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Setting up the physical principles of resilience in a model of the Earth System

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Abstract

Resilience is a property of social, ecological, social-ecological and biophysical systems. It describes the capacity of a system to cope with, adapt to and innovate in response to a changing surrounding. Given the current climate change crisis, ensuring conditions for a sustainable future for the habitability on the planet is fundamentally dependent on Earth System (ES) resilience. It is thus particularly relevant to establish a model that captures and frames resilience of the ES, in also physical terms. In this work we propose that resilience can serve as a theoretical foundation when unpacking and describing metastable states of equilibrium and energy dissipation in any realistic dynamic description of the variables that characterise the ES. Since the impact of the human activities can be suitably gauged by the planetary boundaries (PBs) and the planet's temperature is the net result of multiple PBs interactions, such as CO_2 concentration and radiative forcing, atmospheric aerosol loading, atmospheric ozone depletion, etc, then resilience features arise once conditions to avoid a runaway of the PBs are setup. In this work it is shown that this runaway can be provided by the presence of metastable states and dynamic friction built out of the interaction among the PB variables. As humanity moves away from Holocene conditions, we argue that resilience features arising from metastable states might be crucial for the ES to follow sustainable trajectories in the Anthropocene.

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I. INTRODUCTION

Over the past decades the human imprint on the Earth System (ES) has been exceptional (Jouffray et al. 2020, Steffen et al. 2015a). While the mass of humans is only about 0.01% of the total biomass, we have become a dominant force in shaping the face of Earth, including its atmosphere, biosphere, hydrosphere and lithosphere (Crutzen 2002, Ellis 2011, Foley 2011, Nyström et al. 2019, Vitousek et al. 1997), and as of 2020 the global human-made mass surpasses the dry-weight of all living biomass (Elhacham et al. 2020). Thus, humans have become a hyper-keystone species (Worm & Paine 2016), which rivals geological forces in influencing the trajectory of the ES (Steffen et al. 2018)

A major concern of these changes is the risk of crossing of so-called tipping-points, which refer to the critical threshold at which a small change or event triggers a significant and potentially irreversible (regime) shift in a system. Tipping-points have been observed in various systems, such as ecosystems (e.g. food webs, benthic communities), social systems (e.g. norms policy), economic systems (e.g. market-based economy) and technological systems (e.g. steam engine, smartphone, artificial intelligence) (Nyborg et al. 2016, Scheffer 2009, Scheffer et al. 2001). Over the past couple of decades there have been raising concerns around the existence of tipping-elements, which are large-scale components (subsystems) of the ES that may transgress a tipping-point (Barnosky et al. 2012, Lenton et al. 2008). Example of such tipping-elements include, the Greenland Ice Sheet, the Atlantic Meridional Overturning Circulation (AMOC), permafrost, monsoon systems, and the Amazon rainforest. Importantly, these tipping-elements interact, which can lead to a cascading behaviour of the entire ES (Wunderling et al. 2024). The consequences of these dynamics for humanity are likely to be colossal (Steffen et al. 2018).

Clearly, knowledge about tipping-points, where they are located, when they are approached and identifying ways to navigate away from them, are key challenges for humanity (Barnosky et al. 2012, Scheffer et al. 2012). Two broad frameworks that could help assist in this regard are planetary boundaries and resilience theory. The two are complementary in the sense that the planetary boundaries provide a quantitative assessment whereas the resilience framework adds a strong theoretical underpinning.

The planetary boundaries (PB) framework (Richardson et al. 2023, Rockström et al. 2009, Steffen et al. 2015b) has been used to define global and regional limits in biophysical

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processes – 'safe operating environmental space' – that must not be crossed if humanity is to stay away from systemic and potentially irreversible shifts in the ES. As such, the planetary boundaries framework serves as a "global dashboard," tracking humanity's collective impact on key environmental factors that threaten the Earth's ability to sustain human life. More recently, focus has been directed towards exploration of how different boundaries can interact and potentially cascade, thereby shrinking the safe operating space for future human impacts on the ES (Lade et al. 2020).

The resilience concept describes the extent to which a system can develop with change by absorbing recurrent perturbations, deal with uncertainty and risk, and still sustain its key properties (Folke 2006, Holling 2001). It links to the planetary boundaries framework as it embraces the existence of tipping-points (or thresholds), multiple states (or regimes) and self-reinforcing feedback mechanisms (i.e. hysteresis). Resilience has also been suggested as a conceptual framework that could assist in developing paths towards sustainability (Folke et al. 2016). Hence, it can serves as a theoretical and practical foundation for the planetary boundaries framework. An important point to bear in mind, however is that resilience is a property of a system and is neither "good" nor "bad" per se. It can help maintain the current state of a system no matter whether it is deemed desirable or undesirable.

Bearing in mind the resilience concept and its importance we aim in this work to specify, in the context of a thermodynamical model of the ES, what are the physical properties that manifest themselves collectively as resilience features of the ES. Our starting point is a thermodynamical model of the ES from Holocene state conditions to other potentially stable states, which can be regarded as phase transitions and admit a description through the Landau-Ginzburg Theory (LGT) (Barbosa et al. 2020, Bertolami & Francisco 2018, 2019). The LGT is a theoretical framework used in physics to describe phase transitions, such as when a material changes from a solid to a liquid state or a magnetic material loses its magnetism. Here we use the LGT to describe the transitions the ES has gone throughout the history of Earth. In this approach, the thermodynamic description of the system is obtained through the Helmholtz free energy, F, which can be written as an analytic function of an order parameter, ψ, which is chosen to be the reduced temperature relative to Holocene average temperature, $\langle T_{\mathrm{H}} \rangle$, $\psi := (T - \langle T_{\mathrm{H}} \rangle)/\langle T_{\mathrm{H}} \rangle$.

As we shall review in the next section, this framework allows for determining the equilibrium states of the ES in terms of the planet's biophysical subsystems or processes that are,

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due to the impact of the human activities, the driving forces that dominate its evolution In the Anthropocene, here collectively denoted by H. In the phase-transition model discussed in Refs. (Barbosa et al. 2020, Bertolami & Francisco 2018, 2019), H was considered an external field, however, in the present work, we admit that it has a dynamics on its own, meaning that human impacts modify the topographic landscape of possible Anthropocene trajectories.

As previously discussed, the proposed Landau-Ginzburg model allows for getting the evolution equation of the ES, the so-called Anthropocene equation, and to associate the sharp rise of the physical parameters that characterise the ES to the great acceleration of the human activities (Bertolami & Francisco 2018), which became conspicuous from the second half of the 20th century and onwards (Steffen et al. 2015a). As we shall see, resilience can be associated to the existence of metastable states and retroactive mechanisms, which allow for the ES to settle in a stable/metastable state and show a considerable "resistance" to move away from this state.

A pleasing feature of the proposed description is that it allows for drawing trajectories of the ES in the phase space of model's variables. By considering that the PBs and the ensued temperature have a dynamics on their own, two well defined and distinct sets of trajectories were identified upon assumptions about the evolution of the PB: a linear growth of the human activities, $H(T) = H_0 t$, where H_0 is an arbitrary constant, from which follows that all ES trajectories starting at the Holocene are led to "Hot-House Earth" state (Steffen et al. 2018) with a necessarily higher temperature than the Holocene average temperature (Bertolami & Francisco 2019); if instead, the increase of the human activities impact on the ES obey a discrete logistic map (Jakobson 1981, Kingsland 1995, May 1976), trajectories can display bifurcations or chaotic behaviour (Bernardini et al. 2025). Of course, as human activities are bounded by the finiteness of resources, the logistic map might be a more accurate description of its behaviour, although it is not quite clear what is the time span elapsed between successive steps of the logistic map. In any case, it is relevant to keep in mind that a too fast increase might give origin to trajectory bifurcations or even chaotic behaviour, which, of course, precludes predictions and control measures on the evolution of the ES.

In this work we extend the previous studies of the ES model carried out in Refs. (Barbosa et al. 2020, Bernardini et al. 2025, Bertolami & Francisco 2018, 2019) on various aspects.

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More fundamentally, we consider that the 9 identified PBs denoted as h_i, $i = 1, ..., 9$, have a dynamics of their own and seek for implementing resilience in the the eleven dimensional space $(\psi, h_i, F(\psi, h_i))$. Resilience can be regarded as a set of measures that prevent or delay the evolution of the ES towards a "Hot-House Earth" state and ensuring that this state is as close as possible to the Holocene. This can be implemented by creating metastable states to avoid a runaway situation due to a barrier that arises as higher-order terms into the Helmholtz free energy are introduced (cf. discussion below). A further requirement is dynamic friction to restrict the change of state in the phase space. The specific conditions for the ES to acquire effective resilience features will be discussed below. Trajectories of the ES without and with resilience are depicted in Figs. 1 and 2 respectively (cf. a detailed discussion below). .

This paper is organised as follows: in section II we review the cardinal aspects of the LGT of the ES and discuss the most relevant features of the dynamical system emerging from the model; in section III, we discuss the implementation of the resilience features in the model and connect them to properties that any realistic model of the ES should have. Finally, in section IV we present our conclusions and discuss how our work can be extended to address several issues concerning features and transformations of global social-ecological Human-Earth System.

II. A THERMODYNAMICAL MODEL FOR THE EARTH SYSTEM

We first review the main features of the proposed model for the ES (Bertolami & Francisco 2018) and discuss in the next section the conditions to extend it in order to explicitly exhibit resilient properties.

The proposal of Ref. (Bertolami & Francisco 2018) is to regard transitions of the ES as phase transitions which can be described by the LGT through an order parameter, ψ, and natural parameters (astronomical, geophysical, internal). In the Anthropocene, the natural forces average out to zero and the system is driven by the strength of the human activities, collectively denoted by H. The description of the system is achieved via the Helmoholtz free energy function in terms of an order parameter ψ. In the Anthropocene, disregarding the spatial variation of ψ, one can write (Bertolami & Francisco 2018, 2019):

$$F(\psi, H) = F_0 + a\psi^2 + b\psi^4 - \gamma H\psi, \tag{1}$$

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where F_0, a, b and γ are constants.

The strength of the human activities are probed by their impact via the PBs (Rockström et al. 2009, Steffen et al. 2015b), h_i, $i = 1, 2, ..., 9$ with respect to their Holocene values. Given that the PB can interact among themselves, the most general expression for H is given by (Bertolami & Francisco 2019):

$$H = \sum_{i=1}^{9} h_i + \sum_{i,j=1}^{9} g_{ij} h_i h_j + \sum_{i,j,k=1}^{9} \alpha_{ijk} h_i h_j h_k + \ldots, \tag{2}$$

where $[g_{ij}]$ is a non-degenerate, $\det[g_{ij}] \neq 0$ 9×9 matrix. Similar conditions should be imposed on the coefficients α_{ijk} and β_{ijkl} of the higher-order interaction terms. In principle, these interactions terms are sub-dominating, however, their importance has to be established empirically. As pointed out in Ref. (Bertolami & Francisco 2019), the interaction terms may lead to new equilibrium states and suggest some mitigation strategies depending on their sign and strength in the matrix entries (Bertolami & Francisco 2019). This will be explicitly discussed in the next section. In Ref. (Barbosa et al. 2020), it was shown that the interaction term between the climate change variable (CO_2 concentration), say, h_1, and the oceans acidity, say, h_2, was non-vanishing and contributed to about 10% of the value of the individual contributions themselves.

In order to introduce resilience features into the model we have to consider, contrary to previous works (Barbosa et al. 2020, Bernardini et al. 2025, Bertolami & Francisco 2018, 2019), that the PBs are dynamical variables. This means that the phase space of the model is specified through the variables $(\psi, \dot{\psi}, h_i, \dot{h}_i)$. Thus, for a given set of initial conditions, corresponding to a state $(\psi(0), \dot{\psi}(0), h_i(0), \dot{h}_i(0))$ in the phase space, one can, in principle, obtain the trajectories, *orbits*, in the phase space after solving the initial value problem through the evolution equations of the system. The equations of motion are obtained through the Lagrangian or equivalently through the Hamiltonian formalism. The latter, yielding to first order differential equations, is more suitable to establish a dynamical system in its canonical form.

The Lagrangian function must include, besides the potential, which is given by the free energy, a set of kinetic energy terms for the canonical coordinates. The simplest possible kinetic term is a quadratic term proportional to the squared first derivative of each coordinate.

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153 Thus, we can write the following Lagrangian:

$$\mathcal{L}(q,\dot{q}) = \frac{\mu}{2}\dot{\psi}^2 + \frac{\nu}{2}\sum_{i=1}^{9}\dot{h_i}^2 - F_0 - a\psi^2 - b\psi^4 + \gamma H\psi, \tag{3}$$

154 where μ and ν are arbitrary constants and the dots stand for time derivatives. The constant
155 ν is assumed to be the same for all PB variables.

Aiming to get the Hamiltonian function, we evince the relevant canonical conjugate momenta associated to ψ and to a generic PB variable, h_i:

$$p_\psi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \mu\dot{\psi}, \tag{4}$$

$$p_{h_i} = \frac{\partial \mathcal{L}}{\partial \dot{h_i}} = \nu\dot{h_i}, \tag{5}$$

from which follows the Hamiltonian function

$$\mathcal{H}(\psi,p) = \frac{p_\psi^2}{2\mu} + \sum_{i=1}^{9}\frac{p_{h_i}^2}{2\nu} + F_0 + a\psi^2 + b\psi^4 - \gamma H\psi, \tag{6}$$

and Hamilton's equations,

$$\dot{\psi} = \frac{\partial \mathcal{H}}{\partial p_\psi}, \quad \dot{p_\psi} = -\frac{\partial \mathcal{H}}{\partial \psi}, \tag{7}$$

$$\dot{h_1} = \frac{\partial \mathcal{H}}{\partial p_{h_i}}, \quad \dot{p_{h_i}} = -\frac{\partial \mathcal{H}}{\partial h_i}. \tag{8}$$

The equations of motion read, considering for while just the contribution from the lowest order terms in Eq. (2):

$$\mu\ddot{\psi} = -2a\psi - 4b\psi^3 + \gamma H \tag{9}$$

and

$$\nu\ddot{h_i} = \gamma\psi. \tag{10}$$

To exemplify the behaviour of variables ψ and h_i, let us obtain the resulting solutions for the simple case considered in Ref. (Bertolami & Francisco 2019). For $b \simeq 0$, we can neglect the cubic term in the equation of motion for ψ to get the equation of an harmonic oscillator under the action of an external force, $H(t)$. This yields for the simple case of an initial linear time evolution,

$$H(t) = H_0 t, \tag{11}$$

for an equilibrium initial state, $\dot{\psi}(0) = 0$, the analytical solution:

$$\psi(t) = \psi_0\cos(\omega t) + \alpha t, \tag{12}$$

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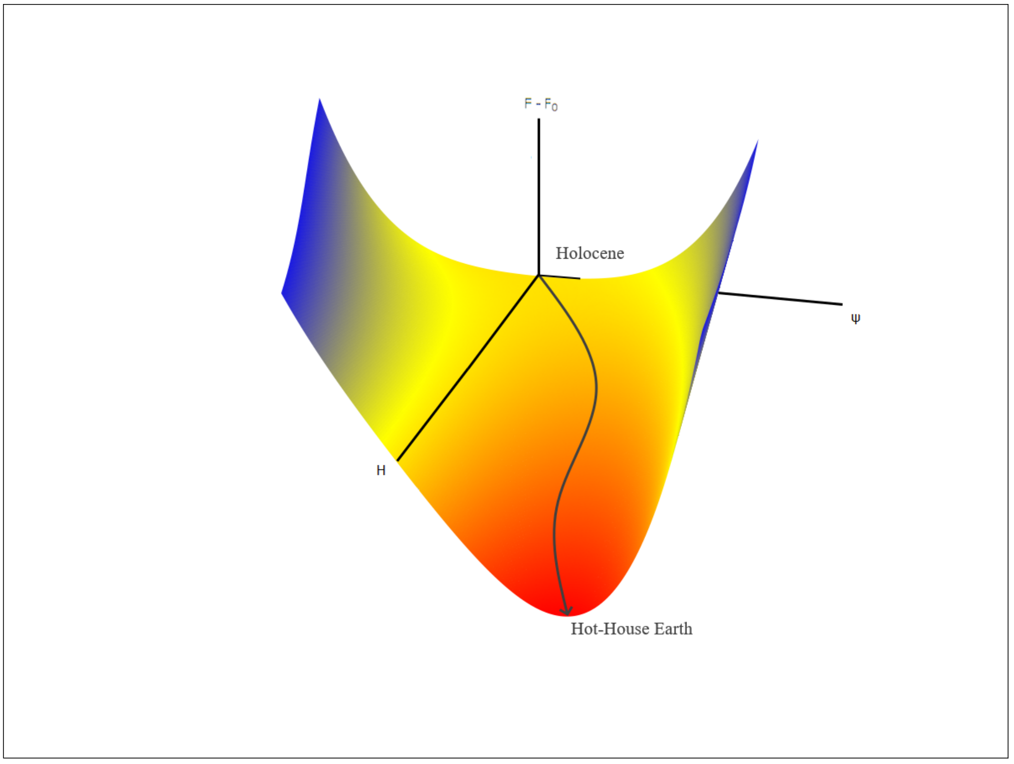

FIG. 1: Free energy in function of the temperature and of the intensity of the human impact on the PBs.

where $\omega = \sqrt{2a/\mu}$ is an angular frequency, $\alpha = \gamma H_0/2a$ and ψ_0 is an arbitrary constant fixed by the initial conditions.

The solution for the impact on the PB, $h_i(t)$, which initially behaves collectively as Eq. (11), that is $\sum_{i=1}^{9} h_i(t \simeq 0) = H_0$, quickly evolves to a cubic growth in time:

$$h_i(t) = A\cos(\omega t) + Bt^3 + \alpha_i t, \tag{13}$$

where $A = -\gamma\psi_0/\nu\omega^2$, $B = \alpha\gamma/6\nu$, for an arbitrary α_i.

These solutions clearly show that if the temperature ψ grows from an initial linear collective behaviour of the PBs, H, then quickly turns the h_is to have a cubic growth. Clearly, this model shows no resilience features as depicted in Figure 1, where one clearly sees that from the Holocene, Anthropocene trajectories inevitably evolve towards a "Hot-House Earth" state.

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In what follows we shall consider the introduction into the free energy function of a cubic term for ψ and higher than linear order terms in h_i as these will allow for metastable states to arise, thus leading to bounded solutions for ψ and h_i. In fact, the conditions for the appearance of metastable states were already discussed in a completely different context, namely in a proposal to classify rocky planets (Bertolami & Francisco 2022), using the ideias developed in Refs. (Barbosa et al. 2020, Bertolami & Francisco 2018, 2019) to describe the ES.

Before concluding this discussion it is worth stressing once again that the behaviour of the ES depends crucially on the assumptions about the evolution of the PB. Indeed, as pointed out in the introduction, the supposition that human activities grow linearly as in Eq. (11) implies, as exemplified above, that ES trajectories lead to the "Hot-House Earth" state (Bertolami & Francisco 2019) as discussed by Ref. (Steffen et al. 2018). However, if the human activities impact on the ES behaves as a discrete logistic map, as suggested in Ref. (Bernardini et al. 2025), then trajectories will depend the rate of growth of human activities as solutions admit regular trajectories as well as trajectories that present bifurcations and even chaotic behaviour.

III. SETTING UP THE PHYSICAL PRINCIPLES OF RESILIENCE

As mentioned above, resilience features are associated to bounded trajectories in the Anthropocene and these ask for the existence of metastable states. In the LGT the metastable states arise by intruding cubic terms on the free energy. As pointed out in Ref. (Bertolami & Francisco 2022), the introduction of a cubic term allows for a richer variety of equilibrium states. Indeed, consider the free energy:

$$F(\psi, H) = F_0 + a\psi^2 - c|\psi|^3 + b\psi^4 - \gamma H\psi, \tag{14}$$

where we assume that constants b, c and γ are positive, while constant a can be negative.

The existence of extrema are given by two conditions. The first one reads:

$$\frac{\partial F(\psi, H)}{\partial \psi} = 0 = 2a\psi - 3c\psi^2 + 4b\psi^3 - \gamma H. \tag{15}$$

The resulting cubic equation admits at least one real solution, say, ψ_M, meaning that there are at least two metastable states, ψ_M and $-\psi_M$. Clearly, $\psi_M \neq 0$ as far as $H \neq 0$.

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However, the unboundedness of the evolution of the variables (ψ, h_i) is due to the unboundedness of the PBs. Recent assessment of the PBs has shown that 6 out of the 9 PBs have gone beyond their Holocene values where they were at equilibrium, a state usually referred to as Safe Operating Space (SOS).

The motion in the eleven-dimensional configuration space, $(\psi, h_i, F(\psi, h_i))$, is quite complex, so in order to simplify the analysis we consider one single generic PB, h_i, and assume that the remaining ones are unchanged[1]. The free energy can be written explicitly in terms of the high order contributions in H depicted in Eq. (2). Therefore, we get:

$$F(\psi, H) = \hat{F}_0 + a\psi^2 - c|\psi|^3 + b\psi^4 - \gamma(h_i + g_i h_i^2 + b_i h_i^3)\psi, \tag{16}$$

where we have aggregated all contributions to the quadratic and cubic terms in h_i within the constants g_i and b_i. To ensure boundedness it is necessary that g_i is negative and that b_i is positive.

Thus, from Eq. (16), one gets the condition:

$$\frac{\partial F(\psi, h_i)}{\partial h_i} = 1 + 2g_i h_i + 3b_i h_i^2 = 0, \tag{17}$$

which admits real non-vanishing solutions, h_{iM}. as far as $g_i^2 > 3b_1$ for $b_i \neq 0$ or $h_{iM} = \frac{1}{-2g_i}$ if $b_1 = 0$.

The general conditions to ensure that the extremum (ψ_M, h_{iM}) corresponds to a minimum and hence to a metastable state are given by:

$$\frac{\partial^2 F(\psi_M, h_{iM})}{\partial \psi^2} \frac{\partial^2 F(\psi_M, h_{iM})}{\partial h_i^2} - \left(\frac{\partial^2 F(\psi_M, h_{iM})}{\partial \psi \partial h_i}\right)^2 > 0. \tag{18}$$

and

$$\frac{\partial^2 F(\psi_M, h_{iM})}{\partial \psi^2} > 0, \tag{19}$$

which yield the following relationships:

$$g_i < -3b_i h_{iM} \tag{20}$$

and

$$2a - 6c|\psi_M| + 12b\psi_M^2 > 0. \tag{21}$$

Satisfying these conditions imply the ES can settle in a the metastable state, (ψ_M, h_{iM}), that is, the system shows resilience and does not runaway towards the "Hot-House Earth" state as depicted in Figure 2 as far as $3b_i < g_i^2 < 9b_i^2 h_{iM}$. .

[1] Notice that the analysis of two-variables case is quite relevant as the Kolmogorov-Arnold representation theorem.establishes that any continuous function of several variables can be constructed out of a finite sum of two-variable functions.

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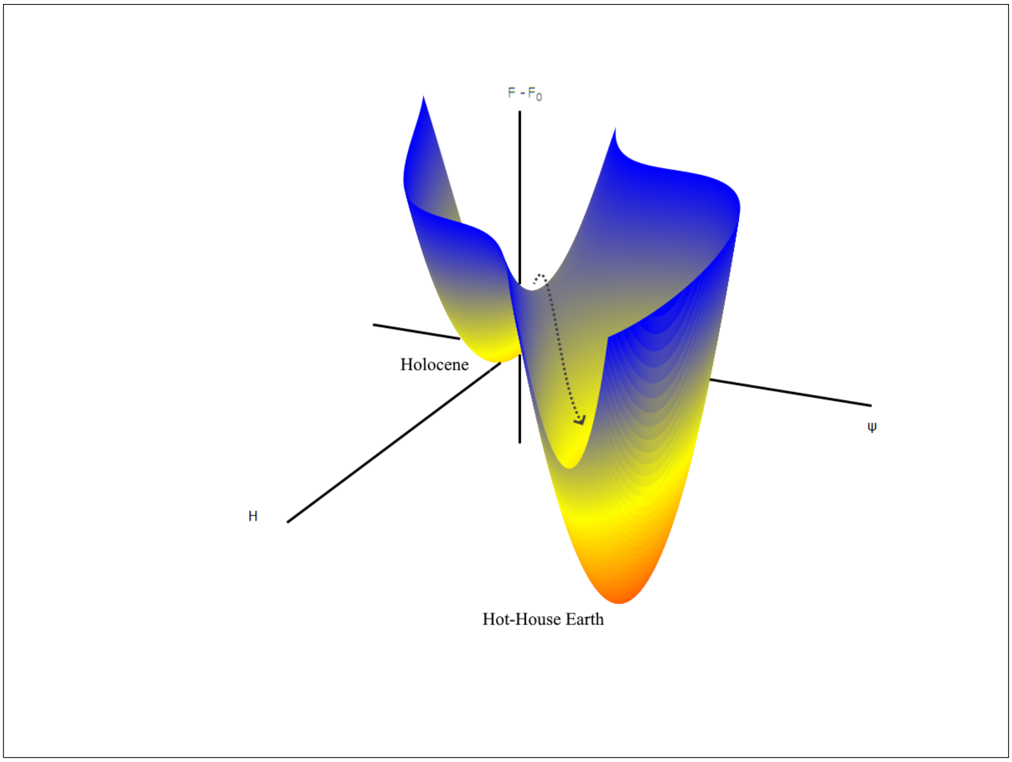

FIG. 2: Free energy in function of the temperature and resilience features.

Notice that the conditions for the existence of a metastable state can be met if $g_i < 0$ even if coefficients b_i vanish. This is quite welcome as these coefficients are associated to higher-order interaction terms, which from phenomenological considerations, are presumably small. On the other hand, a non-vanishing and negative contribution from the quadratic term h_i^2 is absolutely necessary. Actually, the concrete case studied in Ref. (Barbosa et al. 2020) shows that this is indeed the case. Furthermore, condition Eq. (21) can be satisfied if $a < 0$.

Another feature associated to resilience is the "inertia" that the ES shows in changing from a given state to another. This feature can be identified with the ubiquitous dissipation of energy present in any realistic physical system. Most often dynamical dissipation processes can be described through velocity-proportional frictional forces. In the Lagrangian formalism for a particle, the effect of these forces can be accounted through the Rayleigh dissipation function, $R = -\kappa p^2/m$, where κ is a constant, p is the canonical conjugate momentum and

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m the mass of the particle.

For the ES, introducing dissipation through the Rayleigh function implies that the left hand side of the equations of motion (9) and (10) acquire extra terms $-\kappa_\psi \dot{\psi}$ and $-\kappa_{h_i} \dot{h}_i$, respectively. The effect of these terms is to reduce the amplitude of the motion of the ES once it goes from one state to another, thus acting as a resistance of the system to the change of its state. This can be clearly associated to resilience.

These considerations are sufficient for setting the physical conditions for the resilience of the ES. As we have seen, a metastable state corresponding to the solution (ψ_M, h_{iM}) of equations (17), (18), and (19), whose free energy (16) coefficients satisfy the conditions (20) and (21) together with the unavoidable dynamic friction that exists in any realistic system are the physical properties that endow the ES for having a resilient behaviour.

Since the Holocene, the ES has been subjected to a great stress. From the Great Acceleration of the second half of the last century, which presumably sparked the Anthropocene, the hyper expansion of human activities resulted that the safe operating space has been crossed for 6 of the 9 PBs (Richardson et al. 2023) and created all sorts of tensions, whose ongoing climate change crisis is the most persistent consequence for the ES. The tipping of some of the major ecosystems that compose the ES, such the Amazon rainforest and the Pacific Coral reefs, are already visible. As to the question of knowing if we have already inflicted an irreversible damage on ES or are close to it, only the understanding the mechanisms of residence can provide us with a knowledgeable answer. We hope that our work can help in providing a modest help in this respect.

IV. CONCLUSIONS

In this work we have considered the physical principles to ascertain the conditions of resilience in a LGT model of the ES. In order to implement resilience features we have endowed and considered modifications of the free energy so to ensure the existence of metastable states. Furthermore, we have modelled the ES capability to remain in an equilibrium state by arguing that it can be suitably prevented to runway towards the "Hot-House Earth" state by the presence of metastable states and the unavoidable dissipation of energy during the evolution of the relevant variables.

Indeed, we have shown that, thanks to the PBs interactions, a metastable state (ψ_M, h_{iM})

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exists if the conditions, Eqs. (20) and (21), for the coefficients of the free energy, Eq. (16), are satisfied. As pointed out in the above discussion, these conditions can be satisfied even if coefficients b_i vanish as far as $g_i < 0$.

Based on the observational data, it is possible to infer that the metastable state found above might correspond either to an actual state that the ES is close to reach or to a state that can be reached through engineering measures designed to drive the ES away from the Anthropocene traps it seems to be currently entangled in (see. Ref. (Søgaard Jørgensen et al. 2023) for a description of the 14 major Anthropocene traps).

A recent assessment has shown that 6 out of the 9 PBs have been crossed (Richardson et al. 2023) meaning that the evolution of most of the PBs is uncontrolled. It is unclear if the ES has already reached a point of no return, but it is evident that urgent measures to reverse the current development are needed. In fact, no single set of measures seems to be sufficient to halt the evolution of the PBs beyond the SOS. Two of the PBs that deserve particular attention are climate change and biosphere integrity. Both are deemed "core" because their essential role in the ES. The climate system reflects the distribution and balance of energy at the Earth's surface, while the controls material and energy flows, helping to strengthen the systems's resilience against both rapid and long-term changes. This calls for a concerted action involving stewardship measures (Bertolami 2022, Steffen et al. 2015a, 2011), internalising the workings of the PBs (see eg. Ref. (Bertolami 2024)) and making them become part of revised economic paradigms (Sureth et al. 2023), mitigation strategies that may include technological carbon sequestration (see eg. (Bertolami 2025, Bertolami & de Matos 2024) and refs. therein), and storage as means to curb climate overshoot, to avoid irreversible changes to the ES that will compromise the navigation space for the future generations.

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V. AUTHOR CONTRIBUTIONS

OB and MN conceptualised the study. OB performed the formula calculations. OB and MN wrote and edited the paper.

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