals and isotopic systems, providing a tool to decipher the thermal history of a region recorded in isotopic data.

1 Introduction

Geochronology is essential for understanding Earth's history, as it constrains the timing and duration of geological events and processes. One of the most widely utilised geochronometers is the mineral zircon (ZrSiO_4) due to its ability to preserve radiogenic uranium–lead (U–Pb) isotopic systems over a wide range of pressures and temperatures throughout geological time (Rubatto, 2017)(Strutt, 1909; Kulp, 1955; Liu and Liou, 2011; Gehrels, 2014; Rubatto, 2017). U–Pb zircon geochronology has been crucial in unravelling the evolution of metamorphic and igneous terranes by providing key chronological constraints.

Radioactive decay of uranium and the concurrent ingrowth of radiogenic lead underpin the U–Pb dating method. When zircon contains uranium isotopes, it gradually accumulates radiogenic lead, a process that occurs over the long half-lives of ^{238}U and ^{235}U , making the U–Pb system a reliable chronological tool for dating geological events (Gehrels, 2014; Jaffey et al., 1971) (e.g. Gehrels, 2014; Jaffey et al., 1971). However, post-crystallization processes, such as the diffusion of uranium and lead isotopes at high temperatures, can alter U–Pb ratios in zircon and complicate age determinations as well as the reconstruction of a region's geological history (Wetherill, 1963; Wasserburg, 1963).

Temperature plays a central role in modification of isotopes due to diffusion. Temperatures that exceed a mineral's closure temperature facilitate diffusion, causing lead loss that yields discordant measurements that skew age estimates. The closure temperature marks the threshold above which an isotopic system is open to diffusion, and below which they become closed, thereby recording the timing of cooling through modification of isotopes due to diffusion (Dodson, 1973 where the closure temperature can potentially be decreased due to radiation damage (Cherniak et al., 1991; Cherniak and Watson, 2001). Under prolonged (ultra-)high temperature conditions ($> 900^\circ\text{C}$), diffusion can be so extensive that it resets the U–Pb clock altogether (Clark et al., 2011; Cherniak and Watson, 2001). Previous studies have quantified diffusion rates for both uranium (Cherniak et al., 1997; Lee et al., 1997) and lead (Lee et al., 1997; Cherniak and Watson, 2001; Cherniak et al., 1991; Cherniak and Watson, 2003), providing a basis for understanding these effects in zircon.

Although diffusion experiments have provided valuable insights (Cherniak et al., 1991; Cherniak and Watson, 2001; Cherniak et al., 1997; Cherniak and Watson, 2003; Bea and Montero, 2013), a key challenge remains. These experiments have not been combined with numerical models that can replicate the complete temperature–time evolution and lead loss experienced by zircon crystals in geological terranes. This limitation introduces uncertainties in interpreting discordant U–Pb data and restricts the ability to reconstruct detailed thermal histories. Integrating experimental and analytical data with numerical simulations is essential to address these issues.

In this study, we evaluate the interplay between diffusion, radioactive decay, and daughter isotope ingrowth in zircon to improve the interpretation of U–Pb geochronological data and provide temperature–time constraints based on analytical observations. We pursue have two main objectives: (1) develop and validate a two-dimensional numerical model that captures the coupled diffusion–decay–ingrowth processes, and (2) apply the model to a case study of the Trivandrum block in southern India to assess how a prolonged high-temperature event affects the zircon U–Pb record. Numerical simulations are compared to analytical U–Pb data to obtain estimates of the temperature–time path and peak temperatures during the metamorphic event.

We employ the This study employs the *Underworld3* numerical code to simulate the diffusion–decay–ingrowth process in zircon. The model is processes in zircon, facilitating the development of a dedicated package, *UWDiffusion* (Knight, 2025b), to streamline model setup and execution. This initial iteration focuses on diffusion, decay-ingrowth and diffusion-decay-ingrowth models, with plans to incorporate additional complexities, such as multicomponent diffusion. The scripts are designed to be flexible, with adjustable parameters including and meshing including varying zircon crystal size, multiple growth events, diffusion coefficients, and temperature–time paths. It can be modified to investigate diffusion-decay-ingrowth effects in other minerals and their impact on additional isotopic systems, thereby offering a flexible tool for geochronological modelling.

2 Methods

2.1 Diffusion-Decay-ingrowth equations

The diffusion-decay and diffusion-ingrowth equations for uranium and lead are implemented in the finite element code *Underworld3* (Moresi et al., 2025a) developed by the underworld group (Moresi et al., 2003; Mansour et al., 2020), that leverages the *PETSc* computational framework (Balay et al., 1997, 2019). The diffusion-decay and diffusion-ingrowth equations describe the spatial and temporal evolution of the concentration of isotopes undergoing diffusion, along with either decay or ingrowth (production). For uranium (the parent isotope (*PI*)), the decay-diffusion equation is expressed as:

$$\frac{\partial C_U}{\partial t} = D \nabla^2 C_U - \lambda C_U$$

$$\frac{\partial C_{PI}}{\partial t} = D \nabla^2 C_{PI} - \lambda C_{PI} \quad (1a)$$

$$\frac{\partial C_{DI}}{\partial t} = D \nabla^2 C_{DI} + \lambda C_{PI} \quad (1b)$$

Whilst the diffusion-ingrowth equation for lead (Pb) is expressed as:-

$$\frac{\partial C_{Pb}}{\partial t} = D \nabla^2 C_{Pb} + \lambda C_U$$

where: where:

C is the concentration of either uranium (U) or lead (Pb) is the concentration of either parent or daughter isotope,

t is the time is the time in s ,

D is the diffusion coefficient is the diffusion coefficient in $m^2 s^{-1}$,

$\nabla^2 C$ is the Laplacian of the function C ,

λ is the decay constant is the decay constant of the parent isotope in s^{-1} .

In 2D, $\nabla^2 C$ represents $\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}$ which are the second partial derivatives of C with respect to x and y , respectively. The inclusion of decay or ingrowth (λC_{PI}) is included as a source term.

75 Zircon (ZrSiO_4) naturally incorporates uranium atoms into its crystal structure as a substitute for zirconium but excludes lead (almost) entirely when it forms (Krogh, 1993). This makes zircon an ideal candidate for U–Pb dating because any lead found in a zircon crystal is likely due to the radioactive decay of uranium, where the U–Pb ratio can be used for dating.

~~Both~~ When modelling diffusion-decay or diffusion-ingrowth, both U and Pb concentrations are set to zero at the boundary, enforcing a Dirichlet (essential) boundary condition that represents an infinite reservoir into which U and Pb can diffuse. This aligns with closure temperature estimates, above which the daughter product (Pb) is expected to escape from zircon (Dodson, 1973). However, previous studies have shown that radiogenic Pb can be retained within zircon at temperatures exceeding the closure temperature due to surrounding minerals or melt reducing Pb flux at the zircon grain boundary (Bea and Montero, 2013; Bea et al., 2018). Consequently, a zero concentration boundary condition represents the maximum possible Pb loss through diffusion. ~~Underworld3~~ also supports-accommodates Neumann (natural) boundary conditions, ~~where the flux across the boundary ($\frac{\Delta C}{\Delta n}$) can be enforced~~ which specify the normal derivative (n) of the concentration field at the boundary. By enforcing ($\mathbf{n} \cdot \nabla C = 0$), the model ensures that there is no net flux of concentration across the boundary, effectively representing an impermeable or insulated surface. This can be used to model the flux between minerals by utilising partition coefficients of U and Pb between zircon and other minerals, although it is not explored here.

The temperature-dependent diffusion coefficient is calculated by:

$$90 \quad D = D_0 e^{-\frac{E_a}{RT}} \quad (2)$$

where:

D_0 is the pre-exponent factor in $\text{m}^2 \text{s}^{-1}$.

E_a is the activation energy in kJ mol^{-1} .

R is the gas constant in $\text{J mol}^{-1} \text{K}^{-1}$.

95 T is the temperature in K.

To solve ~~for~~ time-dependent diffusion problems, ~~either the forward-Euler ($\theta=1$), backward-Euler ($\theta=0$), or the Crank-Nicolson ($\theta=0.5$) method are used. The generic form of the theta method for solving the diffusion equation is given by:-~~

$$\underline{C^{n+1} = C^n + \Delta t [\theta D \nabla^2 C^{n+1} + (1 - \theta) D \nabla^2 C^n]}$$

the first-order backward differentiation formula (BDF1), also known as the backward Euler method, may be employed. The update equation for BDF1 is:

$$100 \quad \underline{\frac{C^{n+1} - C^n}{\Delta t} = D \nabla^2 C^{n+1}} \quad (3)$$

~~where:~~ where:

C is the concentration of the scalar field.

C^n and u is the value of C at the current time step n .

105 C^{n+1} are the numerical solutions at time steps t_n and t_{n+1} , respectively. C is the value of C at the next time step $n + 1$.

Δt is the time step size. t is time increment between steps n and $n + 1$.

The model supports time integration using the backward differentiation formula (BDF) of up to third order. We use an unstructured irregular (triangle) mesh and use linear shape functions by defining C with a polynomial degree of 1, as we use the first order BDF which restricts the convergence of higher order shape functions. All mesh geometries to replicate the zircon mineral shape are created using *gmsh* (Geuzaine and Remacle, 2009).

110

Element	Pre-exponent [D_0] (m^2s^{-1})	Activation energy [E_a] (J mol^{-1})	Reference
Uranium	1.63	$726 (\pm 83) \times 10^3$	(Cherniak et al., 1997)
Lead	0.11	$550 (\pm 30) \times 10^3$	(Cherniak and Watson, 2001)

Table 1. Variables used to determine the diffusion coefficients of U and Pb in zircon.

Diffusion values for U and Pb are calculated using Eq. 2 and values outlined in Table 1. The values in Table 1 represent a zircon with low uranium content that has not exceeded its alpha damage percolation threshold which would result in faster diffusion due to radiation damage. Other work has utilised radiation-damaged zircons (Cherniak et al., 1991; Ullah et al., 2023) that have much higher diffusion coefficients, resulting in rapid Pb loss and lower closure temperatures.

115 ~~All mesh geometries to replicate the zircon mineral shape are created using *gmsh* (Geuzaine and Remacle, 2009).~~

3 Benchmarking the diffusion-decay-ingrowth implementation

3.1 Decay and ingrowth

Radioactive decay is the spontaneous decay of a parent isotope into a daughter isotope (Hodges, 2013), where the half-life is the time it takes for half of the parent isotope population to decay. The half-life is ~~encapsulated in part of~~ the decay constant (λ), which is equal to the natural logarithm of 2 over the half-life of the decaying element 2, where the ~~and can be used to determine the~~ total amount of decayed material over a time-step ~~can be calculated numerically as by:~~

120

$$\frac{\partial C_U}{\partial t} = -\lambda C_{U_{PI}} = C_{U_{PI_0}} e^{-\lambda t} \quad (4)$$

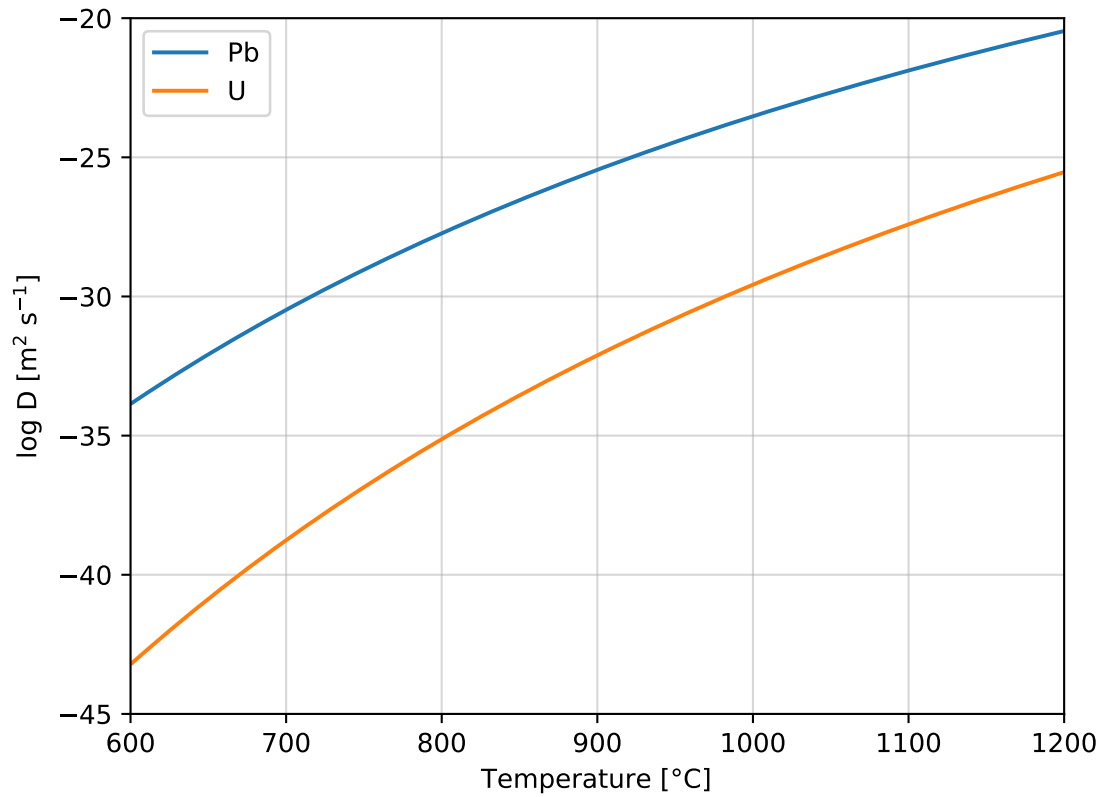


Figure 1. Diffusion coefficients for uranium (U) and lead (Pb) from values in Table 1 and eq. 3 over a temperature range of 600°C to 1200°C

$$\frac{\partial C_{DI}}{\partial t} = \lambda C_{PI} = C_{PI_0} e^{-\lambda t} \quad (5)$$

Table 2. Half-lives and decay constants (λ) for isotopes ^{235}U and ^{238}U from Jaffey et al. (1971).

Isotope	Half-life (yr)	Decay constant λ (yr ⁻¹)
^{235}U	7.038e8	9.849e-10
^{238}U	4.468e9	1.551e-10

125 In the decay and ingrowth calculation the decay chain is ignored and is assumed to be under secular equilibrium, where the parent decays directly into the stable daughter without any intermediate decay products. This is commonly assumed in

uranium–lead (U–Pb) dating as intermediate decay products in the uranium decay series are short lived and do not significantly impact the overall age calculations at geological timescales (Ludwig, 1998; Schoene, 2014).

130 To test the implementation of the decay and ingrowth equation, the ^{238}U to ^{206}Pb and ^{235}U to ^{207}Pb decay chains are tested, as both are ~~required~~ commonly used to determine the age of a ~~sample~~ zircon. The required ratios can be calculated at a given point in time (t) as follows:

$$\frac{^{206}\text{Pb}}{^{238}\text{U}} = (e^{\lambda_{238}t} - 1) \quad (6)$$

$$\frac{^{207}\text{Pb}}{^{235}\text{U}} = (e^{\lambda_{235}t} - 1) \quad (7)$$

$$\frac{^{238}\text{U}}{^{206}\text{Pb}} = \frac{1}{(e^{\lambda_{238}t} - 1)} \quad (8)$$

$$135 \quad \frac{^{207}\text{Pb}}{^{206}\text{Pb}} = 0.0072 \frac{(e^{\lambda_{235}t} - 1)}{(e^{\lambda_{238}t} - 1)} \quad (9)$$

Where 0.0072 is the current ratio of ^{235}U to ^{238}U (Hiess et al., 2012).

Plotting the ratios of uranium and lead isotopes ~~over a range of time generates a concordia plot~~ as a function of time produces a Concordia diagram, which is ~~used for determining the~~ a key tool for visualising results and determining the crystallization age of zircon ~~crystals, which are typically constructed using either a Wetherill (Wetherill, 1963) or Tera-Wasserburg (Tera and Wasserburg, 1972)~~ plot. If the zircon mineral has remained in a closed system since the time of its crystallization—, meaning that neither uranium nor lead isotopes have been gained or lost due to geological processes—, the isotope ratios will plot along a curve known as the ~~concordia~~ Concordia line. This method relies on the principle that zircon minerals retain uranium and exclude lead at formation and accumulate lead over time as uranium decays. The ~~concordia~~ Concordia plot is constructed by plotting the ratio of two isotopes of uranium, ^{238}U and ^{235}U , against the ratios of their respective lead decay products, ^{206}Pb and ^{207}Pb . By comparing
145 the measured ratios of uranium and lead isotopes in a zircon sample with this curve, the age of the zircon can be determined. Any deviations from the ~~concordia~~ Concordia line (discordance) can indicate episodes of lead loss or gain, providing insights into the thermal history and geological events that the zircon has experienced. No boundary conditions are set, enforcing zero flux across all boundaries ($\mathbf{n} \cdot \nabla C = 0$).

Figure 2 shows a comparison between the ~~concordia~~ Concordia plot and the results obtained from the numerical solution at
150 selected ages. ~~The results align along the concordia line when diffusion is not considered, signifying that the decay~~ We model decay of parent isotopes into daughter isotopes in a box domain, with zero flux boundaries applied to every wall. We determine the stable timestep for both decay chains by using the isotope with the shortest half-life (^{235}U) as we found using differing timesteps between chains results in inaccurate results. We start by calculating the expected ^{238}U and ingrowth calculations have

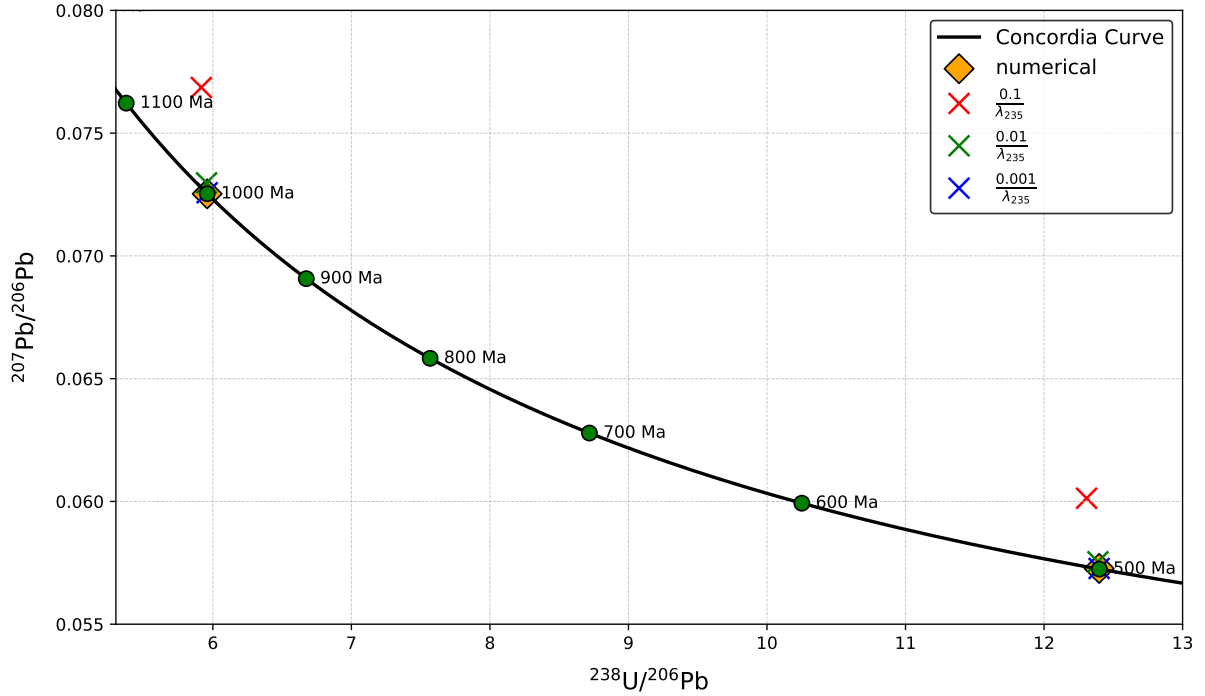


Figure 2. Tera-Wasserburg plot of $\frac{^{238}\text{U}}{^{206}\text{Pb}}$ against $\frac{^{207}\text{Pb}}{^{206}\text{Pb}}$ showing the concordia line (solid black line) 100 Ma increments between 1000 Ma and 500 Ma (green dots) and results of models (between 1050 at 1000 Ma and 550 and 500 Ma, red crosses (various symbols) solving for the decay and ingrowth only plot on the concordia line based on different timesteps when including the decay or ingrowth as a source term (Eqs. 1a - 1b), or calculating it numerically (Eq. 15).

dt	$\frac{^{207}\text{Pb}}{^{206}\text{Pb}}$ (1000 Ma)			$\frac{^{207}\text{Pb}}{^{206}\text{Pb}}$ (500 Ma)			$\frac{^{238}\text{U}}{^{206}\text{Pb}}$ (1000 Ma)			$\frac{^{238}\text{U}}{^{206}\text{Pb}}$ (500 Ma)		
	N	A	% error	N	A	% error	N	A	% error	N	A	% error
$\frac{0.1}{\lambda}$	0.0769	0.0725	5.98%	0.0601	0.0572	5.05%	5.92	5.96	0.708%	12.3	12.4	0.738%
$\frac{0.01}{\lambda}$	0.0730	0.0725	0.619%	0.0575	0.0572	0.528%	5.95	5.96	0.0722%	12.4	12.4	0.0753%
$\frac{0.001}{\lambda}$	0.0726	0.0725	0.0588%	0.0573	0.0572	0.0492%	5.96	5.96	0.00710%	12.4	12.4	0.00730%

Table 3. Numerical (N) vs analytical (A) ratios at 1000 Ma and 500 Ma for three timestep sizes, where timestep is determined by taking the largest decay constant (λ) between ^{235}U and ^{238}U . Values rounded to 3 significant figures. Percent error = $\left| \frac{\text{numerical} - \text{analytical}}{\text{analytical}} \right| \times 100$.

155 been implemented correctly. ^{235}U at the model start time (in Ma). We then solve either Eq. 1a for the parent or Eq. 1b with $D = 0$, to determine the accuracy of the decay and ingrowth implementation. We find that the accuracy of decay and ingrowth when decay and ingrowth is included as a source term is dependent on the time-step size, where $dt \ll \frac{1}{\lambda_{235}}$. Errors outlined in Table 3 show the $\frac{^{207}\text{Pb}}{^{206}\text{Pb}}$ ratio is much more sensitive to the time-step size than the $\frac{^{238}\text{U}}{^{206}\text{Pb}}$ ratio. Our results show that the

time-step should be limited to $\frac{0.001}{\lambda_{235}}$ to minimise the error when included as a source term. Alternatively, the decay-ingrowth can be calculated numerically over a timestep and be unconditionally stable over any size. λ_{235} is the decay constant of ^{235}U and λ_{238} is the decay constant of ^{238}U .

$$x_{235} = \lambda_{235} \Delta t \quad (10)$$

$$x_{238} = \lambda_{238} \Delta t. \quad (11)$$

Using the exact analytic solution for a parent and daughter over the interval Δt , the updates are

$$^{235}\text{U}(t + \Delta t) = ^{235}\text{U}(t) e^{-x_{235}}, \quad (12)$$

$$^{207}\text{Pb}(t + \Delta t) = ^{207}\text{Pb}(t) + ^{235}\text{U}(t)(1 - e^{-x_{235}}), \quad (13)$$

$$^{238}\text{U}(t + \Delta t) = ^{238}\text{U}(t) e^{-x_{238}}, \quad (14)$$

$$^{206}\text{Pb}(t + \Delta t) = ^{206}\text{Pb}(t) + ^{238}\text{U}(t)(1 - e^{-x_{238}}). \quad (15)$$

Which is quicker to calculate the amount of parent decay and daughter ingrowth when running models as it is stable over any timestep size. For this reason, we utilise Eq. 15 to determine the amount of decay and ingrowth.

3.2 Diffusion

3.2.1 Convergence of diffusion solver

~~The heat pipe~~ The advection-diffusion benchmark outlined in Zhang et al. (2022) of a of a rectangular pulse benchmark is utilised to determine the order of convergence of the diffusion solver. To solve for diffusion only, the advection term is removed, which results in the analytical solution for a diffusing one-dimensional heat pipe diffusion only (Crank, 1975):

$$U_a = 0.5 \left(\text{erf} \left(\frac{x_1 - x}{2\sqrt{D \cdot t}} \right) + \text{erf} \left(\frac{-x_0 + x}{2\sqrt{D \cdot t}} \right) \right) \quad (16)$$

where:

U_a is the analytical solution,

x_0 is the start of the initial hot pipe region, is the start of the initial elevated concentration region,

x_1 is the end of the initial hot pipe region, is the end of the initial elevated concentration region,

180 x is the x coordinate,

t is the time,

D is the thermal diffusivity.

For the ~~heat pipe rectangular pulse diffusion~~ benchmark, the initial conditions are: $x_0 = 0.3$ ~~0.35~~ and $x_1 = 0.6$ ~~0.65~~, t is ~~10⁻⁴~~
185 ~~and k is 1. Starting from a t value other than 0 allows the model to start from a diffused state, which provides smoothing to sharp boundaries at the interface between high and low concentrations compared to when $t = 0$, which are difficult for the numerical framework to resolve, and $D = 1$.~~ The benchmark is performed in a square box (Figure 3Aa), with the 1D hot pipe analytical solution valid across the entire domain as it is extended in the vertical direction, with diffusion occurring in the horizontal plane ~~-(Figure 3b). No boundary conditions are enforced, with zero flux across the boundary ($\mathbf{n} \cdot \nabla C = 0$).~~

190 The results presented in Figure 3 demonstrate ~~Underworld3~~ underworld3 is able to accurately model diffusion across the domain. The results indicate that errors are concentrated near the diffusive interface (Figure 3E ~~Figures 3c-3d~~). At larger cell sizes (>0.1), corresponding to lower resolution, ~~Underworld~~ the numerical model produces a less accurate result due to inadequate spatial discretisation. ~~At higher polynomial degrees (>1) at low resolution, small spurious oscillations are also observed close to the concentration boundary due to interpolating from the integration points to the regularly spaced profile points (Runge's Phenomenon) (Figure 3D).~~ However, as the cell size decreases to around 0.025 or below, the ~~use of higher-order polynomials improves the accuracy of the interpolation onto the regularly spaced line used to plot the profiles (Figure 3F-G result improves as the diffusing interface is better constrained (Figures 3c-3d).~~ In these cases, a ~~clear second-order first-order~~ convergence is observed across all tested polynomial degrees (Figure 3D) ~~-f). We find that the CFL condition imposed does not have a major influence on the accuracy (Figure 3e).~~

200 Due to these results, all following benchmarks are performed with a cell size of at least ~~0.025~~ 0.01 and a polynomial of ~~4~~ 2 to balance the accuracy of the solution and the time to solve the problem. ~~The analytical solution using the error function 16 cannot be used to benchmark the zircon mesh geometry as it does not contain boundary conditions. Instead, to test the performance of the numerical implementation in subsequent tests, the numerical solution is compared with an explicit finite difference method for solving the diffusion equation in 1D:-~~

205
$$U_i^{n+1} = U_i^n + \frac{k\Delta t}{(\Delta x)^2} (U_{i+1}^n - 2U_i^n + U_{i-1}^n)$$

where:-

U_i^n is the unknown at point i and time step n ,

k is the thermal conductivity,

Δt is the time step size,

210 Δx is the spatial step size.

which produces the same profile as the analytical solution for the heat pipe benchmark regardless of cell size (Figure 3E-G).

3.2.2 Isotropic diffusion

The diffusion of uranium and lead in zircon has been observed to be isotropic (Cherniak and Watson, 2003). The diffusion
215 component of the solver is benchmarked by isolating the diffusion process from decay and ingrowth effects, with ~~initial values~~
~~of both lead and uranium set to 1.~~ the initial distribution of U and Pb matching that used in the diffusion benchmark (Figure
3a). The initial benchmarking is conducted under isothermal conditions, at temperatures of 750°C, 800°C, and 850°C, for a
duration of 500 million years (Myr). We benchmark the results using the analytical solution from Eq. 16 by substituting D for
U or Pb at the given temperature using Eq. 2 and the values from Table 1.

220 To test the numerical models ability to replicate the diffusion process in geological conditions, i.e. as temperate decreases
over time, models are also conducted where the temperature decreased linearly from 850 to 750 C over 500 Myr. ~~The 1D~~
~~numerical solution is calculated at each timestep to~~ We utilise the analytical solution presented in Eq. 16 to benchmark
the model, with a modification to account for the ~~varying diffusivity over time, allowing the 1D solution and Underworld~~
~~numerical solution to be compared to determine the accuracy of the Underworld solution.~~ time-dependent diffusivity arising
225 from temperature variations. The cumulative diffusivity (D_c), defined as the time integral of D over the time-step dt . This
integrated value replaces the constant diffusivity (D) in Eq. 16, thereby enabling Eq. 16 to accurately reflect the evolving
diffusion for both U and Pb as a function of the thermal history. We set zero flux ($\mathbf{n} \cdot \nabla C = 0$) on all boundaries for the
benchmark.

The numerical results match those produced by the ~~1D analytical~~ solution, with the model replicating the expected diffusion
230 profile in the horizontal plane (Figure 4). The consistency of results ~~between the underworld model and 1D profile~~ (Figure 4)
~~show demonstrate~~ the numerical model captures the transport of uranium and lead isotopes within the zircon matrix due to
diffusion reliably and the model can be used to simulate long-term isotope diffusion over geological timescales.

A minor amount of diffusion of lead is observed ~~close to the rim of the zircon~~ at 750°C over 500 Myr (Figure ~~4e~~4a), whilst
at 850°C (Figure ~~4a~~4c), diffusion affects ~~all but the core of the zircon ($\sim 10 \mu m$)~~ most of the initially elevated region, whilst the
235 uranium concentration remains unaffected up to 850°C for 500 Myr. These results emphasise the varying diffusion rates for
uranium and lead, with ~~slight~~ increases in temperature significantly alter the distribution of lead isotopes across a zircon crystal
over geological time due to the exponential dependence of the diffusion coefficient on temperature (Eq. 2). The results also

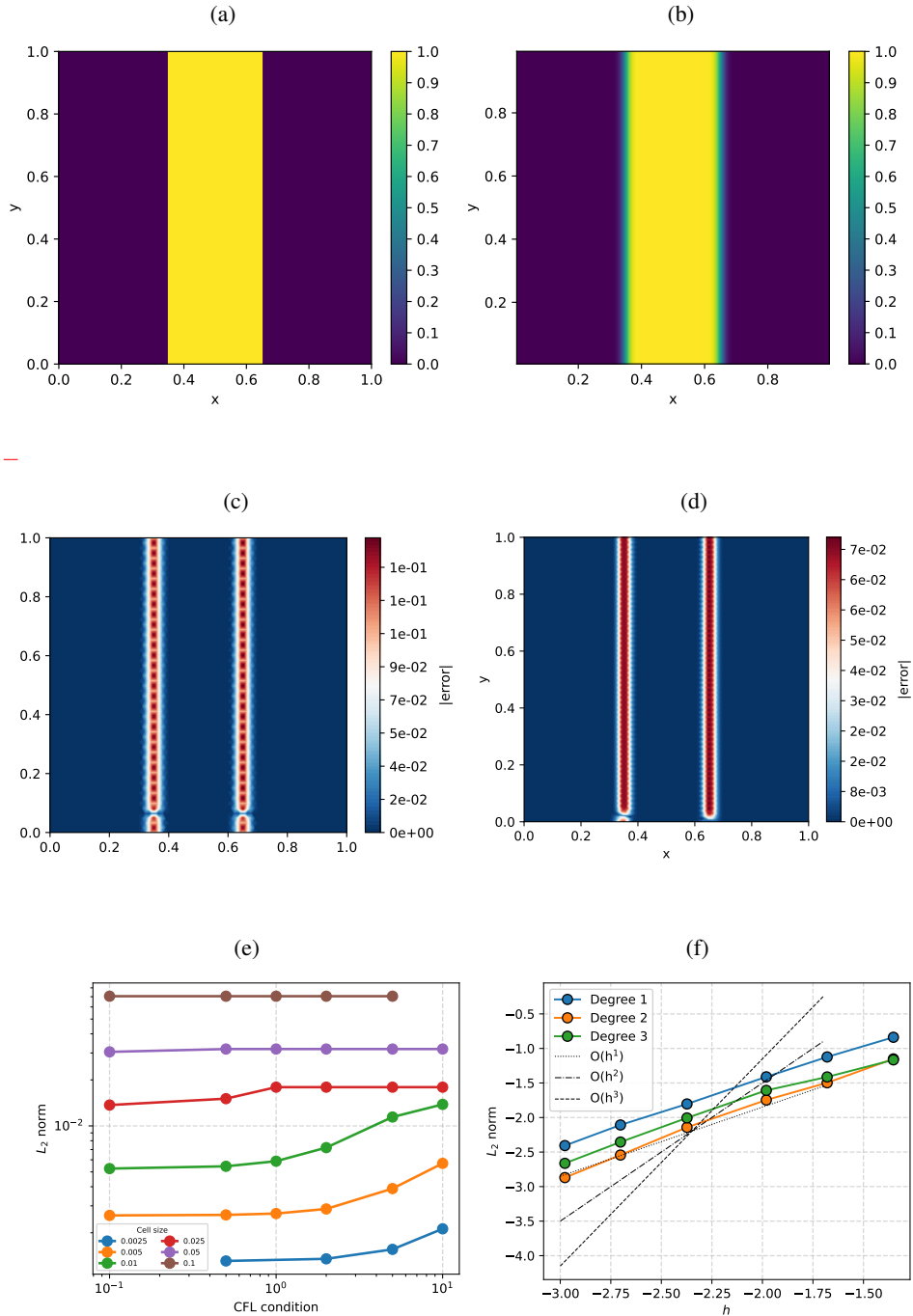


Figure 3. Diffusion convergence order from heat-pipe-rectangular-pulse benchmark. a) Initial distribution of unknown at $t=10^{-4}$. b) Final distribution of unknown at $t=21 \times 10^{-4}$. c) L2-norm when cell size is 0.01. Profiles comparing the analytical and numerical solution at $y=0.5$ for a cell size of 0.01. d) 0.01. e) 0.025 shows the L2-norm based on cell size and factor of CFL condition. f) 0.01. g) Solution Shows the convergence of diffusion benchmark for different unknown degrees for different cell sizes the solver.

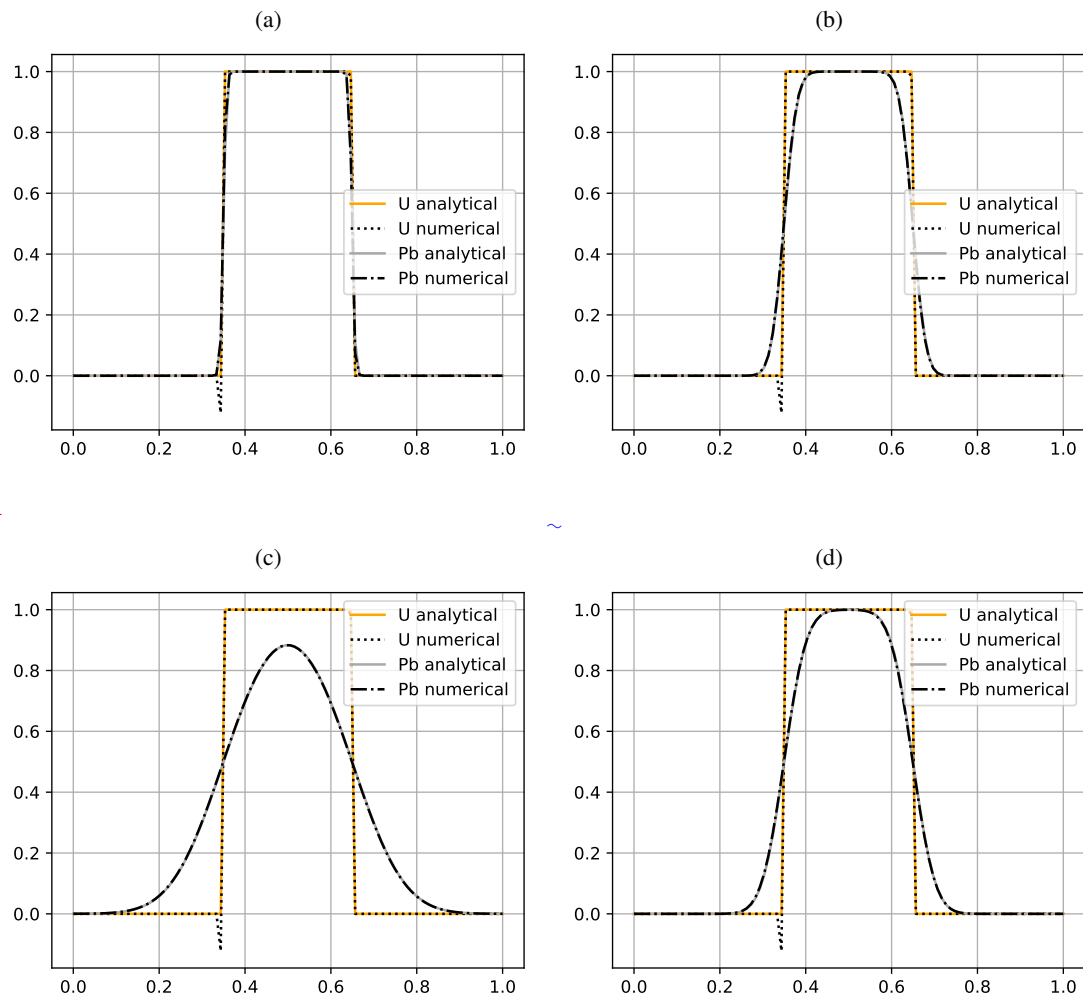


Figure 4. Diffusion benchmark results in a zircon geometry with cell size of 0.01 and degree of 2 for both U and Pb. Errors observed in U profile at the interface are interpolation errors rather than numerical errors. Diffusion profile after 500 Myr at a) 750°C, b) 800°C, c) 850°C; d) 750°C. Diffusion profile after decreasing temperature from 850°C to 750°C over 500 Myr. e) Anisotropic diffusion profiles over 500 Myr. Errors are concentrated at 800°C the diffusing front, as observed in Figures 3c - 3f.

highlight that lead is the primary isotope lost due to diffusion as the diffusion rates of lead are much higher than uranium at the same temperature (Figure 1).

240 3.2.3 Anisotropic diffusion

An additional benchmark is also run to highlight the ability to model anisotropic diffusion in *Underworld3*. Although anisotropic diffusion has not been observed in lead or uranium in zircon, it has been observed in other elements in zircon, including titanium (Bloch et al., 2022), helium (Cherniak et al., 2009; Anderson et al., 2020; Gautheron et al., 2020; Reich et al., 2007) and neon (Gautheron et al., 2020). The model can also be modified and applied to other minerals that may exhibit anisotropic diffusion.

245 To model anisotropic diffusion, the 800°C model is repeated with the lead diffusion coefficient doubled in the y axis. The model reproduces the benchmark results of Figure 4b in the x axis, whilst higher diffusion rates in the y axis match the 1D numerical solution (Figure ??). The robustness and flexibility of *Underworld3* allow isotropic and anisotropic diffusion to be modelled in 2D and can be extended to 3D.

3.3 Diffusion-decay-ingrowth

250 The diffusion-decay ~~benchmark involves repeating example models~~ the isotropic diffusion ~~benchmarks and including and~~ the decay of uranium to lead over a period of 500 million years (Myr). Unlike the diffusion-only ~~benchmarks benchmark~~ that focus solely on the movement of isotopes through the ~~zircon matrix crystal~~, these models integrate decay and ingrowth with diffusion. During decay and ingrowth, the concentration of uranium isotopes decrease (~~decay~~) while lead isotopes start from zero and increases (~~ingrowth~~), whilst the concentrations are also altered due to diffusion of uranium and lead across the zircon.

255 All boundaries are set to 0 for both U and Pb.

At 750°C maintained over 500 Myr, diffusion ~~is of both U and Pb~~ slow. As a result, the isotopic concentrations within the zircon are governed by the decay of uranium and ingrowth of ~~radiogenic~~ lead. This ensures that the sample locations across the zircon plot on the concordia line, thereby providing an accurate age constraint on the growth of the zircon (Figure 5a & 5d). When the temperature is elevated to 800°C for the same duration, the distribution of lead isotopes across the profile begins to

260 exhibit a distinctive ~~diffusion (bell-shaped) curve (diffusive front close to the rim of the zircon~~ (Figure 5b). This pattern is due lead loss close to the rim of the zircon, while the core preserves lead values. ~~Consequently, this distribution leads to discordant ages close to the rim ($r \geq 25 \mu m$) (Figure 5d).~~

Elevating the temperature to 850°C over a period of 500 million years significantly enhances the diffusion of lead within the zircon. Under these conditions, the lead isotopes display ~~a pronounced diffusion profile~~ pronounced diffusion, indicative

265 of considerable lead loss, whereas the uranium isotopes remain undisturbed. The core region of the zircon (radius < 10 μm) remains unaffected, whilst lead values decrease towards the rim due to diffusion (Figure 5c). The effect of diffusion is observed in both ratios but is more pronounced in the $\frac{^{238}\text{U}}{^{206}\text{Pb}}$ $\frac{^{238}\text{U}}{^{206}\text{Pb}}$ ratio. This highlights the spatial variation in isotopic ratio within the zircon crystal due to diffusion at elevated temperatures. Three sample spots each with a ~~radius diameter~~ of 23 μm - comparable to data acquisition from laser ablation sampling - are also selected. The results presented in Figure 5 highlight the

270 effect of diffusion and sample location on discordant ages within a zircon.

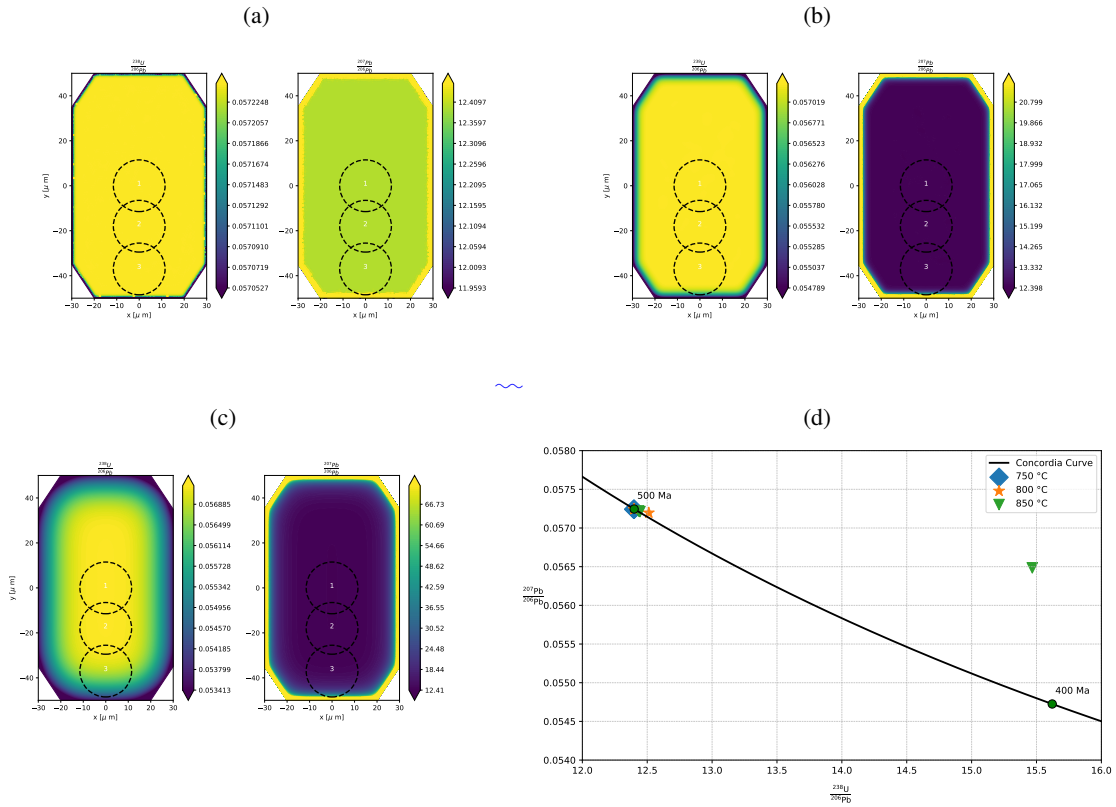


Figure 5. Diffusion-decay and diffusion-ingrowth benchmark test results in a zircon mesh geometry for a cell size of 0.01 and degree of 2 for uranium and lead. U–Pb isotope profiles after 500 Myr at a) 750°C, b) 800°C, c) 850°C. d) Effect of sample location on age data. Plots Discordance increases closest to the initial-age are from the centre, rims and furthest-away are closest to the edge. e) $^{238}\text{U}/^{206}\text{Pb}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ across the modelled zircon after 500 Myr at 850°C with sample spot locations elevated temperatures for longer.

These experiments reflect the significant influence temperature can have on zircon ages due to the diffusion of lead within zircon. Higher temperatures result in increased diffusion rates, which results in disparities in $^{238}\text{U}/^{206}\text{Pb}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ ratios between the core and rim of the zircon, creating discordant values. This discordance is crucial for interpreting the thermal history and subsequent geological events that the zircon has undergone.

275 4 Applications to geological problems

4.1 Partial and full lead loss events

The benchmarks presented above demonstrate *Underworld3* can accurately represent the evolution of radiogenic uranium and lead isotopes in zircons, accounting for radioactive decay, ingrowth, and diffusion across geological timescales and tempera-

tures. The lead loss at temperatures above 800°C are responsible for the discordant values observed above, with the degree of
280 lead loss dependent on the intensity and duration of the thermal event, where higher temperatures accelerate diffusion rates, potentially leading to significant lead loss, or prolonged thermal events can also result in lead loss.

Partial lead loss in the models tested above occurs with temperatures above 800°C over 500 Myr, with higher lead loss experienced at higher temperatures (Figure 5). However, regions do not experience constant elevated temperatures for extended periods of time, instead these thermal events may occur over a wide range of time and reach varying peak temperatures,
285 depending on the geological conditions. To simulate partial and full lead loss events due to varying temporal and thermal conditions, four temperature-time paths are tested. The zircon is assumed to have formed at 1500 Ma and cooled to below 800°C over the first 100 Myr, which then experiences a subsequent thermal pulse-event that peaks at 500 Ma, with different peak temperatures and duration's durations, but all result in an average temperature of 770°C over the entire evolution (Figure 6a). Temperatures are decreased to 750°C which is below the closure temperature for zircon and no diffusion will occur in the
290 mineral.

The findings illustrated in Figure 6-6b highlight the influence of peak temperature and duration of a thermal pulse-event on isotopic ratios, for a zircon measuring 60 μm in width and 100 μm in height. The results highlight peak temperatures of 1000°C (profile-T path 1) over a 20 Myr can result in complete lead loss of the zircon, with the ages recorded in the zircon representing the time at which the closure temperature is passed, which vary from core to rim, resulting a spread of ages that
295 appear concordant. In contrast, elevated temperatures up to 850°C (profile-T path 4) result in minimal lead loss, with some discordant values towards the rim of the zircon and original concordant ages close to the core, however all values fit along the discordant line between 1500 Ma and 500 Ma as the zircon exceeds the closure temperature for a short amount of time during the thermal pulseevent. Temperatures up to 950°C (profiles-T paths 2 and 3) leads to a considerable amount of lead loss, and produces a curved discordant line. This is similar to profile-T path 1 and is a result of varied lead loss from core to rim, with
300 the rim values converging towards the closure temperature age whilst core values lie along the discordant line between 1500 Ma and 500 Ma.

From the modelling results, the distribution of discordant values on a Tera-Wasserburg diagram (Figure 6b) can be used to infer minimum peak temperatures during subsequent heating events. However, peak temperature impacts estimated age due to varied amounts of lead loss from core to rim, which creates a curved array on (Figure 6b) at temperatures above the closure
305 temperature (800°C) but below the total lead loss temperature (1000°C) for the zircon size tested. The U-Pb ratios can be coupled with Ti-in-zircon thermometry (Ferry and Watson, 2007; Crisp et al., 2023) to determine potential Temperature-time paths for zircon-bearing rocks (e.g.-Clark et al. (2024))(e.g. Clark et al., 2024), however this may be complicated due to the potential diffusion heterogeneity of Ti observed in zircon (Bloch et al., 2022).

4.2 Multiple zircon growth events and intra-grain diffusion

310 Temperature profile-path four is re-run, where the zircon forms at 1500 Ma that subsequently experiences a thermal pulse-event up to 850°C at 500 Ma. In this model, a new zircon nucleates around the pre-existing grain when the temperature reaches 850°C at 500 Ma. The mesh contains an inner boundary, with the Dirichlet boundary condition for uranium and lead isotopic

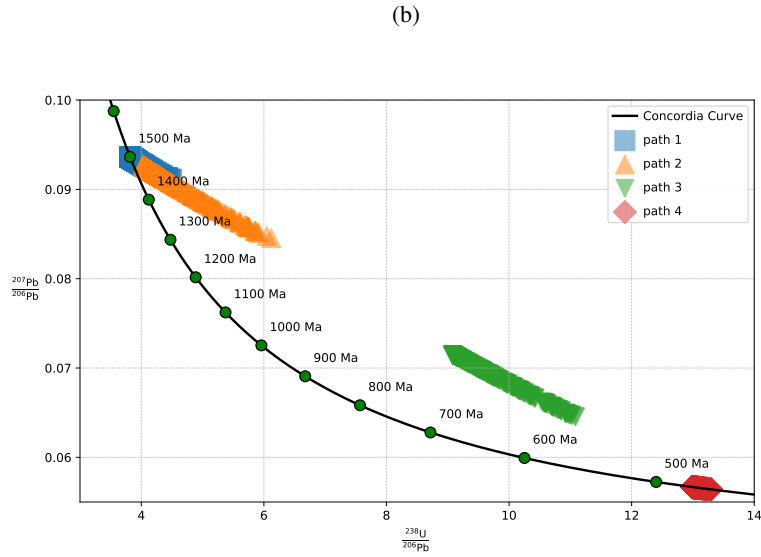
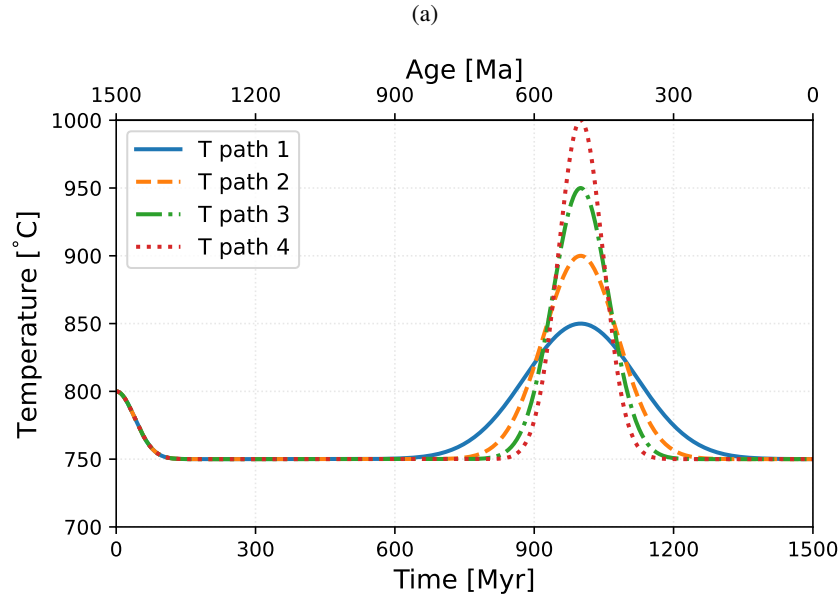


Figure 6. a) Different temperature-time (Tt) paths tested. b) Effects of different temperature-time paths on U–Pb ratios and ages based on sample location. [Spot locations are presented in Figure S1.](#)

concentrations set to 0 between the formation age (1500 Ma) and second growth age (500 Ma). At the second growth, the inner boundary condition is removed and the uranium and lead isotope values are updated in the outer region with the uranium isotopic values for formation of a zircon at 500 Ma, with the Dirichlet boundary then applied to the outer zircon edge. This approach highlights that [Underworld3](#) can handle the diffusion of heterogenous isotope values across a crystal and is used to investigate the effects of a second growth event as well as intra-grain diffusion on discordant U–Pb isotopic values. The U–Pb ratios between zircons that have undergone a single growth event and those that have experienced double growth along the specified temperature–time (T–t) path are assessed, ensuring that the dimensions of the inner zircon remain consistent with those of the single growth zircon.

Figure 7a illustrates how a second growth event influences U–Pb ratios across the zircon and the corresponding ages observed on the Tera–Wasserburg plot (Figure 7b). Sample spots from the inner zircon cluster near the formation age, with minor diffusion occurring across the original 60 μm zircon over the first 1000 Myr. These spots plot similarly to the single-event values (Figure 7b), though the single-growth zircon experiences slightly more diffusion due to its smaller size over the entire model duration. Sample spots from the interface span the formation and second growth ages, reflecting a mix of inner and outer zircon material. However, they are biased toward the second growth age due to prior Pb^{206} diffusion and subsequent lead mixing across the inner and outer zircon after the second growth event. This mixing results in decreased $^{238}\text{U}/^{206}\text{Pb}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ ratios near the inner boundary. The outer zircon exhibits some diffusion at the rim, but it does not penetrate significantly and remains unsampled in the selected spot locations. Consequently, the measured ages primarily reflect the zircon growth event on the Tera–Wasserburg plot (Figure 7b). These findings emphasize how secondary zircon growth can preserve the metamorphic age while also influencing the original growth age near the intra-grain boundary.

4.3 Assessing the metamorphic history of the Trivandrum Block, southern India

The Trivandrum Block in southern India provides an excellent case study for applying U–Pb geochronology modelling to estimate peak temperatures during high temperature metamorphism. The Trivandrum Block experienced granulite-facies metamorphism during the late Neoproterozoic to Early Cambrian, coinciding with the amalgamation of Gondwana. High temperature metamorphism is thought to have occurred between 580 and 490 Ma based on U–Pb ages from zircon and monazite (Taylor et al., 2014; Blereau et al., 2016; Praharaj and Rekha, 2022; Kadowaki et al., 2019), with peak temperatures in the range of 800–1000°C, primarily based on overlapping peak mineral assemblage fields from phase equilibria modelling (Blereau et al., 2016; Praharaj and Rekha, 2022; Kadowaki et al., 2019).

To reconstruct the temperature–time history of the Trivandrum Block, four temperature–time profiles–paths are analysed to evaluate the distribution of U–Pb ratios from zircons in sample TB14-093. This sample was selected because all zircons are interpreted to be magmatic, as indicated by their steep heavy rare-earth element profiles, with analysed U–Pb values suggesting zircon formation between 2.0 and 1.8 Ga. This sample also preserves little evidence for widespread partial melting associated with the high temperature event as it is a more enderbitic (orthopyroxene + plagioclase) composition. This minimises the effect of fluid/melt–rock interactions that facilitates Pb-loss via dissolution-precipitation processes i.e. diffusion-related processes are

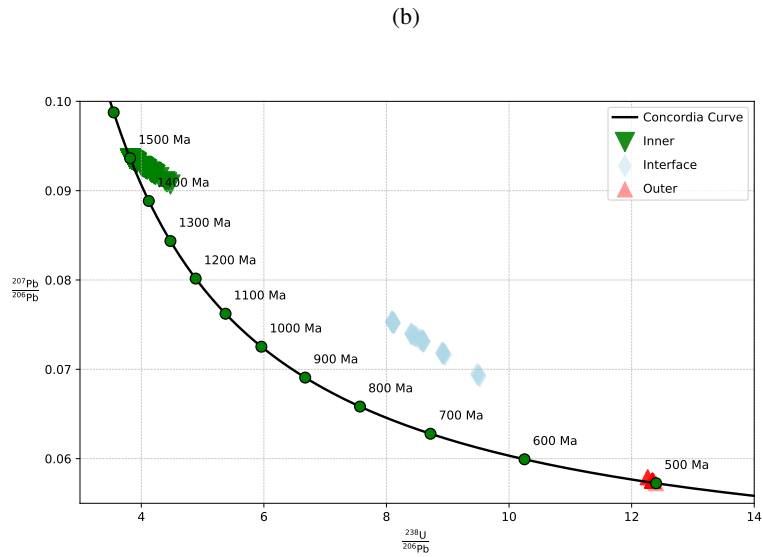
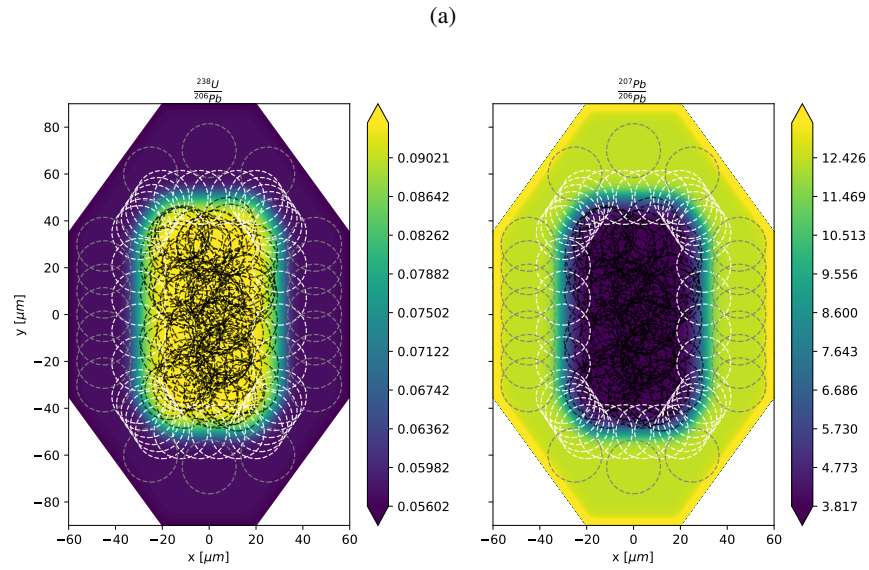


Figure 7. Effects of secondary zircon growth on U–Pb ratios and age based on sample location. a) Dual growth zircon sample spot locations. b) Tera-Wasserburg plot from sample spots.

the dominant mechanism for Pb mobility rather than reaction mechanisms. See the supplementary material for a full sample description and analytical methods associated with the zircon geochronology.

Using the Ti-in-zircon thermometer of Ferry and Watson (2007), and assuming an α_{SiO_2} of 1.0 and α_{TiO_2} of 0.7 ([activity of Silica and Titanium, respectively](#), based on the stability of ilmenite in the sample rather than rutile), zircons in sample TB14-093
350 record temperatures ranging from approximately 725°C to 900°C, with most recording a temperature of 800°C ([Fig-Figure 8a](#)) at formation. Minor variations in the estimated temperatures (within tens of degrees) are observed when adjusting α_{TiO_2} . These temperatures are interpreted to represent the majority of zircons forming at 800°C, although the range of Ti-in-zircon temperatures and U–Pb ratios suggest zircon formation between temperatures of 700°C and 1000°C that may have occurred over a period of 200 million years (Taylor et al., 2014). For the modelling, we assume that all zircons form at 1.95 Ga at an
355 initial temperature of 800°C as most of the U–Pb ratios fit along a discordant array between 1950 Ma and 555 Ma, which represents the estimated time of peak metamorphism.

The four temperature-time paths ([Figure 8b](#)) represent the expected range of peak temperatures of the metamorphic event at 555 Ma (Kadowaki et al., 2019), which range between 850°C and 1000°C with a temperature above 800°C between 600 and 500 Ma (Praharaaj and Rekha, 2022; Taylor et al., 2014). Temperatures are decreased to 750°C as this is below the closure
360 temperature, with the same results obtained with lower minimum temperatures as diffusion effectively stops below 800°C using the parameters outlined in Table 1.

Four zircon geometries are tested, 70x70, 80x120, 90x160 and 100x200 μm , which encompasses the range of zircon grains analysed. The estimated duration of metamorphism is in agreement with limits placed by various geochronological ages. An upper age estimate of metamorphism is constrained by depleted HREE zircon growth at 582 ± 17 Ma (inferred temperature
365 $>810^\circ$) (Kadowaki et al., 2019) and monazite formation at 585 Ma, characterised by variable REE profiles (Taylor et al., 2014). The depleted zircon HREE signature reflects the simultaneous growth of zircon and garnet during prograde metamorphism (Kadowaki et al., 2019). The lower age limit is marked by the growth of REE-enriched zircons at 489 ± 12 Ma, interpreted to be due to the breakdown of garnet during retrograde metamorphism (Taylor et al., 2014; Kadowaki et al., 2019).

The results indicate that the diffusion-decay-ingrowth modelling is able to reproduce some of the observed U–Pb ratios
370 from TB14-093, but none of the Tt paths completely match the analytical data. Models peaking at 950°C ([Fig-Figure 9c](#)) and 1000°C ([Fig-Figure 9d](#)) show partial agreement with analytical data, with [Fig-Figure 9c](#) capturing the range of discordant U–Pb ratios between formation at 1950 Ma and peak metamorphism at 555 Ma, whilst [Fig-Figure 9d](#) shows full lead loss and resetting of ages in the small modelled zircon grains and close to the rims in the larger grains.

The model not being able to replicate the data from sample TB14-093 completely may be due to the Tt path not accurately
375 representing the thermal history. To replicate the discordant ages observed between 600 and 500 Ma, temperatures may need to be higher or sustained for a longer period, exceeding 900°C between 600 and 500 Ma, rather than the 800°C assumed in the models. However, a longer duration of elevated temperatures does not agree with the geochronological constraints from monazite, zircon and garnet, which are outlined above. Alternatively, additional processes not accounted for in the modelling may be able to explain the complete lead loss. The zircons grains analysed from sample TB14-093 that show complete lead loss
380 and 'concordant' ages between 600 and 500 Ma typically have high uranium content ([Fig-Figure 9](#)). The high uranium content

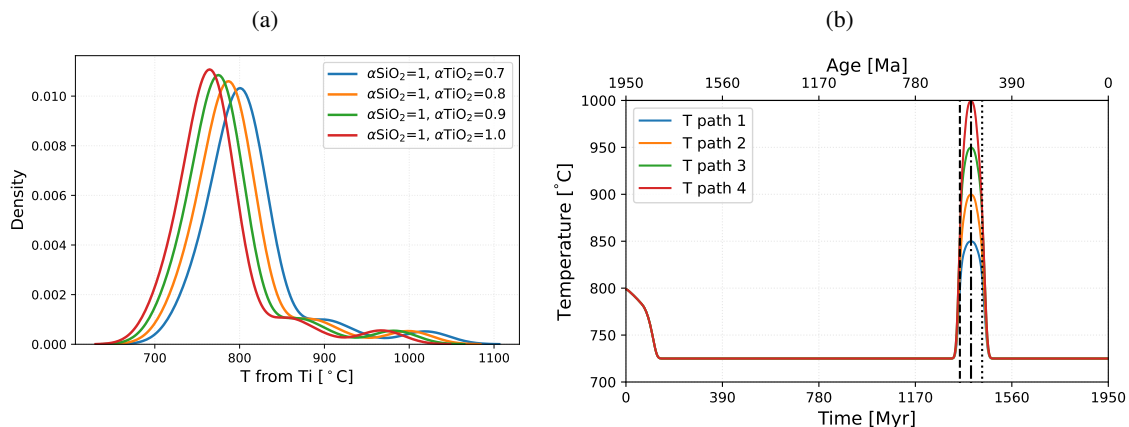


Figure 8. a) kernel density estimate (KDE) plot showing the bulk of zircons record a temperature of 800°C at formation based on the Ti-in-zircon thermometer. b) Range of temperature-time paths tested for the Trivandrum block based on Ti-in-zircon thermometry and analytical data. Dotted lines represent the estimated start (600 Ma), peak (555 Ma) and end (520 Ma) of UHT metamorphism.

can cause radiation damage, damaging the crystal structure and enhancing lead diffusion rates beyond those considered in the modelling, potentially resulting in complete lead loss during metamorphism. This has been observed in experimental data used to determine the diffusion coefficient of Pb using zircons that have experienced significant radiation damage (Cherniak et al., 1991). However, the effect of radiation damage on diffusion rates may be mitigated by annealing at elevated temperatures, with critical amorphization temperature (i.e. the threshold above which Pb diffusion is unaffected by radiation damage from high U content) is estimated at 360°C for a U concentration of 1,000 ppm and approximately 380°C for a U concentration of 10,000 ppm (Cherniak and Watson, 2003). These temperatures are well below the estimated peak temperature, however the radiation damage may result in lead loss at much lower temperatures. Alternatively, the zircon crystals may have undergone deformation, resulting in a damaged crystal structure that results in fast diffusion pathways through the crystal. Both processes have the ability to enhance diffusion that may have resulted in the complete lead loss observed in some of the sampled zircons.

We propose that diffusion decay and ingrowth modelling of U and Pb and the resulting isotopic ratios can be a viable method to estimate peak temperature conditions during metamorphism. Modelling of the U–Pb system suggests that peak metamorphic temperatures in the Trivandrum Block may have reached 950°C and potentially 1000°C during peak UHT metamorphism 555 Ma. 950°C is necessary to reproduce the discordant array observed between protolith formation at 1950 Ma and subsequent metamorphism at 555 Ma, whilst temperatures of 1000°C are needed to explain the complete lead loss observed when not taking other processes into account.

Our results suggest higher peak temperatures than some previous estimates, with proposed maximum temperatures below 900°C (Blereau et al., 2016; Praharaj and Rekha, 2022), based on peak stable mineral assemblages derived from thermodynamic modelling of charnockites. However, our results are consistent with higher temperature estimates of 950°C to 1000°C

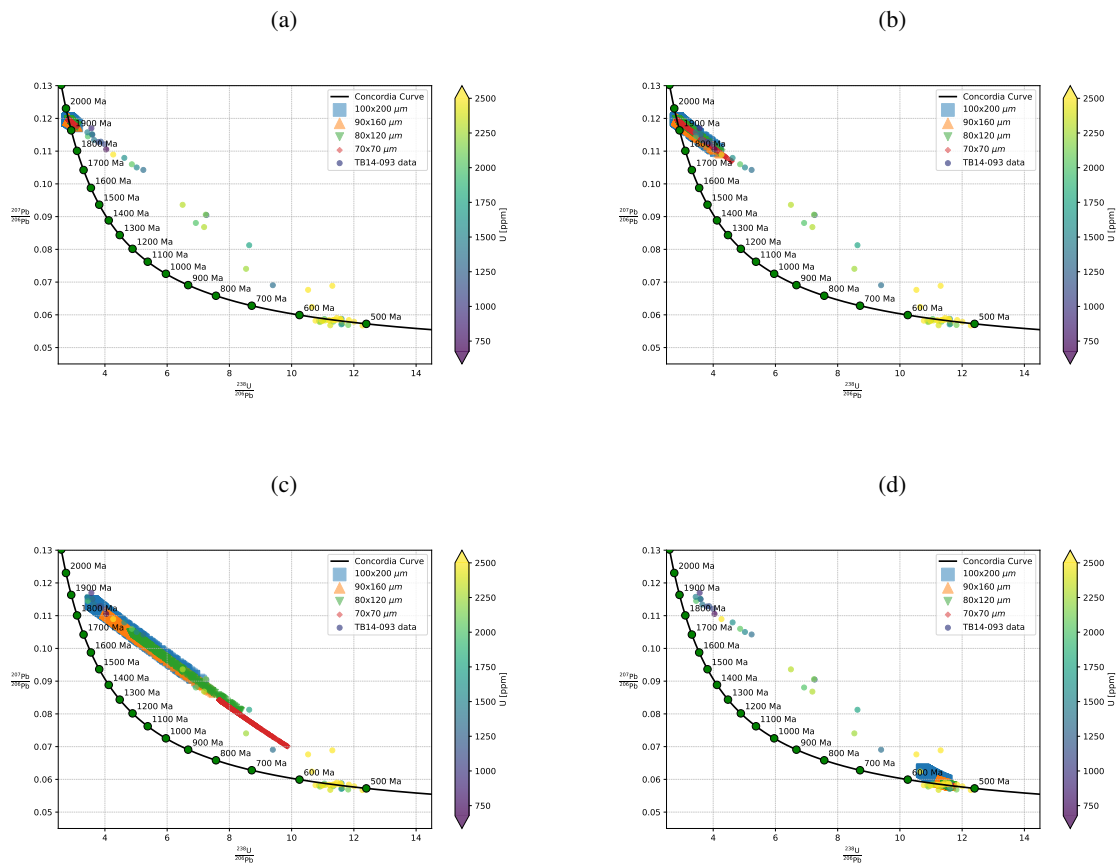


Figure 9. Comparison of analytical data sampled from TB13-093 and modelled zircons. a) $T_{peak} = 850$ °C, b) $T_{peak} = 900$ °C, c) $T_{peak} = 950$ °C, d) $T_{peak} = 1000$ °C.

from Taylor et al. (2014) which is based on heavy rare earth element partitioning between zircon and garnet, with partitioning values close to those observed in high temperature (1000°C) experimental estimates from Rubatto and Hermann (2007). The higher temperatures have also been estimated from thermodynamic modelling of khondalite found in the region Kadowaki et al. (2019), with peak temperatures estimated to be between 920°C and 1030°C.

5 Conclusions

This study benchmarks the diffusion-decay and diffusion-ingrowth using *Underworld3*, focusing on the U–Pb isotopic system within zircons that may experience modification in lead values due to diffusion or a secondary growth event. Our results demonstrate the model effectively captures the complex interplay of diffusion and radiogenic decay and ingrowth in zircons subjected to varied thermal and temporal histories. The model is able to accurately reconstruct U–Pb ratios that undergo

lead loss due to diffusion at elevated temperatures, providing insight into how the thermal and temporal history of a region
410 can modify isotopic ratios due to diffusion that are crucial for precise age dating in geochronological studies. Additionally,
secondary zircon growth events can have a the significant impact on U–Pb isotopic ratios, especially close to the intragrain
boundary. The double growth model highlights the broadening of U–Pb discordant values between the first and second growth
ages, which arise from sampling both the inner and outer zircons as lead transfer between them. This effect underscores the
necessity of considering sample location in zircon geochronology when estimating growth ages. We also highlight how U–Pb
415 geochronology can be used to infer maximum peak temperatures during metamorphic events, providing an upper limit of
950°C to 1000°C in the Trivandrum block during the amalgamation of Gondwana 555 Ma.

Overall, the model presented is a robust tool for modelling the isotopic evolution of U–Pb in zircons under varying thermal
conditions and can be applied to any other ~~isepotie~~-~~isoptotic~~ system in any other mineral. The results presented validate the use
of ~~Underworld3~~underworld3 and UWDiffusion to model isotope diffusion at the mineral scale and can be used to enhance our
420 ability to interpret complex thermal histories and their impact on age estimations from zircon U–Pb dating. The script can be
also easily modified to model diffusion-decay-ingrowth in different mineral systems. Future work will encompass modelling a
population of zircon grains in 3D which are cut at various locations through the crystal to better constrain the Temperature-time
history obtained from analytical measurements.

Code availability. underworld3 v0.99.1 is available from Moresi et al. (2025b) (doi:10.5281/zenodo.16838572) and UWDiffusion is availble
425 from Knight (2025b) (doi: 10.5281/zenodo.17936511), both licensed under LGPL Version 3. Scripts to replicate the research presented are
available at Knight (2025a) (doi: 10.5281/zenodo.17936675), licensed under Creative Commons Attribution 4.0 International. Figures were
made with Matplotlib v3.10.1 (Team, 2025; Caswell et al., 2021), available under the Matplotlib license at <https://matplotlib.org/>.

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References

- Anderson, A. J., van Soest, M. C., Hodges, K. V., and Hanchar, J. M.: Helium diffusion in zircon: Effects of anisotropy and radiation damage revealed by laser depth profiling, *Geochimica et Cosmochimica Acta*, 274, 45–62, 2020.
- Balay, S., Gropp, W. D., McInnes, L. C., and Smith, B. F.: Efficient Management of Parallelism in Object-Oriented Numerical Software Libraries, in: *Modern Software Tools for Scientific Computing*, pp. 163–202, Springer, 1997.
- Balay, S., Abhyankar, S., Adams, M., Brown, J., Brune, P., Buschelman, K., Dalcin, L., et al.: PETSc Users Manual, <https://ora.ox.ac.uk/objects/uuid:fa2b9e7c-1c58-429c-90fd-f780a3c3dc7d>, 2019.
- Bea, F. and Montero, P.: Diffusion-induced disturbances of the U–Pb isotope system in pre-magmatic zircon and their influence on SIMS dating. A numerical study, *Chemical Geology*, 349, 1–17, 2013.
- Bea, F., Montero, P., and Palma, J. F. M.: Experimental evidence for the preservation of U-Pb isotope ratios in mantle-recycled crustal zircon grains, *Scientific Reports*, 8, 12 904, 2018.
- Blereau, E., Clark, C., Taylor, R. J., Johnson, T., Fitzsimons, I., and Santosh, M.: Constraints on the timing and conditions of high-grade metamorphism, charnockite formation and fluid–rock interaction in the Trivandrum Block, southern India, *Journal of Metamorphic Geology*, 34, 527–549, 2016.
- Bloch, E. M., Jollands, M. C., Tollan, P., Plane, F., Bouvier, A.-S., Hervig, R., Berry, A. J., et al.: Diffusion Anisotropy of Ti in Zircon and Implications for Ti-in-Zircon Thermometry, *Earth and Planetary Science Letters*, 578, 117 317, <https://doi.org/10.1016/j.epsl.2021.117317>, 2022.
- Caswell, T. A., Droettboom, M., Lee, A., Sales De Andrade, E., Hoffmann, T., Hunter, J., Klymak, J., Firing, E., Stansby, D., Varoquaux, N., et al.: matplotlib/matplotlib: REL: v3. 5.0, Zenodo, 2021.
- Cherniak, D., Watson, E., and Thomas, J.: Diffusion of helium in zircon and apatite, *Chemical Geology*, 268, 155–166, 2009.
- Cherniak, D. J. and Watson, E. B.: Pb Diffusion in Zircon, *Chemical Geology*, 172, 5–24, [https://doi.org/10.1016/S0009-2541\(00\)00233-3](https://doi.org/10.1016/S0009-2541(00)00233-3), 2001.
- Cherniak, D. J. and Watson, E. B.: Diffusion in Zircon, *Reviews in Mineralogy and Geochemistry*, 53, 113–143, <https://doi.org/10.2113/0530113>, 2003.
- Cherniak, D. J., Lanford, W. A., and Ryerson, F.: Lead diffusion in apatite and zircon using ion implantation and Rutherford backscattering techniques, *Geochimica et cosmochimica acta*, 55, 1663–1673, 1991.
- Cherniak, D. J., Hanchar, J. M., and Watson, E. B.: Diffusion of Tetravalent Cations in Zircon, *Contributions to Mineralogy and Petrology*, 127, 383–390, <https://doi.org/10.1007/s004100050287>, 1997.
- Clark, C., Fitzsimons, I. C., Healy, D., and Harley, S. L.: How does the continental crust get really hot?, *Elements*, 7, 235–240, 2011.
- Clark, C., Brown, M., Knight, B., Johnson, T. E., Mitchell, R. J., and Gupta, S.: Ultraslow cooling of an ultrahot orogen, *Geology*, 2024.
- Crank, J.: *The mathematics of diffusion*, Clarendon Press, Oxford, [Eng], 2d ed edn., 1975.
- Crisp, L. J., Berry, A. J., Burnham, A. D., Miller, L. A., and Newville, M.: The Ti-in-zircon thermometer revised: The effect of pressure on the Ti site in zircon, *Geochimica et Cosmochimica Acta*, 360, 241–258, 2023.
- Dodson, M. H.: Closure temperature in cooling geochronological and petrological systems, *Contributions to Mineralogy and Petrology*, 40, 259–274, 1973.
- Ferry, J. and Watson, E.: New thermodynamic models and revised calibrations for the Ti-in-zircon and Zr-in-rutile thermometers, *Contributions to Mineralogy and Petrology*, 154, 429–437, 2007.

- 475 Gautheron, C., Djimbi, D. M., Roques, J., Balout, H., Ketcham, R. A., Simoni, E., Pik, R., Seydoux-Guillaume, A.-M., and Tassan-Got, L.: A multi-method, multi-scale theoretical study of He and Ne diffusion in zircon, *Geochimica et Cosmochimica Acta*, 268, 348–367, 2020.
- Gehrels, G.: Detrital zircon U-Pb geochronology applied to tectonics, *Annual Review of Earth and Planetary Sciences*, 42, 127–149, 2014.
- Geuzaine, C. and Remacle, J.-F.: Gmsh: A 3-D finite element mesh generator with built-in pre-and post-processing facilities, *International Journal for Numerical Methods in Engineering*, 79, 1309–1331, <https://doi.org/10.1002/nme.2579>, 2009.
- 480 Hiess, J., Condon, D. J., McLean, N., and Noble, S. R.: $^{238}\text{U}/^{235}\text{U}$ systematics in terrestrial uranium-bearing minerals, *Science*, 335, 1610–1614, 2012.
- Hodges, K.: Thermochronology in orogenic systems, in: *The Crust*, pp. 281–308, Elsevier Inc., 2013.
- Jaffey, A. H., Flynn, K. F., Glendenin, L. E., Bentley, W. C., and Essling, A. M.: Precision Measurement of Half-Lives and Specific Activities of U 235 and U 238, *Physical Review C*, 4, 1889–1906, <https://doi.org/10.1103/PhysRevC.4.1889>, 1971.
- 485 Kadowaki, H., Tsunogae, T., He, X.-F., Santosh, M., Takamura, Y., Shaji, E., and Tsutsumi, Y.: Pressure-temperature-time evolution of ultrahigh-temperature granulites from the Trivandrum Block, southern India: Implications for long-lived high-grade metamorphism, *Geological Journal*, 54, 3041–3059, 2019.
- Knight, B.: bknight1/diffusion_problems: GMD resubmission, <https://doi.org/10.5281/zenodo.17936675>, 2025a.
- Knight, B.: bknight1/UWDiffusion: Diffusion-decay-ingrowth models, <https://doi.org/10.5281/zenodo.17936511>, 2025b.
- 490 Krogh, T.: High precision U-Pb ages for granulite metamorphism and deformation in the Archean Kapuskasing structural zone, Ontario: implications for structure and development of the lower crust, *Earth and Planetary Science Letters*, 119, 1–18, 1993.
- Kulp, J. L.: Isotopic dating and the geologic time scale, 1955.
- Lee, J. K. W., Williams, I. S., and Ellis, D. J.: Pb, U and Th Diffusion in Natural Zircon, *Nature*, 390, 159–162, <https://doi.org/10.1038/36554>, 1997.
- 495 Liu, F. and Liou, J.: Zircon as the best mineral for P–T–time history of UHP metamorphism: a review on mineral inclusions and U–Pb SHRIMP ages of zircons from the Dabie–Sulu UHP rocks, *Journal of Asian Earth Sciences*, 40, 1–39, 2011.
- Ludwig, K. R.: On the treatment of concordant uranium-lead ages, *Geochimica et Cosmochimica Acta*, 62, 665–676, 1998.
- Mansour, J., Giordani, J., Moresi, L., Beucher, R., Kaluza, O., Velic, M., Farrington, R., Quenette, S., and Beall, A.: Underworld2: Python geodynamics modelling for desktop, HPC and cloud, *Journal of Open Source Software*, 5, 1797, 2020.
- 500 Moresi, L., Dufour, F., and Mühlhaus, H.-B.: A Lagrangian integration point finite element method for large deformation modeling of viscoelastic geomaterials, *Journal of computational physics*, 184, 476–497, 2003.
- Moresi, L., Mansour, J., Giordani, J., Knepley, M., Knight, B., Graciosa, J. C., Gollapalli, T., Lu, N., and Beucher, R.: Underworld3: Mathematically Self-Describing Modelling in Python for Desktop, HPC and Cloud, *Journal of Open Source Software*, 10, 7831, <https://doi.org/10.21105/joss.07831>, 2025a.
- 505 Moresi, L., Mansour, J., Giordani, J., Knepley, M., Knight, B., Graciosa, J. C., Gollapalli, T., Lu, N., and Beucher, R.: Underworld3: Mathematically Self-Describing Modelling in Python for Desktop, HPC and Cloud, <https://doi.org/10.5281/zenodo.16838572>, 2025b.
- Praharaj, P. and Rekha, S.: Tectonometamorphic evolution of the Trivandrum and Southern Madurai blocks in the Southern Granulite Terrane, south India: correlation with south-central Madagascar, *Geological Magazine*, 159, 1569–1600, 2022.
- Reich, M., Ewing, R. C., Ehlers, T. A., and Becker, U.: Low-temperature anisotropic diffusion of helium in zircon: implications for zircon (U–Th)/He thermochronometry, *Geochimica et Cosmochimica Acta*, 71, 3119–3130, 2007.
- 510 Rubatto, D.: Zircon: The metamorphic mineral, *Reviews in mineralogy and geochemistry*, 83, 261–295, 2017.

- Rubatto, D. and Hermann, J.: Experimental zircon/melt and zircon/garnet trace element partitioning and implications for the geochronology of crustal rocks, *Chemical Geology*, 241, 38–61, 2007.
- Schoene, B.: 4.10-u–th–pb geochronology, *Treatise on geochemistry*, 4, 341–378, 2014.
- 515 Strutt, R. J.: The accumulation of helium in geological time.-II, *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 83, 96–99, 1909.
- Taylor, R. J., Clark, C., Fitzsimons, I. C., Santosh, M., Hand, M., Evans, N., and McDonald, B.: Post-peak, fluid-mediated modification of granulite facies zircon and monazite in the Trivandrum Block, southern India, *Contributions to Mineralogy and Petrology*, 168, 1–17, 2014.
- 520 Team, T. M. D.: Matplotlib: Visualization with Python, <https://doi.org/10.5281/zenodo.14940554>, 2025.
- Tera, F. and Wasserburg, G.: U-Th-Pb systematics in three Apollo 14 basalts and the problem of initial Pb in lunar rocks, *Earth and Planetary Science Letters*, 14, 281–304, 1972.
- Ullah, M., Klötzli, U., Wadood, B., Khubab, M., Islam, F., Shehzad, K., and Ahmad, R.: Pb diffusion in perfect and defective zircon for thermo/petro-chronological investigations: An atomistic approach, *Chemical Geology*, 640, 121 750, 2023.
- 525 Wasserburg, G.: Diffusion processes in lead-uranium systems, *Journal of Geophysical Research*, 68, 4823–4846, 1963.
- Wetherill, G.: Discordant uranium-lead ages: 2. Discordant ages resulting from diffusion of lead and uranium, *Journal of Geophysical Research*, 68, 2957–2965, 1963.
- Zhang, Z., Li, Z., and Wu, Y.: Advection–Diffusion lattice Boltzmann method with and without dynamical filter, *Frontiers in physics*, 10, 875 628, 2022.