Comments from Referee #1, followed by the authors' responses

Referee #1 - This paper presents a novel sea ice model that represents the mixed phase between grease ice and consolidated ice floes, including both dynamic and thermodynamic components. The work investigates the response of this model to idealized wave forcing for a sea ice configuration derived from SAR imagery. Simpler configurations are also considered as sensitivity experiments. The key diagnostics are the mean viscosity and local strain rate as a function of wave properties and ice floe concentration relative to grease ice. This work represents a useful advance in modelling sea ice within marginal ice zones and its response to wave forcing. My specific comments are listed below.

Referee #1 - Eq. 6: Does the vertical component of wave velocity play a role in these simulations? The rheology seems to be described for a 2-D simulation only (section 2.1.2).

Authors' response - The vertical component of wave velocity does not play a role in the simulations, however, for completeness, we report it in Eq. 6. We will clarify this in the revised manuscript.

Referee #1 - L138: What does 'apical plane' mean?

Authors' response - The apical plane refers to the upper surface of the ice, which is exposed to the atmosphere and lies parallel to the xy-plane. We also recall that the basal plane refers to the lower surface of the ice, which is in contact with the ocean. We will clarify this in the revised manuscript.

Referee #1 - Table 1: Can you describe how these values were chosen?

Authors' response - The parameter values in Table 1 can be organised into three categories. Ice floe rheology parameters are based on the model by Hibler III (1979) and related formulations, such as Mehlmann and Richter (2017). Notably, the limit of the effective strain rate (Leppäranta and Hibler III, 1985) was deliberately reduced by one order of magnitude to ensure numerical stability. While this adjustment influences the solidity of the ice floes by altering their viscosity, it was found that the overall simulation results remain unaffected. The values for grease ice rheology were derived from literature sources, such as Paul et al. (2021), and further refined through empirical tuning via iterative simulations. The ice-ocean turning angle is set to zero, reflecting that the water drag on the ice acts purely along the flow direction. Drag coefficient values related to wave characteristics are based on Smedsrud (2011) and Alberello et al. (2019). Finally, a range of wave parameters was selected to conduct a sensitivity analysis. These details will be added to the revised version of the manuscript.

Referee #1 - L238: Why can we expect a higher growth rate for frazil ice? Is it because it is thinner than the floes?

Authors' response - The net surface heat flux, F_0, is thickness dependent: a smaller ice thickness results in a higher F_0, which in turn leads to a greater rate of change in thickness. Furthermore, the latent heat of fusion of frazil ice is lower than that of ice floes, also contributing to a higher rate of change in thickness. We will add these details in the revised manuscript.

Referee #1 - Fig 2: Why is there not much happening in the first and last parts of the curves in Fig. 2a and the first part of Fig. 2b?

Authors' response - In Fig. 2a, the thickness of both sea ice and snow is zero during the summer months, i.e. no ice, as the air temperature exceeds the threshold for ice formation and growth. Ice begins to form and its thickness increases in winter, around June, when the air temperature drops. The sea ice thickness returns to zero in spring, i.e. sea ice is melting. Note that in the model, Fig. 2b, snow growth commences only once the minimum snow threshold ($h_{s,\text{min}} = 0.02\,\text{m}$) has been exceeded.

Referee #1 - Fig 2: What are the radiative and snow forcings used to obtain these plots? And what is used for the simulations presented in the results section?

Authors' response - The radiative and snow forcings used to produce the plots depend on the net surface heat flux, F_0, which is the resultant of all fluxes applied in the thermodynamic model (Hunke et al., 2015). These include the sensible heat flux, latent heat flux, incoming and outgoing longwave radiation, and incoming shortwave radiation. The thermodynamic model is forced by ERA5 as stated in the manuscript. In the revised manuscript, we will extend the description by adding the full list of atmospheric inputs used to derive the fluxes.The conditions used to produce Fig 2, which shows the output of the thermodynamic model run as stand-alone, are the same as those used for the simulations presented in the results section. This will be clarified in the revised manuscript.

Referee #1 - L306-307: How was the thickness of ice determined in the SAR image?

Authors' response - The SAR image gives the intensity of the reflected radar signal, which can be assimilated to surface properties. This allowed us to distinguish different types of ice cover by assuming an intensity threshold to obtain an initial layout of the two different ice types: ice floes and grease ice. It is not possible to derive thickness information from this image, and we used reference visual observations from the SCALE-WIN22 research expedition (reference cited in the text). The thicknesses of the ice floes in the simulations were randomly assigned, starting from the minimum threshold of 0.1 m that was observed in the field for pancakes. The thickness assigned to grease ice was chosen to be slightly smaller than the smallest ice floe thickness. These details will be added to the revised version of the manuscript.

Referee #1 - L307: The minimum threshold for what exactly?

Authors' response - Most sea ice models, e.g. Rousset et al. (2015), do not simulate the process of ice formation starting from frazil ice aggregation, e.g. Smedsrud (2011); Smedsrud and Martin (2015). New ice formation in open water is considered only when heat losses are equal to or greater than the enthalpy required to form this sea ice thickness. This will be clarified in the revised manuscript.

Referee #1 - L312: If I understand correctly, the sub-domains are defined for diagnosis purpose only. Mentioning them here caused some confusion for me, as not much information was provided about what they represented. One solution could be to not mention them here, but describe them later in the context of Fig. 8.

Authors' response - We agree with the referee's suggestion and will move the introduction of the subdomains closer to Fig. 8 in the revised manuscript.

Referee #1 - L317: 'that gradually increase in thickness'. This could be misinterpreted as an increase in time. I suggest rephrasing.

Authors' response - We will rephrase sentence L317 to read: 'Two circular floes of different sizes and thicknesses are linked by a narrow connection, with thickness spatially varying from one floe to the other.'

Referee #1 - Fig 5: Could you please explain the motivation behind the configurations shown in panels b-e, including the wiggles in panel e?

Authors' response - In Fig. 5(b-e), the emphasis is on the orientation of the narrow connection relative to the imposed wave, which originates from the west. We found that the domain-averaged sea ice viscosity significantly depends on the connection's orientation (Fig. 14). Wiggles were included in Fig. 5e to demonstrate that the irregular shape induces more pronounced oscillations in the curves, as observed in Fig. 14d. This clarification will be added to the revised manuscript.

Referee #1 - L338-339: 'The viscosity is locally affected by the propagating wave, since the rheology of the grease ice and ice floes is described as a function of thickness (Eq. (15) and (18)).' I don't understand how the first part of the sentence follows from the second part. Could you please explain in more detail?

Authors' response - We agree with the referee that this sentence is unclear. The intended meaning was that the viscosity of both ice floes and grease ice depends on thickness, which is modified by waves and consequently affects viscosity. However, this effect cannot be observed in Fig. 6; therefore, we have decided to remove the sentence from the revised manuscript.

Referee #1 - L361-364: 'Figure 8(e) and (f) illustrate the relative difference in shear viscosity between the highest and lowest wave periods. The viscosity values in the case with waves from the west are significantly higher than those from the south, indicating a greater response to wave periods.' I don't understand how this indicates a greater response to wave periods. Could you clarify?

Authors' response - By 'greater response', we mean that the spatially averaged viscosity exhibits an increased sensitivity to wave periods when waves originate from the west, compared to waves coming from the south. In the revised manuscript, we will explicitly add the phrase 'increased sensitivity' to L361-364.

Referee #1 - L369-370: The north-south orientation of what?

Authors' response - This refers to the north-south orientation of the incoming wave. We will add this clarification to the revised manuscript.

Referee #1 - L370: Can this linear relationship be derived analytically? If so, this may be another useful output of the paper regarding parameterization of these effects in climate models.

Authors' response - We do not think such a relationship can be derived analytically through first principles, which was our driver for building this more complex numerical model. As stated in the manuscript, the linear relationship is an emerging empirical property from the numerical simulation results. This linear relationship between sea ice viscosity and ice floe percentage can be expressed analytically and used in parameterisations. Indeed, in the discussion (L522), we stated that the sea ice viscosity for 100% ice floes can be derived using Eq. 13, while the viscosity for 0% ice floes can be obtained from Eq. 18. Since the relationship with sea ice concentration is linear, these two data points (100% and 0% ice floes) are sufficient to define it.

Referee #1 - L376-377: 'Based on the linear relationship presented in Fig. 9, we assume that the model resolves the smaller scales of the heterogeneous field, allowing us to extract properties at larger scales.' How does the linear relationship allow you to make this assumption?

Authors' response - Fig. 9 is based on the percentage of ice floes in each subdomain. Due to the strong scale invariance of mean sea ice viscosity from $840\,\mathrm{m} \times 840\,\mathrm{m}$ (smaller scale) to $5040\,\mathrm{m} \times 5040\,\mathrm{m}$ (larger scale), we made this assumption. We acknowledge that the sentence is unclear and convoluted, and more of a corollary of this section. We will first move the sentence L376-377 to L388, after discussing the scaling analysis to better frame the discussion. Additionally, we will explicitly mention the strong scale invariance in this sentence, which would then read: 'Based on the inclusion of smaller scale processes that we assume realistic, the emergence of the linear relationship presented in Fig. 9 and the strong scale invariance of the mean viscosity of sea ice, we can use the model to extract properties at larger scales.'

Referee #1 - L380-381: 'Therefore, we considered subdomain groups of increasing size from 1 to 6, with the latter corresponding to the full domain (see Fig. 8(a))'. This is not entirely clear to me. What are the sizes of these groups and how are they chosen spatially? What units does '1 to 6' have in the above sentence and what does it represent exactly? Perhaps a diagram would help?

Authors' response - The entire domain is partitioned into 36 subdomains each of dimension $840\,\mathrm{m} \times 840\,\mathrm{m}$, as shown in Fig. 8a. By subdomain groups of increasing size from 1 to 6, we refer to grouping the subdomains to form progressively larger ones. Ultimately, the full domain is covered by only one tile with size corresponding to 6×6 subdomains. We refer to grids of $1 \times 1 = 1$ subdomain (e.g. subdomain number 1), $2 \times 2 = 4$ subdomains (e.g. subdomain numbers $1 - 2$, $7 - 8$), $3 \times 3 = 9$ subdomains (e.g. subdomain numbers $1 - 3$, $7 - 9$, $13 - 15$), $4 \times 4 = 16$ subdomains (e.g. subdomain numbers $1 - 4$, $7 - 10$, $13 - 16$, $19 - 22$), $5 \times 5 = 25$ subdomains (e.g. subdomain numbers $1 - 5$, $7 - 11$, $13 - 17$, $19 - 23$, $25 - 29$) and $6 \times 6 = 36$ subdomains (subdomain numbers $1 - 36$). The use of 'e.g.' here is intentional because these represent only one of the possible combinations for each grid size of size $n \times n$ within the entire domain. The total number of possible combinations per grid size is listed in the second column of Table 3.

We acknowledge that L380-381 are not entirely clear. Therefore, in the revised manuscript, we will replace the term 'subdomain groups' with 'grid sizes', referring explicitly to sizes from 1×1 (i.e. $840\,\mathrm{m} \times 840\,\mathrm{m}$) to 6×6 (i.e. $5040\,\mathrm{m} \times 5040\,\mathrm{m}$). Additionally, we will modify and refer to Fig. 8a, to illustrate one example combination for each grid size.

Referee #1 - L382: 'The results are presented in Table 3, showing a strong scale invariance from 800 m up to 5 km. The 800 m scale is already sufficient to capture the heterogeneity of the ice cover, and variations in ice type patterns do not affect the mechanical response at the larger scales up to 5 km.' Where do the numbers 500 m and 5 km come from? I am guessing they result from the size of each group, but this is hard to infer from just Table 3. Also, for clarity, please state explicitly what is invariant with scale.

Authors' response - Correct — to be precise, a grid size of 1×1 is equivalent to $840\,\mathrm{m} \times 840\,\mathrm{m}$, and a grid size of 6×6 is equivalent to $5040\,\mathrm{m} \times 5040\,\mathrm{m}$. The mean viscosity is invariant with scale. We will add this clarification to the revised manuscript.

Referee #1 - L497: 'in the dynamic model by 3% in $t = 24\,\mathrm{h}$'. Should it be 'at $t = 24\,\mathrm{h}$'?

Authors' response - We agree with the referee that the 3% difference is reached only at $t = 24\,\mathrm{h}$. In the revised manuscript, we will update the text accordingly.

Referee #1 - L507-508: 'degree of heterogeneity' is a bit ambiguous to me. I think ice floe percentage relative to grease ice would be clearer.

Authors' response - To avoid ambiguity, we will replace 'degree of heterogeneity' with 'ice floe percentage within each subdomain' in the revised manuscript.

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