

Response to comments of Referee #1 on “MONKI: a three-dimensional Monte Carlo simulator for transfer of polarised light in planetary atmospheres” by Victor Trees et al.

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We thank the reviewer for his/her careful reading and for the comments and suggestions, which have improved the manuscript. Below, we give in *blue italic* the reviewer’s comment, in black our response, in *black italic* copied text from the manuscript, and in *red italic* the changed or new text in the manuscript.

5 *First of all, I apologize to the authors and the editor(s) for the delay in producing this referee report.*

The manuscript describes a new implementation of the Monte Carlo (MC) method to solve the vector radiative transfer equation. The manuscript confirms recent evidence that MC methods can combine both flexibility and accuracy. The first had long been clear, and indeed MC methods had been the usual choice for complex geometries. The second attribute, however, had
10 *been less obvious to the radiative transfer community. The current paper gives further evidence that MC methods are "exact" and, when properly implemented, can achieve arbitrary accuracies if one is willing to pay the cost of running sufficiently long simulations. In my opinion, this is the main point of the paper and I believe it's an important one.*

The manuscript provides enough context to understand the implementation of the MC method, and demonstrates sufficiently its
15 *performance with comparisons against standard benchmark cases. This said, my main criticism of the manuscript, however, is that the implementation is based on "intuition", i.e. on tracking what happens to the simulated photons at each collision event. That makes it difficult to understand, for example, why neglecting the polarization state of the photon may lead to solutions that fail to convergence. In the same vein, it's difficult to understand why the implementation neglects variance reduction techniques. The integral formulation described in Garcia Munoz & Mills, 2015 (based on O'Brien, JQSRT, 1992) gives a more*
20 *straightforward framework to tackle these issues.*

We thank the reviewer for these important comments. We agree that an implementation-level description based on following simulated photons through successive collision events is intuitive, but does not by itself provide the clearest framework for

understanding why certain Monte Carlo formulations converge and others do not. We have therefore substantially revised the manuscript in this respect.

25 In particular, we added Appendices G and H, in which we derive the forward and backward Monte Carlo equations for polarised radiative transfer directly from the vector radiative transfer equation, including the analytical formulation used to sample the polarised scattering direction. This addition strengthens the methodological contribution of the paper, because it provides a complete and unified theoretical framework for polarised Monte Carlo radiative transfer in both forward and backward modes, rather than only an implementation-oriented description of the code. Related derivations exist in the literature, 30 for example in Marshak and Davis (2005) and Deutschmann et al. (2011), who neglected polarisation, or in Marchuk et al. (1980), Deutschmann (2015), and Wang et al. (2019), who included polarisation but did not start their derivations from the vector radiative transfer equation and also neglected polarisation-dependent sampling of the scattering direction. In addition, those derivations are distributed across different chapters, sometimes omit intermediate derivation steps, or have inconsistent physical units in their equations. Therefore, they do not provide the same complete and internally consistent derivation in the 35 form presented here. We believe this framework is useful not only for understanding the implementation of MONKI itself, but also for readers who wish to validate or extend polarised Monte Carlo radiative transfer methods.

We also added a dedicated subappendix, App. G9, in which we discuss the classical or alternative sampling approach often used in Monte Carlo codes, such as MYSTIC. There we show explicitly that, in this approach, the scattering direction is sampled from the PDF corresponding to unpolarised incident radiation, after which a correction factor is applied. This 40 subappendix makes clear why neglecting the incident photon’s polarisation state during the sampling step can lead to large variance, apparent bias in finite simulations, and poor convergence in optically thick and strongly polarising atmospheres.

In addition, the revised manuscript now demonstrates this point numerically in new Figs. 5 and 6b by comparing MONKI with this classical alternative approach (see also Figs. 2 and 3 below). These new results show that MONKI converges well in both forward and backward modes for optically thick and strongly polarising atmospheres, whereas the classical approach 45 shows apparent bias in finite simulations and poor convergence. García Muñoz and Mills (2015) already demonstrated this for a backward Monte Carlo formulation. Other Monte Carlo codes that take into account polarisation-dependent sampling of scattering directions are 3DMCPOL (Cornet et al., 2010), SMART-G (Ramon et al., 2019), and the code of Bartel and Hielscher (2000), but only do so in forward mode. To the best of our knowledge, MONKI is the first Monte Carlo code that consistently uses polarisation-dependent sampling successfully in both forward and backward modes.

50 We therefore believe that the revised manuscript now addresses the reviewer’s concern more satisfactorily. The added Appendices G and H provide the formal integral framework that was missing from the original version, and App. G9 shows explicitly why neglecting the photon’s polarisation state during the sampling step can lead to large variance, apparent bias in finite simulations, and poor convergence. Combined with the new comparison in Figs. 5 and 6b, this makes the rationale of the implementation much clearer and places it in the broader methodological framework suggested by the reviewer.

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We added the following sentences to the abstract: *"Unlike classical Monte Carlo approaches that rely on post-sampling weight corrections and can therefore produce rare but disproportionately large photon weights, MONKI samples scattering directions*

from the full polarisation-dependent distribution, consistently in both forward and backward Monte Carlo modes. We provide the corresponding theoretical framework in the appendices. Because of this exact polarised sampling approach, the code converges robustly for optically thick and/or strongly polarising atmospheres."

And to the introduction, line 87: "Other Monte Carlo codes that take into account polarisation-dependent sampling of scattering directions are 3DMCPOL (Cornet et al., 2010), SMART-G (Ramon et al., 2019), and the code of Bartel and Hielscher (2000), but only do so in forward mode."

And to the introduction, line 95: "By fully tracking the polarisation state history of photons along their trajectories and sampling scattering directions from the corresponding polarisation-dependent distribution, MONKI converges in both forward and backward modes, including for optically thick and strongly polarising planetary atmospheres. The code has been written in modern Fortran. In addition to presenting the code itself, this paper provides, we believe for the first time, a complete derivation of the underlying forward and backward Monte Carlo equations and polarisation-dependent sampling formulae, thereby offering a unified theoretical framework for readers who wish to understand, validate, or extend polarised Monte Carlo radiative transfer methods."

And we modified line 98 of the introduction: "The appendices contain detailed explanations of the formulae implemented in MONKI, together with the derivation of the forward and backward Monte Carlo equations for polarised radiative transfer."

We changed the paragraph in the Conclusion starting on line 480 into:

"Although MONKI is not the first code of its kind, it has fundamental methodological differences from other Monte Carlo codes. A common limitation of Monte Carlo codes is their failure to converge in optically thick and strongly polarising atmospheres. This issue arises when approximations are made in the sampling of scattering directions, by neglecting the photon's polarisation state and correcting afterwards by weight factors. MONKI instead samples scattering directions from the full polarisation-dependent distribution while fully tracking the polarisation history of each photon along its trajectory. As demonstrated in this paper, this exact polarised sampling approach converges in both forward and backward modes, including for optically thick and strongly polarising planetary atmospheres. In addition, the appendices provide a complete derivation of the corresponding forward and backward Monte Carlo equations and polarisation-dependent sampling formulae, thereby offering a unified theoretical framework for polarised Monte Carlo radiative transfer."

The manuscript is very well written. References in the manuscript seem complete.

I didn't check, but the MC implementation appears to be publicly available, which will help further disseminate the ideas developed here.

The code is indeed publicly available, and we agree that this will help further disseminate the ideas developed here.

I recommend the manuscript for publication, after addressing the following very minor comments.

95

Methods. It took me a while to figure out that the MC implementation is valid only for plane-parallel atmospheres, perhaps because the introduction mentions the application of polarimetry to disk-integrated observations. I suggest making it clear in the Introduction (#1) or Methods (#2) sections.

In the introduction, we changed line 90 from 'MONKI can perform simulations in both forward and backward modes (with reversed photon directions) for horizontally homogeneous and inhomogeneous (3D cloudy) scenes.' into 'MONKI can perform simulations in both forward and backward modes (with reversed photon directions) for horizontally homogeneous and inhomogeneous (3D cloudy) *plane-parallel atmospheres.*'

** line 16. w.r.t. is not standard, I think.*

105 We changed 'w.r.t.' into '*with respect to*'

** line 20. "at the top of at the atmosphere". Language issue.*

We changed 'at the top of at the atmosphere (TOA)' into '*at the top of the atmosphere (TOA)*'

110 ** Fig. 1. In my printout, angles theta' and phi' become corrupt. It may just be a problem with my pdf reader though. Further, it would help if you could show the actual geometry of the problem, with the cells in both the x and y directions. This would make it clearer that the current geometry is plane parallel.*

We thank the reviewer for pointing out that the angles became corrupt in their printout. We did not, however, encounter this problem. As requested, we made a sketch of the 3D rectangular computational domain, and a photon path in forward and backward mode as shown in Fig. 1. We have also added this figure to the paper.

** In Methods section, please state how $V > 0$ is defined.*

We added the following explanation to Stokes vector introduction of the Method section: '*In particular, for $Q > 0$ ($Q < 0$), the direction of polarisation is parallel (perpendicular) to the reference plane. For $U > 0$ ($U < 0$), the direction of polarisation is rotated by $+45^\circ$ (-45°) from the direction parallel to the reference plane, with positive rotations defined as anti-clockwise when looking in the propagation direction of the light. For $V > 0$, there is an excess of right-handed circular polarisation, that is, the electric field vector rotates clockwise when looking in the propagation direction of the light.*'

125 ** line 166. Having the code decide between scattering by a gas molecule or an aerosol particle (or different types of them, if different aerosol types coexist) is inefficient in a MC method. It would be notably faster if the optical (scattering, absorption) properties are properly averaged and assigned to "average" particles. Could the authors please comment on this?*

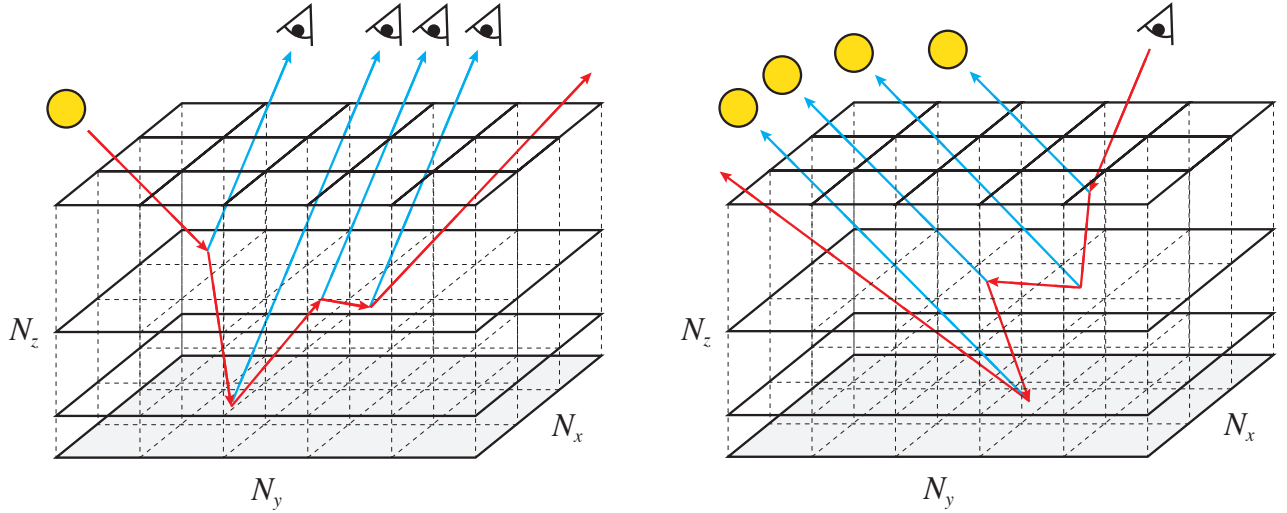


Figure 1. Sketches of the 3D computational domain of MONKI and of a photon trajectory (red arrows) in forward mode (left) and backward mode (right). Here, the numbers of columns in the x and y directions are $N_x = 3$ and $N_y = 5$, respectively, while $N_z = 3$ is the number of layers. The domain is periodic in the horizontal directions. At every scattering event (i.e. each change in direction of the red arrow), the probability of scattering toward the detector (in forward mode) or toward the Sun (in backward mode) is computed using the local estimation method (blue arrows).

We thank the reviewer for this suggestion. Indeed, the separation of photon paths into gas or aerosol (type) scattering is, in terms of path sampling, less efficient than considering average optical properties (phase matrices and single scattering albedos, weighted by their relative scattering optical thicknesses).

130 However, the dominant computational cost lies in sampling scattering directions from the single-scattering phase matrix. The construction and sampling of cumulative distribution functions (CDFs) for aerosol phase matrices are computationally expensive. In MONKI, these CDFs are precomputed once per aerosol type and are independent of local number densities (see Appendix F of the MONKI paper). If gas and multiple aerosol types were mixed into an effective scatterer, the phase matrix, and thus its CDF, would in general differ for each grid cell due to spatially varying composition, especially in 3D
 135 inhomogeneous scenes. This would require either storing a large number of CDFs or recomputing them on the fly, both of which are computationally inefficient.

Moreover, Rayleigh scattering is treated separately using an efficient rejection-sampling approach (Appendix E of the MONKI paper). Mixing Rayleigh and aerosol scattering into a single phase matrix would remove this simplicity without a clear performance gain.

140 For these reasons, we chose to sample gas and aerosol scattering events separately, which offers a favourable balance between computational efficiency, memory usage, and algorithmic clarity.

* pg. 8. Footnote. Setting $w=1e-16$ to terminate a simulation seems unnecessarily stringent. My guess is that the calculation converges for $w=1e-6$. Could the authors please confirm this statement?

145 Indeed, using a larger cut-off value for w would reduce the computation time. However, this would also introduce a small negative bias in the simulated radiances, because photons would then be terminated prematurely rather than being followed until their weight becomes numerically negligible. The magnitude of this bias is proportional to the chosen cut-off value. In this paper, we aimed to present numerically unbiased results, and therefore chose the smallest practical value, $w = 10^{-16}$, close to machine precision.

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* pg. 9. Just wondering, how easy would it be to extend the surface treatment to non-Lambertian reflection?

There are two ways to implement non-Lambertian surface reflection in a Monte Carlo code (see, e.g., Mayer, 2009).

The first and simplest approach is to sample the reflection direction from a simple angular distribution (i.e., isotropic and cosine-weighted over the hemisphere, as in Appendix C) and to scale the photon weight (and, when including polarisation, the
155 normalised Stokes vector) by the surface BRDF (or surface reflection matrix) evaluated for the chosen incident and reflected directions. This approach is unbiased but may lead to large variance when the BRDF is strongly peaked (e.g., for specular reflection from a flat water surface), because directions close to the specular peak are only rarely sampled, while those rare events carry very large weights.

A more efficient, but more complex, approach is to interpret the reflected intensity as a probability density function and to
160 sample the reflection angles from the corresponding cumulative distribution functions. This can be done using direct inversion if an analytic CDF exists (as for Lambertian reflection; Appendix C), via rejection sampling (Appendix E), or by using pre-computed CDF tables (Appendix F), analogous to the treatment of atmospheric scattering already implemented in MONKI. Consequently, extending MONKI to non-Lambertian surfaces mainly requires providing a suitable PDF (based on the reflected intensity, Appendix D) and applying the corresponding reflection matrix to update the photon's Stokes vector.

165

* line 261. "In the backward mode, every photon that is not completely absorbed contributes to the result, assuming that the light source is at an infinite distance." I was a bit puzzled to read this. My understanding is that the backward model works best when the observer has a narrow entry cone or, equivalently, when the observer is very far. For the backward implementation, it is no problem if the star subtends a finite solid angle.

170 In MONKI we use the local estimation method. For each scattering or surface reflection event, the contribution to the signal is computed from the probability of scattering toward the detector direction, multiplied by the direct transmittance along that path. When the detector is at infinite distance in forward mode, we compute the radiance in a single viewing direction, represented by a collimated bundle of plane-parallel rays. In this case, every scattering location can, in principle, be connected to the detector in that viewing direction, and the corresponding scattering probability can be evaluated for all events. When the detector is at a
175 finite distance in forward mode, it has a specific location in 3D space and a finite field of view. Then, some scattering locations may lie outside the detector's field of view and cannot be connected to the detector by a straight line, and those events do not

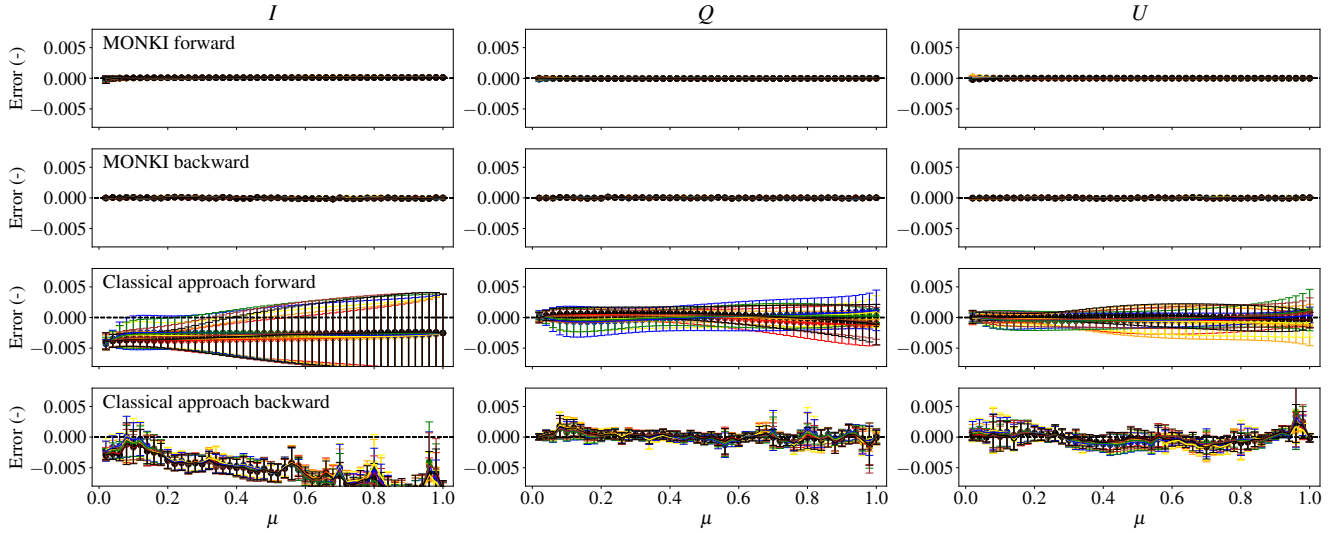


Figure 2. Differences between the MONKI results and those of Natraj and Hovenier, for MONKI in forward mode (first row) and in backward mode (second row), fully taking into account polarisation of light when sampling scattering directions. The third and fourth rows show the results when using instead the classical scattering direction sampling approach in MONKI, as used by, e.g., MYSTIC (see App. G9). The error bars indicate the ± 2 standard deviations of the simulated results. Those error bars, and the seven different colours referring to the different values of φ , are virtually indistinguishable for the MONKI forward and backward modes.

contribute in forward mode. In backward mode this geometric restriction does not occur, because photons are launched from the detector itself.

Finally, the statement in the manuscript refers to the illumination geometry: MONKI assumes a collimated light source at
180 infinite distance such that in backward mode all scattering locations are directly illuminated and can, in principle, be connected to the light source direction and contribute.

We revised the first paragraph of the backward mode section: *'MONKI can be used in both forward and backward modes. In backward mode, photon paths are traced from the detector towards the light source (the Sun), as illustrated in Fig. 1. The
185 detector radiance Stokes vector is again computed using the local estimation method, here with the option to evaluate multiple solar zenith angles ϑ_0 at a fixed solar azimuth angle φ_0 simultaneously.'*

*The backward mode is particularly useful when the detector is at a finite distance from, or even inside, the atmosphere-surface system. Although this application is not treated in the present paper, it is important for future extensions of MONKI to spherical atmospheres in combination with a detector at a finite distance. In such cases, the path from a scattering event to the
190 detector depends on the location of the event, and the detector cannot be represented by a single collimated viewing direction throughout the domain. Consequently, in forward mode, only a small fraction of photons launched from the source would reach the detector's field of view, severely limiting the code's efficiency. In backward mode, by contrast, photons are launched from*

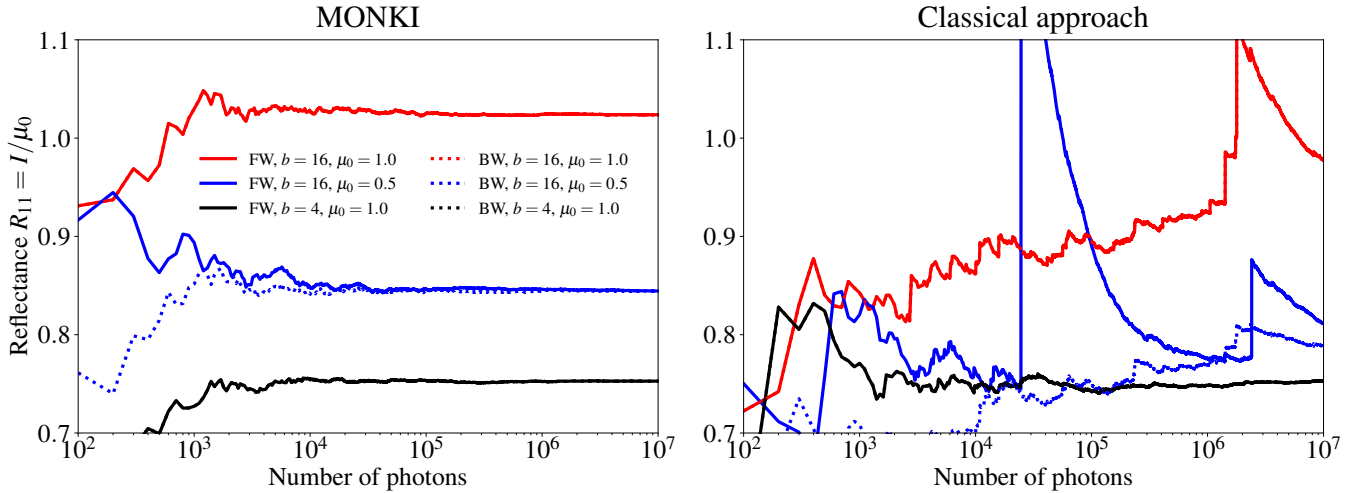


Figure 3. Left figure: convergence history of the TOA reflectance, R_{11} , by MONKI in forward mode (solid lines) and backward mode (dotted lines) for a Rayleigh scattering atmosphere with $b = 4$ and $\mu_0 = 1.0$ (black), $b = 16$ and $\mu_0 = 0.5$ (blue), and $b = 16$ and $\mu_0 = 1.0$ (red). The viewing geometry is nadir ($\mu = 1.0$). For $\mu_0 = 1.0$, the solid and dotted lines overlap. Right figure: similar, but using the classical scattering direction sampling approach in MONKI, as used by, e.g., MYSTIC (see App. G9). The jumps are caused by infrequent but large photon weights.

the detector, so that all traced photon paths lie within its field of view. Assuming collimated illumination from a light source at infinite distance, every backward-traced photon that is not completely absorbed can, in principle, be connected to the source direction and therefore contributes to the local estimate. We refer to Appendix G8 for the derivation of the backward Monte Carlo equations for polarised light as used in this section.'

** line 324. Did the authors "switch off" the direction sampling based on the polarization state of the photon and, in that case, did they run into problems to converge the model? I feel curious to know the answer.*

We have included Fig. 2 below in the paper, which shows the differences between the MONKI results and the benchmark results for the thick Rayleigh atmospheres of Natraj and Hovenier (2012), in forward and backward modes (top two rows). The two bottom rows show the results obtained when “switching off” the direction sampling based on the polarisation state of the photon. More precisely, we use there the classical approach in MONKI, as is also used in MYSTIC: neglecting the polarisation state of the incident photon when sampling a new scattering direction, and correcting for this neglect by adjusting the photon’s weighting factor. We have included the new subappendix G9 in the paper, explaining the formulae involved in this classical approach and why it may fail to converge for optically thick and strongly polarising atmospheres.

The reason for the larger apparent bias and variance in the classical approach is its poor convergence for these cases, as pointed out by García Muñoz and Mills (2015). We added the following paragraph to Section 3.2:

210 *'MONKI fully accounts for the photon's polarisation history when sampling scattering directions. However, when the classical scattering-direction sampling approach is used in MONKI instead, Fig. 2 (two bottom rows) shows clear deviations and a much larger variance. The classical approach (see Collins et al., 1972) neglects the polarisation state of the incident photon when sampling a new scattering direction, and corrects for this neglect by adjusting the photon's weighting factor (see Appendix G9). Examples of codes that use this approach are MYSTIC (Emde et al., 2010), MSCART (Wang et al., 2017, 2019), SPARTA (Barlakas et al., 2016), IVMCM (Gay et al., 2010), and the backward mode of SMART-G (Ramon et al., 2019). García*
215 *Muñoz and Mills (2015) pointed out that for thick, strongly polarising atmospheres, such as those used by Natraj and Hovenier (2012), those codes may fail to converge. This is because, upon multiple scattering, these correction factors are multiplied repeatedly. In the unlikely event that a photon path contains several strongly polarising scattering directions, this can produce infrequent but very large photon weights (see Appendix G9).'*

We also demonstrate the non-convergence of the classical approach by adding the convergence history of that approach to
220 Fig. 3 above.

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Response to comments of Referee #2 on “MONKI: a three-dimensional Monte Carlo simulator for transfer of polarised light in planetary atmospheres” by Victor Trees et al.

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We thank the reviewer for his/her careful reading and for the comments and suggestions, which have improved the manuscript. Below, we give in *blue italic* the reviewer’s comment, in black our response, in *black italic* copied text from the manuscript, and in *red italic* the changed or new text in the manuscript.

5 *General comments:*

This paper presents a new 3D atmospheric radiative transfer model based on Monte-Carlo Method.

10 *It presents the methods commonly used to track the photons through the atmosphere and to compute the reflectances. Several validation cases are then presented showing that the MONKI code can correctly reproduce previous results. Finally, a simulation of the Earth and Venus atmosphere were computed and analysed, demonstrating that the code is able of handling different optical properties.*

The paper is well written and clear. However, it does not really contribute to anything new.

15 We thank the reviewer for this comment. We agree that MONKI is not the first three-dimensional Monte Carlo radiative transfer code for polarised light. However, the present paper does make a clear methodological contribution.

First, MONKI samples scattering directions from the full polarisation-dependent distribution, fully accounting for the polarisation history of the photon. This is fundamentally different from the classical Monte Carlo approach of Collins et al. (1972) used by, e.g., MYSTIC, MSCART, SPARTA, IVMCM, and the backward mode of SMART-G, in which polarisation is neglected in the scattering-direction sampling and the resulting approximation is corrected afterwards by weight factors. García Muñoz and Mills (2015) did take polarisation fully into account during the sampling in their backward Monte Carlo code, and showed that the classical approach can lead to poor convergence in optically thick and strongly polarising atmospheres. Other

Monte Carlo codes that take into account polarisation-dependent sampling of scattering directions are 3DMCPOL (Cornet et al., 2010), SMART-G (Ramon et al., 2019), and the code of Bartel and Hielscher (2000), but only do so in forward mode.

25 To the best of our understanding, MONKI is the first Monte Carlo code that consistently uses polarisation-dependent sampling successfully in both forward and backward modes.

In the revised paper, we have included Figs. 5 and 6b (Figs. 1 and 2 below), which now also show results obtained with the classical approach in MONKI. MONKI converges well in both forward and backward modes, whereas the classical approach shows large variance and apparent bias in finite simulations. We have included Appendix G9, which explains the formulae

30 involved in the classical approach and why it does not converge for thick polarising atmospheres.

Second, we added Appendices G and H, in which we derive the forward and backward Monte Carlo equations for polarised radiative transfer directly from the vector radiative transfer equation, including the analytical formulation used to sample the polarised scattering direction. This addition strengthens the methodological contribution of the paper, because it provides a complete and unified theoretical framework for polarised Monte Carlo radiative transfer in both forward and backward

35 modes, rather than only an implementation-oriented description of the code. Related derivations exist in the literature, for example in Marshak and Davis (2005) and Deutschmann et al. (2011), who neglected polarisation, or Marchuk et al. (1980), Deutschmann (2015), and Wang et al. (2019), who included polarisation but did not start their derivation from the vector radiative transfer equation, and also neglected polarisation-dependent sampling of the scattering direction. In addition, those derivations are distributed across different chapters, sometimes omit intermediate derivation steps, or have inconsistent physical

40 units in equations. Therefore, they do not provide the same complete and internally consistent derivation in the form presented here. We believe this framework is useful not only for understanding the implementation of MONKI itself, but also for readers who wish to validate or extend polarised Monte Carlo radiative transfer methods.

Third, the paper presents a precomputation strategy for optical paths to the detector, which improves computational efficiency without changing the result. To our knowledge, this feature has not been described in this form for polarised local-estimation

45 Monte Carlo radiative transfer.

We have revised the manuscript to make these contributions more explicit. The novelty of the paper therefore lies not merely in presenting another Monte Carlo code, but in showing that exact polarised direction sampling can be implemented in both forward and backward Monte Carlo modes, in demonstrating the resulting robust convergence for optically thick and strongly polarising atmospheres, and in providing a complete and unified theoretical framework for polarised Monte Carlo radiative

50 transfer.

We added the following sentences to the abstract: *"Unlike classical Monte Carlo approaches that rely on post-sampling weight corrections and can therefore produce rare but disproportionately large photon weights, MONKI samples scattering directions from the full polarisation-dependent distribution, consistently in both forward and backward Monte Carlo modes. We provide*

55 *the corresponding theoretical framework in the appendices. Because of this exact polarised sampling approach, the code converges robustly for optically thick and/or strongly polarising atmospheres."*

And to the introduction, line 87: *"Other Monte Carlo codes that take into account polarisation-dependent sampling of scattering directions are 3DMCPOL (Cornet et al., 2010), SMART-G (Ramon et al., 2019), and the code of Bartel and Hielscher (2000), but only do so in forward mode."*

And to the introduction, line 95: *"By fully tracking the polarisation state history of photons along their trajectories and sampling scattering directions from the corresponding polarisation-dependent distribution, MONKI converges in both forward and backward modes, including for optically thick and strongly polarising planetary atmospheres. The code has been written in modern Fortran. In addition to presenting the code itself, this paper provides, we believe for the first time, a complete derivation of the underlying forward and backward Monte Carlo equations and polarisation-dependent sampling formulae, thereby offering a unified theoretical framework for readers who wish to understand, validate, or extend polarised Monte Carlo radiative transfer methods."*

And we modified line 98 of the introduction: *"The appendices contain detailed explanations of the formulae implemented in MONKI, together with the derivation of the forward and backward Monte Carlo equations for polarised radiative transfer."*

We changed the paragraph in the Conclusion starting on line 480 into:

"Although MONKI is not the first code of its kind, it has fundamental methodological differences from other Monte Carlo codes. A common limitation of Monte Carlo codes is their failure to converge in optically thick and strongly polarising atmospheres. This issue arises when approximations are made in the sampling of scattering directions, by neglecting the photon's polarisation state and correcting afterwards by weight factors. MONKI instead samples scattering directions from the full polarisation-dependent distribution while fully tracking the polarisation history of each photon along its trajectory. As demonstrated in this paper, this exact polarised sampling approach converges in both forward and backward modes, including for optically thick and strongly polarising planetary atmospheres. In addition, the appendices provide a complete derivation of the corresponding forward and backward Monte Carlo equations and polarisation-dependent sampling formulae, thereby offering a unified theoretical framework for polarised Monte Carlo radiative transfer."

In the conclusion of the article, three contributions of the MONKI code are listed, but are not convincing as they stand : 1- line 481 "common limitation of Monte Carlo codes is their failure to converge in optically thick and strongly polarising atmospheres". Can the authors be more specific about the values beyond which the convergence does not occur and give an example of such atmosphere.

We have added the convergence history when using the classical approach as used by, e.g., MYSTIC, to Fig. 2 below. As can be seen, with a Rayleigh-scattering optical thickness of 4, the classical approach approaches the correct value, albeit with slower convergence, but with an optical thickness of 16 severe problems occur. An example of a planet with such a thick Rayleigh-scattering atmosphere is Venus, with a surface pressure of approximately 92 bar (see Fig. 9 of the paper). Indeed, for a detector in space, the thick clouds high in the atmosphere often scatter the light more strongly than the gas (depending on

the wavelength, and the illumination and viewing geometry), as shown in Fig. 13 of the paper. However, a detector below the clouds (mounted on a probe or located on the surface) would be more sensitive to scattering by the gas and requires simulations
 95 by an accurate code such as MONKI.

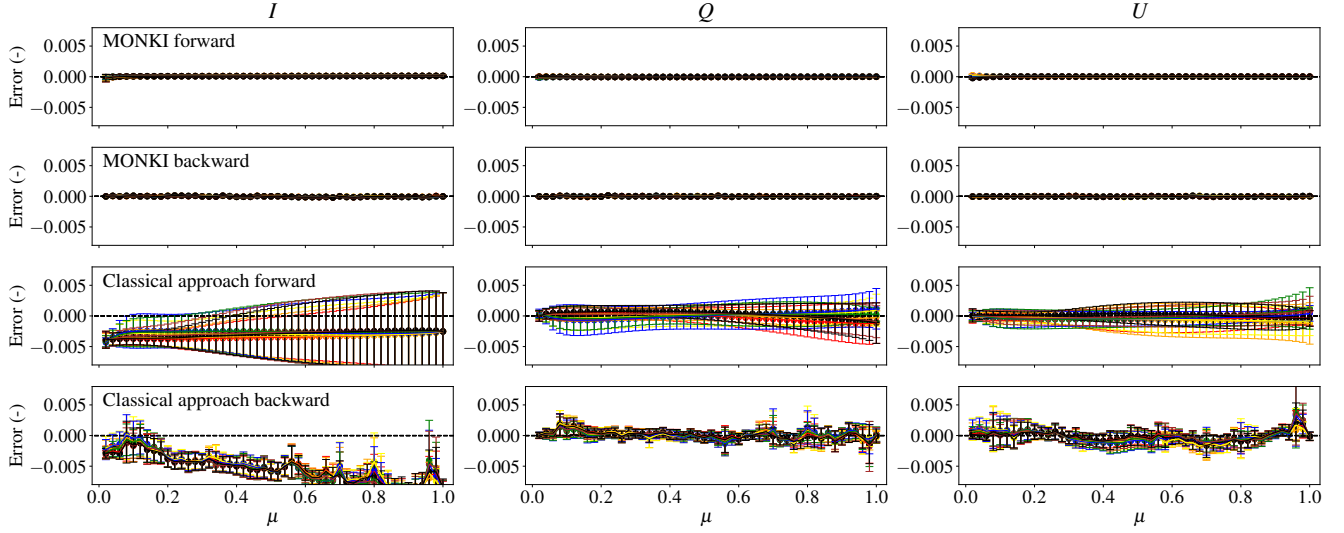


Figure 1. Differences between the MONKI results and those of Natraj and Hovenier, for MONKI in the forward mode (first row) and in the backward mode (second row), fully taking into account polarisation of light when sampling scattering directions. The third and fourth rows show the results when using instead the classical scattering direction sampling approach in MONKI as used by, e.g., MYSTIC (see App. G9). The error bars indicate the ± 2 standard deviations of the simulated results. Those error bars, and the seven different colours referring to the different values of φ , are virtually indistinguishable for the MONKI forward and backward modes.

2- *"The current version of MONKI does not use variance reduction techniques". I suppose other codes may choose whether or not to use variance reduction technique, which are often necessary to achieve convergence in complex 3D atmospheres. So, it does not appear to be a real advantage.*

100 MONKI does indeed not use other variance reduction techniques than the local estimation method. We agree that not having the option of extra variance reduction techniques may not be an advantage for users who use MONKI only as a tool. However, this design choice has advantages for us as code developers and for users who want to understand the details of the code, as explained on line 489: *"By omitting those variance reduction techniques, MONKI produces unbiased results while preserving readability and simplicity, and thus adaptability when future extensions of the code are to be implemented."* Simultaneously, we
 105 have presented an alternative method to speed up the simulations, as explained on line 491: *"Instead of using such techniques, we increase the number of photons to achieve higher precision when needed."*

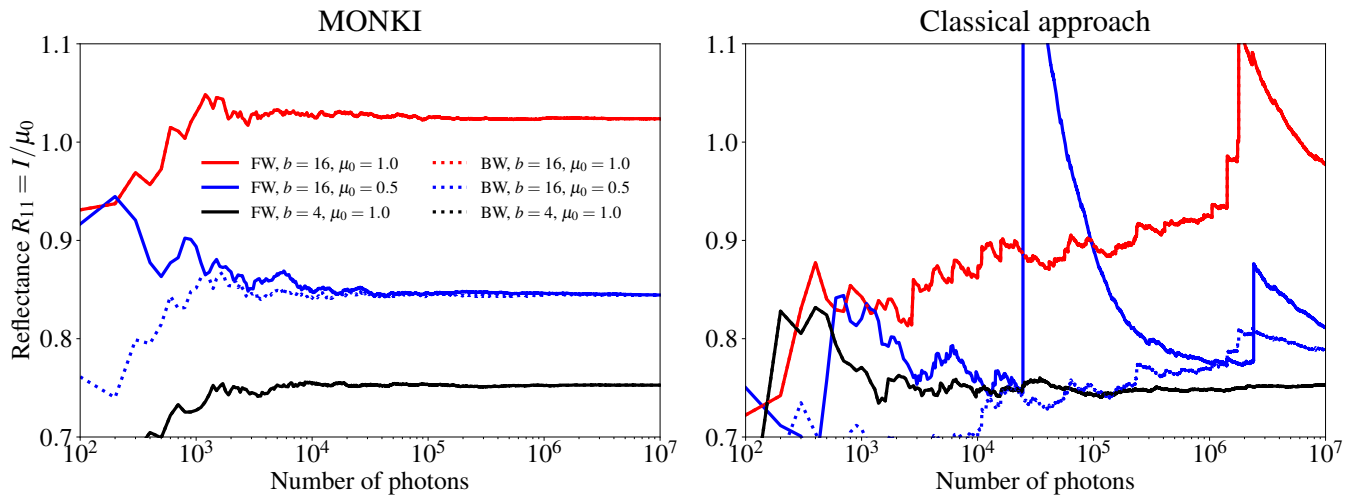


Figure 2. Left figure: convergence history of the TOA reflectance, R_{11} , by MONKI in forward mode (solid lines) and backward mode (dotted lines) for a Rayleigh scattering atmosphere with $b = 4$ and $\mu_0 = 1.0$ (black), $b = 16$ and $\mu_0 = 0.5$ (blue), and $b = 16$ and $\mu_0 = 1.0$ (red). The viewing geometry is nadir ($\mu = 1.0$). For $\mu_0 = 1.0$, the solid and dotted lines overlap. Right figure: similar, but using the classical scattering direction sampling approach in MONKI, as used by, e.g., MYSTIC (see App. G9). The jumps are caused by infrequent but large photon weights.

3- “In order to speed up MONKI’s simulations, we implemented and presented a precomputation approach for optical paths to the detector”. This may be a new method for obtaining converging results more quickly but quantitative values showing the gain in computational speed have to be added to the article. Note that adaptative grid was already introduced in Villefranque et al. (2019)

The speed improvement obtained with the precomputation approach depends strongly on the number of grid cells (layers and columns) used in the simulations. More grid cells can give more potential speed improvement, but only if (most of) those grid cells are cloud-free. In addition, the potential speed improvement increases with the number of simultaneous zenith angles (viewing zenith angles in forward mode, and solar zenith angles in backward mode) that are computed, but only if (most of) those optical paths are cloud-free. Therefore, it is difficult to give a single number for the speed improvement. As an example, however, we have added the following footnote to the last sentence of the precomputation approach section:

‘The speed increases with the number of cloud-free grid cells and viewing directions. For example, for the cloudy Earth sample signal of Fig. 11, for $\theta_0 = \varphi = \varphi_0 = 0^\circ$ and 90 viewing zenith angles simultaneously, the speed improvement is a factor 1.9.’

We thank the reviewer for mentioning the adaptive-grid method of Villefranque et al. (2019). Our precomputation approach is fundamentally different. In MONKI, the original atmospheric grid is not modified: photons are still propagated through the same grid during the Monte Carlo simulation. Our speed-up applies only to the evaluation of the optical path from a scattering or surface event to the detector in the local-estimation step. For paths that are cloud- and aerosol-free, we precompute the

125 detector optical thickness and thereby avoid repeated event-to-detector propagations. In contrast, Villefranque et al. accelerate the ray tracing through the full 3-D volume itself by means of hierarchical adaptive grids. In principle, our optical-path pre-computation could be combined with such an adaptive-grid strategy in future work.

130 *4 - “MONKI can import different particle scattering matrices and can, in principle, also simulate light scattered by non-spherical aerosol particles and ice clouds”. The same applies for other code as long as optical properties are known.*

We thank the reviewer for pointing this out. We do not claim that the ability to simulate light scattered by non-spherical particles is unique to MONKI.

135 *To increase the novelty of the paper, I would recommend to go deeply in the analysis of the methods used to sample the scattering direction at each interaction presented in appendices E and F. I may be wrong, but it seems to me that the rejection method is not commonly used in Monte-Carlo Atmospheric radiative transfer. A deeper analysis of this method comparing to the conventionnal one (tabulated cumulative distribution function) in terms of accuracy, convergence and computation time, could be valuable to increase the interest of this publication for the community.*

140 We thank the reviewer for this helpful suggestion. We agree that the choice of sampling method deserves clarification. In the present work, however, the main methodological point is not the generic comparison between acceptance-rejection sampling and tabulated inverse-CDF sampling as numerical techniques, but the fact that the scattering direction is sampled from the full polarisation-dependent probability distribution, rather than from an approximate distribution corrected afterwards by weight factors.

145 We have therefore revised the manuscript to analyse this point more deeply. In particular, Appendices G and H and the new Figs. 1 and 2 (Figs. 5 and 6b in the new paper) now compare MONKI’s exact polarised sampling approach with the classical alternative approach, and show the consequences for bias and convergence in optically thick and strongly polarising atmospheres. This increases the novelty of the paper (see also the reply to the first comment above).

150 Regarding the specific use of the acceptance-rejection method in Appendices E and F: for Rayleigh scattering, we use this method because obtaining the CDF, and especially its inverse, for the full joint polarised PDF is not straightforward when the polarisation of the incident beam is taken into account. For Mie scattering, we likewise use acceptance-rejection, but with an envelope function constructed to closely follow the strongly forward-peaked distribution and thus maintain a high acceptance rate. Moreover, in that case the proposal distribution itself is sampled using the inverse transform method. The adopted sampling strategy is therefore chosen for practicality and efficiency in the context of the full polarised PDF.

155 A systematic benchmark of acceptance-rejection sampling versus tabulated inverse-CDF sampling in terms of computation time, convergence, and implementation cost would certainly be of interest, but we consider that to be a separate algorithmic study that falls outside the main scope of the present paper. The focus of the present manuscript is on the exact polarised sampling formulation itself and its consequences for convergence and accuracy, which we now discuss more explicitly in the revised paper.

160 *In summary, I understand that the authors wish to publish this article to have a reference to cite it for further study or when it will be used by other scientific teams as the code is freely available, which is important to mention. Nevertheless, I would recommend to try to improve the paper by taking the above comments into account.*

Specific comments :

165 – *Equation of line 137 : This is correct only in the principal plane, ie when the scattered radiation is in the same plane of the incident radiation.*

We have changed the sentence on line 135 as follows: "When $U = 0$, we can use an alternative definition that includes the direction of polarisation:" $\rightarrow$ '*For observations in the principal plane, i.e. the vertical plane containing the local zenith direction and the incident solar beam, the viewing direction lies in the same vertical plane as the incident radiation. In this case (with relative azimuth angles of 0° or 180°), the principal plane coincides with the scattering plane. If the local atmosphere-surface system is horizontally homogeneous, symmetry implies that $U = 0$, and we can use the signed degree of linear polarisation...*'

170 – *To avoid confusion, in Equation (10) and (13), authors should be more precise by indicating the direction of the incident radiation and of the view (for eq 10) or scattered (eq 13) direction.*

175 We thank this suggestion. For readability, we did not add the angle dependencies of the incident light. In fact, the incident light depends on the full geometry of the preceding photon trajectory. In the first paragraph of Sect. 2.3, we have added: '*Since $\bar{I}$ changes along the photon trajectory, its dependence on position, direction, and previous events is omitted for readability.*' We also clearly state below the equation that $\bar{I}'$ (with a prime) refers to the incident light. For the signal contribution $\mathbf{W}$, we have added the $\omega^\bullet$ dependence notation to emphasize that $\mathbf{W}$ can be computed simultaneously for multiple detector angles.

180 – *Section 2.6, line 274 : does it means that the χ matrix need to be stored and each scattering in the reverse mode to be apply at the end of the incident Stokes vector ?*

In the backward mode, the multiplication by the phase matrices is indeed applied effectively at the end, before multiplying by the incident Stokes vector. However, the individual phase matrices do not need to be stored. Instead, their cumulative effect is incorporated progressively in the detector-response matrix, which is updated at each scattering event, as explained in the backward mode section of the paper.

185 – *Section 3.1, Table 1: The results are in forward or bacward mode ? Can you add the absolute and relative difference in the Table*

The results are for MONKI in forward mode. We have clarified this in the caption:

190 "*Comparison of the TOA radiances computed using MONKI in forward mode with ...*"

In addition, we modified the text on line 302 as follows:

"The MONKI simulations were done in forward mode and used 10^7 photons, ..."

In addition, we have now included the comparison tables with de Haan et al. (1987) for MONKI's backward mode as well; see Tables S4 to S7 of the supplement.

We have decided not to include the differences in the tables, because in principle that information would be redundant and would make the tables unnecessarily large. We note that almost all differences fall within two standard deviations of the Monte Carlo noise, and will converge further to the exact result of de Haan et al. (1987) with an increasing number of photons.

- *Section 3.3, Figure 6 : Why do you use the standard deviation of 3DMCPOL and not the one of MYSTIC that it is used to calculate the difference*

We use the standard deviation of 3DMCPOL for consistency with Fig. 10 of Emde et al. (2018). (See the caption of their Fig. 10: "The grey area in the difference plots corresponds to 2σ of 3DMCPOL (Monte Carlo model without variance reduction).").

- *Section 3.3, Figure 6: Why divided the results by $1000/E_0$ and not using the same quantity than in the IPRT comparison (Emde et al. 2018) ?*

We use the same quantity as used in the IPRT comparison. See the caption of Fig. 10 of Emde et al. (2018): "The Stokes vector components are normalised to $1000/E_0$."

- *Section 4 : I do not know Venus atmosphere, is it common to have clouds between 50 and 70 km ? Do you have some references to support the configuration ?*

Yes, the clouds on Venus are approximately located between 50 and 70 km altitude. See, e.g., Fig. 6 of Haus et al. (2015). We have modified the sentence on line 423 as follows:

"The cloud in the Venus model extends from 50 to 70 km, with a vertically homogeneous optical thickness $b^a = 30$ (see Haus et al., 2015, for more detailed Venus cloud models)."

- *Appendices E and F describe how the rejection method is used for the Rayleigh and Mie scattering instead of the inverse CDF method using tabulated values as is usually done. Have the two methods been compared in terms of accuracy and computation time. These can be an interesting result to add in the paper. See my general comments.*

We indeed use rejection sampling and, in the case of Mie scattering, also inverse sampling from the CDF of the envelope function. This choice is dictated by the form of the probability density function that has to be sampled (Eq. D7), which depends on the incident state of polarisation and on multiple elements of the single-scattering matrix. For this full polarisation-dependent PDF, deriving a tabulated cumulative distribution function, and especially its inverse, is not straightforward. For that reason, we do not currently have an inverse-CDF formulation for the same fully polarised sampling problem, and therefore cannot provide a direct benchmark of rejection sampling versus inverse-CDF sampling for this case.

We note, however, that the main methodological point of the paper is not a generic comparison between two numerical sampling techniques, but the fact that the scattering direction is sampled from the full polarisation-dependent distribution. In the revised manuscript, we have therefore focused the additional analysis on this point, by comparing MONKI's exact polarised sampling approach with the classical alternative approach and by showing the consequences for bias and convergence.

- *Line 505. MONKY is designed to support the preparatory studies for the aforementioned spectropolarimetric missions, which means calculating radiances at different wavelengths. However, no information is given on how the multispectral properties of the atmosphere including gas absorption and particles scattering will be accounted for.*

The multispectral properties of the atmosphere, including gas absorption and particle scattering, enter the simulation via the wavelength-dependent input absorption and scattering cross-sections, as described by Eqs. A10, A12, A16, and A17 in Appendix A. We have added the following sentence to the conclusion, line 500:

"For Earth applications, MONKI uses the same method as DAK (Doubling-Adding KNMI) to compute the input atmospheric optical properties. MONKI can handle spectral variations of those properties, producing simulated spectra by so-called line-by-line computations."

Typos :

line 58 : delete "and"

We have kept "and" here because it reflects the distinction between errors in the geometric albedo and errors in the locally reflected radiances.

Reference :

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