Dear Reviewer#3,

Thank you for your detailed comments about our manuscript. All your suggestions have been considered, and we propose the following changes to address the questions you raised in your review.

In the following point-by-point responses your comments are in normal font and our responses are in *italic*.

Hoping that the proposed improvements will fulfill your expectations,
Best regards,

Lionel Benoit, on behalf of the authors.

Major comments

1. Novelty and positioning
- What is the genuinely new feature developed here? Gaussian models (including some forms of nonstationarity) have been studied for some time (e.g., Alliot 2009). The comparison to prior literature should be expanded to better highlight the novelty.
- l50: “fully non-stationary” — in what sense? Space only, or also time?

The main novelty of this paper is to develop a trans-Gaussian model that is fully non-stationary, in the sense that both the transform function AND the covariance function are non-stationary in space. Another important contribution is to propose a robust strategy for the calibration of the model from sparse rain gauge observations. We are not the first to propose a non-stationary transform function (this has been proposed by e.g., Frazier et al., 2016; Benoit et al., 2021; Lucas et al., 2022) nor a non-stationary covariance (this has been proposed by e.g., Paciorek and Schervish, 2006; Fuglstad et al., 2015; Fouedjio et al., 2016) to model rainfall fields, but to the best of our knowledge this paper is the first to combine these two features and to put the resulting model into practice for the purpose of (1) rainfall mapping and (2) stochastic generation of spatial rainfall fields in a context of strong orographic effects impacting rainfall fields. The introduction will be largely rewritten to better highlight the novelty of the paper and to expand the comparison to prior literature.

Associated references:

Benoit, L., Lucas, M., Tseng, H., Huang, Y. F., Tsang, Y. P., Nugent, A. D., Giambelluca, T. W., and Mariethoz, G. (2021). High space-time resolution observation of extreme orographic rain gradients in a Pacific Island catchment. Frontiers in Earth Science, 8, 546246.
Fouedjio, F., Desassis, N., and Rivoirard, J. (2016). A generalized convolution model and estimation for non-stationary random functions. Spatial Statistics, 16, 35-52.
Frazier, A. G., Giambelluca, T. W., Diaz, H. F., and Needham, H. L. (2016). Comparison of geostatistical approaches to spatially interpolate month-year rainfall for the Hawaiian Islands.
Fuglstad, G. A., Simpson, D., Lindgren, F., and Rue, H. (2015). Does non-stationary spatial data always require non-stationary random fields?. Spatial Statistics, 14, 505-531.
Lucas, M. P., Longman, R. J., Giambelluca, T. W., Frazier, A. G., Mclean, J., Cleveland, S. B., Huang, Y-F., and Lee, J. (2022). Optimizing automated kriging to improve spatial interpolation of monthly rainfall over complex terrain, Journal of Hydrometeorology, 23(4): 561-572.
Paciorek, C. J., and Schervish, M. J. (2006). Spatial modelling using a new class of nonstationary covariance functions, Environmetrics, 17(5):483-506.

2. Modeling assumptions (end of p. 3)
- The choices of Gamma and Matérn are not justified. Why Gamma rather than, say, double exponential or heavier-tailed alternatives? Why Matérn rather than exponential, Gneiting Matérn, or other forms? Please discuss strengths/weaknesses of these choices.

Regarding the marginal distribution, the Gamma distribution has been chosen to model the distribution of rainfall intensity because, once combined with rain types, the mixture of those leads to a very good simulation of the marginal rainfall distribution at all gauged locations (cf. Fig. 3). It allows for the proper modeling and simulation of the whole marginal distribution (cf results of the Wasserstein distance in figure 3) including the high intensities (cf q95% daily rainfall conditional to the presence of rain in figure 3). In addition, in the present case study of Hawai'i, the parameters of the Gamma distribution display "good behavior" for spatial interpolation: their spatial auto-correlation is high and their cross-correlation is relatively low.
In contrast, the tail parameter of heavy tailed distributions is often difficult to estimate from data at a single rain gauge (Evin et al., 2018) and can vary a lot from one gauge to the other. This complicates the interpolation of this parameter and in turn the design of a non-stationary distribution with a heavy tail, which explains why we didn't consider this option in this study.
Regarding the covariance model, we chose a Matérn covariance because this model is known to be flexible and to allow for the control of the spatial dependence at both short and long distances (Porcu et al., 2024). The Gneiting-Matérn model was not considered here because we focus on the spatial dependencies (the temporal variability is modeled only by the succession of rain types) and the Gneiting-Matérn is a space-time covariance model and not a purely spatial model.
In the revised manuscript we will better justify these choices building on the above explanations.

Associated references:
Evin, G., Favre, A. C., and Hingray, B. (2018). Stochastic generation of multi-site daily precipitation focusing on extreme events. Hydrology and Earth System Sciences, 22(1), 655-672.
Porcu, E., Bevilacqua, M., Schaback, R., and Oates, C. J. (2024). The Matern model: A journey through statistics, numerical analysis and
machine learning. Statistical Science, 39(3), 469-492.

3. Definition and use of "trans-Gaussian" (l91)
- The term sounds new here, but the description resembles what several cited works already do (sometimes under labels like "censored Gaussian"). Clarify what is new in your definition/usage.

The introduction will be re-written to better introduce trans-Gaussian models. The following paragraph will be included in the revised introduction:
"When the focus is on spatial patterns and spatial dependencies, models based on Gaussian random fields (GRFs) - also known as geostatistical models - are often chosen because they enable modeling rainfall at any point of the spatial domain while accounting for spatial dependencies (Chilès and Delfiner, 2012). However, the distribution of rainfall intensity is rarely Gaussian at daily to sub-daily resolution, and a parametric transform function is often combined with a GRF to model rainfall at these time scales resulting in so-called trans-Gaussian (or meta-Gaussian) approaches (Leblois and Creutin, 2013; Benoit et al., 2018; Papalexiou and Serinaldi, 2020). The ability for spatial modeling makes trans-Gaussian models well suited for mapping rainfall from sparse rain gauge observations while accounting for interpolation uncertainty (Creutin and Obled, 1982; Lanza, 2000; Kyriakidis et al., 2001), for radar-rain gauge data fusion (Creutin et al., 1988; Foehn et al., 2018), and for the stochastic generation of spatially continuous rainfall fields (Paschalis et al., 2013; Peleg et al., 2017; Vaittinada Ayar et al., 2020; Wilcox et al., 2021). The resulting rainfall datasets can be used to assess input errors in distributed hydrological models (Vischel et al., 2009; Renard et al., 2011) or to conduct sensitivity analysis of spatially explicit hydro-meteorological modeling chains (Moraga et al., 2022; Liu et al., 2024)."

Associated references:
Benoit, L., Allard, D. and Mariethoz, G. (2018). Stochastic Rainfall Modeling at Sub-kilometer Scale, \Water Resources Research, 54, 4108-4130.
Chilès, J. P., and Delfiner, P. (2012). Geostatistics: modeling spatial uncertainty (Vol. 713). John Wiley and Sons.
Creutin, J. D., and Obled, C. (1982). Objective analyses and mapping techniques for rainfall fields: an objective

comparison. Water resources research, 18(2), 413-431.
Creutin, J. D., Delrieu, G., and Lebel, T. (1988). Rain measurement by raingage-radar combination: a geostatistical approach. Journal of Atmospheric and oceanic technology, 5(1), 102-115.
Foehn, A., Hernández, J. G., Schaefli, B., and De Cesare, G. (2018). Spatial interpolation of precipitation from multiple rain gauge networks and weather radar data for operational applications in Alpine catchments. Journal of Hydrology, 563, 1092-1110.
Kyriakidis, P. C., Kim, J., and Miller, N. L. (2001). Geostatistical mapping of precipitation from rain gauge data using atmospheric and terrain characteristics. Journal of Applied Meteorology and Climatology, 40(11), 1855-1877.
Lanza, L. G. (2000). A conditional simulation model of intermittent rain fields. Hydrology and Earth System Sciences, 4(1), 173-183.
Leblois, E., and Creutin, J. D. (2013). Space-time simulation of intermittent rainfall with prescribed advection field: Adaptation of the turning band method. Water Resources Research, 49(6), 3375-3387.
Liu, Y., Wright, D. B., and Lorenz, D. J. (2024). A nonstationary stochastic rainfall generator conditioned on global climate models for design flood analyses in the Mississippi and other large river basins. Water Resources Research, 60(5), e2023WR036826.
Moraga, J. S., Peleg, N., Molnar, P., Fatichi, S., and Burlando, P. (2022). Uncertainty in high-resolution hydrological projections: Partitioning the influence of climate models and natural climate variability. \ Hydrological Processes, 36(10), e14695.
Papalexiou, S.M. and Serinaldi, F. (2020). Random fields simplified: Preserving marginal distributions, correlations, and intermittency, with applications from rainfall to humidity. Water Resources Research, 56(2), e2019WR026331.
Paschalis, A., Molnar, P., Fatichi, S., and Burlando, P. (2013). A stochastic model for high-resolution space-time precipitation simulation. Water Resources Research, 49(12), 8400-8417.
Peleg, N., Fatichi, S., Paschalis, A., Molnar, P., and Burlando, P. (2017). An advanced stochastic weather generator for simulating 2-D high-resolution climate variables. Journal of Advances in Modeling Earth Systems, 9(3), 1595-1627.
Renard, B., Kavetski, D., Leblois, E., Thyer, M., Kuczera, G., and Franks, S. W. (2011). Toward a reliable decomposition of predictive uncertainty in hydrological modeling: Characterizing rainfall errors using conditional simulation. Water Resources Research, 47(11).
Vaittinada Ayar, P., Blanchet, J., Paquet, E., and Penot, D. (2020). Space-time simulation of precipitation based on weather pattern sub-sampling and meta-Gaussian model. Journal of Hydrology, 581, 124451.
Vischel, T., Lebel, T., Massuel, S., and Cappelaere, B. (2009). Conditional simulation schemes of rain fields and their application to rainfall–runoff modeling studies in the Sahel. Journal of Hydrology, 375(1-2), 273-286.
Wilcox, C., Aly, C., Vischel, T., Panthou, G., Blanchet, J., Quantin, G., and Lebel, T. (2021). Stochastorm: A stochastic rainfall simulator for convective storms. Journal of Hydrometeorology, 22(2), 387-404.

- The function Ψ is introduced in Section 2.1 but does not reappear until the start of Section 2.2. Consider re-using or referencing it explicitly at the end of Section 2.1 for continuity.

This is a good idea, we will reference it explicitly at the end of Section 2.1 for continuity.

- l93: Define the separation vector and the norm used. If altitude/mountain effects are important, indicate where they enter the model.

We will define the separation vector and the norm used in the revised manuscript. Regarding the altitude, it is not explicitly accounted for in the model, and orographic effects are rather captured indirectly through the spatial interpolation of model parameters. This will be added in the revised section "Making the trans-Gaussian model non-stationary".

- As I understand it, there are two kinds of non-stationarity: (i) parameter variation by location s

and (ii) possible non-stationarity in the covariance. Should the covariance be non-isotropic?

Yes, this understanding of the two sources of non-stationarity is correct. And the covariance is already anisotropic in the current model (in the anisotropy is also non-stationary).

- Eq. (2) lacks sufficient explanation; please expand.

We will expand the description of Eq. 2 in the revised manuscript.

- Eq. (3): I am not sure N_s was defined. What optimizer is used for l in Eq. (3)?

N_s is the number of rain gauges and was defined only indirectly (l 119). We will define it explicitly in the revised manuscript. Regarding the optimizer we used the default algorithm of the fmincon function in Matlab, which correspond to the interior-point method. We will add this information in the revised manuscript.

- Estimation strategy: Is there a risk in estimating the Gamma parameters first and then C? Should these be estimated jointly? Why use pairwise likelihood, and what guarantees does it provide in this context?

The sequential estimation of the parameters of the marginal distribution and the covariance function is common practice in trans-Gaussian models (e.g., Leblois and Creutin, 2013; Papalexiou and Serialdi, 2020) and to our knowledge it performs well in a frequentist approach (but will lead to under-dispersed posteriors in a Bayesian setting).
The pairwise likelihood is used to avoid the computation of large multivariate (i.e., multi-sites) normal likelihoods and large multivariate normal cdf (to model rainfall occurrence). The pairwise likelihood therefore allows to boost parameter inference. Pairwise likelihood has been proven efficient and unbiased for the estimation of covariance parameters (e.g., Bevilacqua et al., 2012 and Allard and Bourotte, 2015 for an application to rainfall modeling).

Associated references:
Allard, D. and Bourotte, M. (2015). Disaggregating daily precipitations into hourly values with a transformed censored latent Gaussian process, Stochastic Environmental Research and Risk Assessment 29: 453-462.
Bevilacqua, M., Gaetan, C., Mateu, J., and Porcu, E. (2012). Estimating space and space-time covariance functions for large data sets: a weighted composite likelihood approach. Journal of the American Statistical Association, 107(497), 268-280.
Leblois, E., and Creutin, J. D. (2013). Space-time simulation of intermittent rainfall with prescribed advection field: Adaptation of the turning band method. Water Resources Research, 49(6), 3375-3387
Papalexiou, S.M. and Serinaldi, F. (2020). Random fields simplified: Preserving marginal distributions, correlations, and intermittency, with applications from rainfall to humidity. Water Resources Research, 56(2), e2019WR026331.

4. Structure of presentation (l135 and Section 2.4–2.6)
- This is the second mention of “climate zones,” but they have not been presented. Without this context, it is difficult to follow. Either present the model fully and then describe the application (e.g., fitting within each zone), or introduce the application earlier so the assumptions are easier to track.

We agree that the description of the climate divisions must appear earlier in the manuscript. To this end we will create a new section entitled “2. Example dataset: orographic precipitation on the Island of Hawai‘i” just after the introduction. This new section will introduce the climate of Hawai‘i, the dataset we use and the climate divisions. This section will include the material of the former section “3.1 Orographic precipitation on the Island of Hawai‘i” as well as the following new figure (and associated description in the main text) illustrating *the rainfall climate of each climate division, with focus on seasonality and inter-annual variability.*

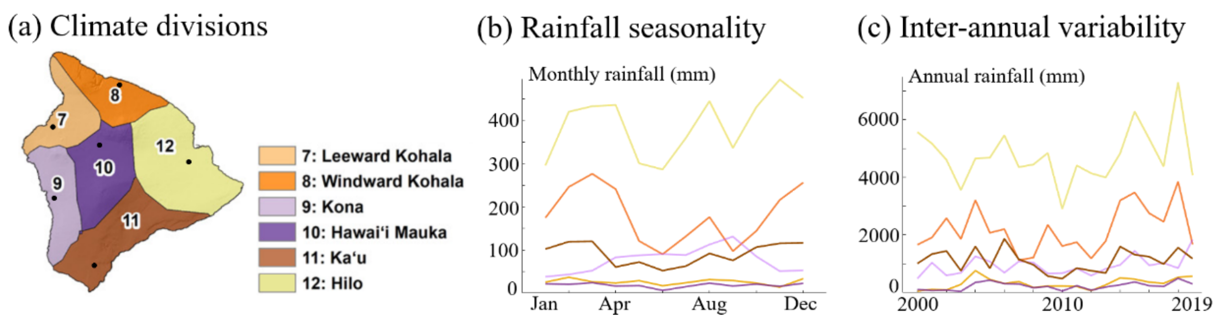


Figure 2. Climate divisions of the Island of Hawai'i. (a) Footprint of the climate zones (adapted from Luo et al. (2024); the numbering over the Island of Hawai'i starts at 7 because the original zonation covers the whole State of Hawaii). (b) Rainfall seasonality and (c) rainfall inter-annual variability for 6 rain gauges spread in the different climate divisions (gauge locations are denoted by black dots in (a)).

- The paragraph before Section 2.4 lacks mathematical detail, which makes it harder to follow; the last sentence is particularly unclear.

This paragraph will be re-written to add details and improve readability.

- Section 2.6 mentions MCMC, but its role is not clearly explained. A brief conceptual explanation would help (and the Appendix would benefit from a few guiding sentences). Clarify the list of steps as well.

We agree that the MCMC used to update the daily maps in order to condition their sum to monthly totals was not clearly introduced and described. Section 2.6 will be re-written to improve the description of the framework we use. In addition, a schematic describing the Metropolis within Gibbs algorithm will be added in the main text to improve the description of the algorithm and make the link with the algorithm description in appendix.

5. Evaluation metrics and visualization

- In Section 3.3, Comparing to another model is a good idea, but including a comparison with a model not authored by the same team would strengthen the case. Also, briefly recap the key differences that make BSNLG2022 distinct.

As proposed by another reviewer the differences between the two models will be presented in the following table placed in appendix, with a brief overview in the main text (NB: the table outlines rainfall modeling conditionally to a pre-defined rain type).

	Benchmark model	**This study**
Simulation locations	Observation sites only	Any location
Marginal distribution	Single Gamma across the island	Gamma at each location (non-stationary)
Spatial dependencies	Empirical copulas	Non-stationary Matérn covariance

Regarding the use of BSNLG2022 as a benchmark, we selected it because this is the only stochastic rainfall generator we know that is specifically designed for tropical islands with strong orographic effects. And since the benchmark of the cross-validation is performed with the widespread Climatologically-Aided Interpolation (CAI) method (Willmott and Robeson, 1995; Hunter and Meentemeyer, 2005) we believe that readers can form an *opinion that is not only centered on the work of our team but also contextualized with an independent and well known spatial modeling approache. We will add the above references in the revised version of the manuscript to make it clear that CAI is a common and state-of-the-art interpolation method for climate variables.*

Associated references:
Hunter, R. D., and Meentemeyer, R. K. (2005). Climatologically aided mapping of daily precipitation and temperature. Journal of Applied Meteorology, 44(10), 1501-1510.
Willmott, C. J., and Robeson, S. M. (1995). Climatologically aided interpolation (CAI) of terrestrial air temperature. International Journal of Climatology, 15(2), 221-229.

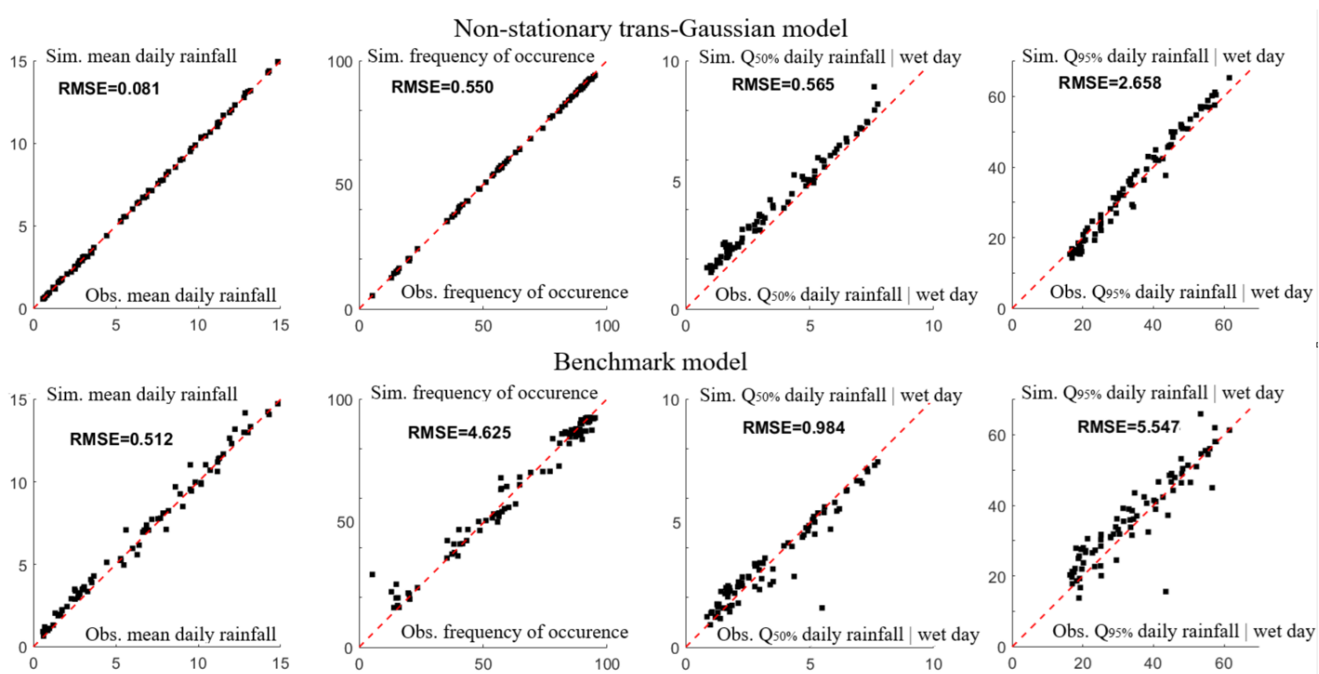
- Figure 3 is promising; the Wasserstein distance is an interesting choice, has it been used elsewhere in the literature of weather generators?

We are not aware of the use of the Wasserstein distance in the literature of weather generators strictly speaking but it has been proposed to apply it to the comparison of climate models (Vissio et al., 2020), which motivated our choice for this metric. We will add the above reference to the revised manuscript.

Associated reference:
Vissio, G., Lembo, V., Lucarini, V., and Ghil, M. (2020). Evaluating the performance of climate models based on Wasserstein distance. Geophysical Research Letters, 47(21), e2020GL089385.

- I would also like to see an observed vs. simulated scatter plot, or an overall RMSE for the chosen statistics. Visual comparison across multiple maps is difficult and the lack of quantitative comparison is a limitation.

This is a good idea. We will add the following scatter plots of observed vs. simulated target statistics and mention overall RMSE in the revised figure 3.



Minor comments
- Clarify the meaning of “local scale” vs. “rain gauge” (l114 and l132).

We will better define what we call the local scale in the revised manuscript.

- l146: “more scalable” — please quantify (e.g., $\mathcal{O}(N)$ vs. $\mathcal{O}(N^2)$).

Spectral approaches have a computational cost scaling linearly with the number of target locations. We will add this information in the reviser version of the paper.