

# Setting up the physical principles of resilience in a model of the Earth System

Orfeu Bertolami<sup>1,\*</sup> and Magnus Nyström<sup>2,‡</sup>

<sup>1</sup>*Departamento de Física e Astronomia,  
Faculdade de Ciências da Universidade do Porto,  
Rua do Campo Alegre s/n, 4169-007, Porto,  
Portugal; Centro de Física das Universidades do Minho e do Porto,  
Rua do Campo Alegre s/n, 4169-007, Porto, Portugal.*

<sup>2</sup>*Stockholm Resilience Centre, University of Stockholm,  
Albanovägen 28, 106 91, Stockholm, Sweden.*

(Dated: December 21, 2025)

## Abstract

Resilience is a property of social, ecological, social-ecological and biophysical systems. It describes the capacity of a system to cope with, adapt to and innovate in response to a changing surrounding. Given the current climate change crisis, ensuring conditions for a sustainable future for the habitability on the planet is fundamentally dependent on Earth System (ES) resilience. It is thus particularly relevant to establish a model that captures and frames resilience of the ES, most particularly in physical terms that can be captured and influenced by human policy<sup>1</sup>. In this work we propose that resilience can serve as a theoretical foundation when unpacking and describing metastable states of equilibrium and energy dissipation in any dynamic description of the variables that characterise the ES. Since the impact of the human activities can be suitably gauged by the planetary boundaries (PBs) and the planet's temperature is the net result of the multiple PB variables, such as CO<sub>2</sub> concentration and radiative forcing, atmospheric aerosol loading, atmospheric ozone depletion, etc, then resilience features arise once conditions to avoid an ES runaway to a state where the average temperature is much higher than the current one. Our model shows that this runaway can be prevented by the presence of metastable states and dynamic friction built out of the interaction among the PB variables once suitable conditions are satisfied. In this work these conditions are specified. As humanity moves away from Holocene conditions, we argue that resilience features arising from metastable states might be crucial for the ES to follow sustainable trajectories in the Anthropocene that prevent it run into a much hotter potential equilibrium state.

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<sup>1</sup> See page 4 for examples of strategies

\*Electronic address: orfeu.bertolami@fc.up.pt

‡Electronic address: Magnus.Nystrom@su.se

## 12 I. INTRODUCTION

13 Over the past decades the human imprint on the Earth System (ES) has been exceptional  
14 (Jouffray et al. 2020, Steffen et al. 2015a). While the mass of humans is only about 0.01% of  
15 the total biomass, we have become a dominant force in shaping the face of Earth, including  
16 its atmosphere, biosphere, hydrosphere and lithosphere (Crutzen 2002, Ellis 2011, Foley  
17 2011, Nyström et al. 2019, Vitousek et al. 1997), and as of 2020 the global human-made  
18 mass surpasses the dry-weight of all living biomass (Elhacham et al. 2020). Thus, humans  
19 have become a hyper-keystone species (Worm and Paine 2016), which rivals geological forces  
20 in influencing the trajectory of the ES (Steffen et al. 2018)

21 A major concern of these changes is the risk of crossing of so-called tipping-points, which  
22 refer to the critical threshold at which a small change or event triggers a significant and  
23 potentially irreversible (regime) shift in a system (Lenton et al. 2008). Tipping-points have  
24 been observed in various systems, such as ecosystems (e.g. food webs, benthic communi-  
25 ties), social systems (e.g. norms, policy), economic systems (e.g. market-based economy)  
26 and technological systems (e.g. steam engine, smartphone, artificial intelligence) (Nyborg  
27 et al. 2016, Scheffer 2009, Scheffer et al. 2001). Over the past couple of decades there have  
28 been raising concerns around the existence of tipping-elements, which are large-scale com-  
29 ponents (subsystems) of the ES that may transgress a tipping-point (Barnosky et al. 2012,  
30 Lenton et al. 2008). Example of such tipping-elements include, the Greenland Ice Sheet, the  
31 Atlantic Meridional Overturning Circulation (AMOC), permafrost, monsoon systems, and  
32 the Amazon rainforest. Importantly, these tipping-elements interact, which may lead to a  
33 cascading behaviour of the entire ES (Wunderling et al. 2024). The consequences of these  
34 dynamics for humanity could be colossal (Steffen et al. 2018).

35 Clearly, knowledge about tipping-points, where they are located, when they are ap-  
36 proached and identifying ways to navigate away from them, are key challenges for humanity  
37 (Barnosky et al. 2012, Scheffer et al. 2012). Two broad frameworks that could help assist  
38 in this regard are planetary boundaries and resilience theory. The two are complementary  
39 in the sense that the planetary boundaries provide a quantitative assessment whereas the  
40 resilience framework adds a strong theoretical underpinning.

41 The planetary boundaries (PB) framework (Richardson et al. 2023, Rockström et al.  
42 2009, Steffen et al. 2015b) has been used to define global and regional limits in biophysical

43 processes – ‘safe operating space’ – that must not be crossed if humanity is to stay away  
44 from systemic and potentially irreversible shifts in the ES. As such, the planetary boundaries  
45 framework serves as a “global dashboard,” tracking humanity’s collective impact on key  
46 environmental factors that threaten the Earth’s ability to sustain human life. More recently,  
47 focus has been directed towards exploration of how different boundaries can interact and  
48 potentially cascade, thereby shrinking the safe operating space for future human impacts on  
49 the ES (Lade et al. 2020). Importantly, while the PB framework highlights the presence of  
50 tipping points in biophysical processes, it does not specify their exact thresholds. Instead,  
51 it delineates two risk zones: a zone of increasing risk and a high-risk zone. In the former,  
52 the further boundary limits are exceeded, the greater the likelihood of causing significant  
53 harm—destabilizing critical Earth system processes and disrupting essential life-support  
54 functions. In the latter, or high-risk zone, there is a substantial risk of severe and potentially  
55 irreversible damage to key planetary functions. In essence, these zones are defined at a  
56 precautionary distance from the estimated locations of potential tipping points.

57 The resilience concept describes the extent to which a system can resist and develop (e.g.  
58 ecosystems or the the entire ES) with change by absorbing recurrent perturbations, deal with  
59 uncertainty and risk, and still sustain its key properties (Folke 2006, Holling 2001). This  
60 conception of resilience is based on the understanding that humans and nature are deeply  
61 interconnected through feedbacks between social and ecological components, which together  
62 influence overall behavior and dynamics (Biggs et al. 2012). This interdependence defines a  
63 social-ecological system (Berkes and Folke 1998) in which human well-being and prosperity  
64 rely on the stability and functioning of the Earth system (Folke et al. 2011). Multiple  
65 states (regimes), tipping-points and self-reinforcing feedback mechanisms (hysteresis) are  
66 a central feature of resilience (Holling 2001). In cases where resilience is high, a powerful  
67 shock – such as, storms, large wildfires, pest outbreaks in ecosystems, or armed conflicts,  
68 trade wars, and supply chain disruptions in social systems – is required to push the system  
69 beyond a tipping-point and into another state. However, gradual (creeping) change – such  
70 as, loss biodiversity, habitat fragmentation and pesticide resistance in ecosystems, or growing  
71 inequality and changing social norms in society – erodes resilience of the current state. This  
72 makes the system vulnerable even to smaller perturbations. Once the system finds itself  
73 in this new state it can be difficult, or even impossible to reverse due to self-reinforcing  
74 feedback mechanisms (Nyström et al. 2019, Scheffer 2009, Scheffer et al. 2001). Within

75 the context of PB variables, species extinction (i.e. biodiversity loss PB) represents an  
76 irreversible process. Resilience has also been suggested as a conceptual framework that  
77 could assist in developing paths towards sustainability (Folke et al. 2016). Hence, it can  
78 serve as a theoretical and practical foundation for the planetary boundaries framework.  
79 An important point to bear in mind, however is that resilience is a property of a system  
80 and is neither "good" nor "bad" per se. It can help maintain the current state of a system  
81 no matter whether it is deemed desirable or undesirable. The Holocene epoch has allowed  
82 development of agriculture, permanent settlements, and the emergence of complex human  
83 societies, so maintaining Holocene-like conditions can be deemed desirable, and safeguarding  
84 of resilience that support these conditions of critical importance for humanity (Steffen et al.  
85 2018).

86 Bearing in mind the resilience concept and its importance we aim in this work to specify,  
87 in the context of a thermodynamical model of the ES, what are the physical properties  
88 that manifest themselves collectively as resilience features of the ES. Our starting point is  
89 a thermodynamical model of the ES from Holocene state conditions to other potentially  
90 stable states, which can be regarded as phase transitions and admit a description through  
91 the Landau-Ginzburg Theory (LGT) (Barbosa et al. 2020, Bertolami and Francisco 2018,  
92 2019). The LGT is a theoretical framework used in physics to describe phase transitions,  
93 such as when a material changes from a solid to a liquid state or a magnetic material loses  
94 its magnetism. Here we use the LGT to describe the transitions the ES has gone throughout  
95 the history of Earth.

96 As we shall review in the next section, this framework allows for determining the equi-  
97 librium states of the ES in terms of the planet's biophysical subsystems or processes that  
98 are, due to the impact of the human activities, the driving forces that dominate its evo-  
99 lution. In the Anthropocene, human activities are here collectively denoted by  $H$ . In the  
100 phase-transition model discussed in Refs. (Barbosa et al. 2020, Bertolami and Francisco  
101 2018, 2019),  $H$  was considered an external field, however, in the present work, we admit  
102 that through policies and actions, the dynamic features of the ES can be altered so to mod-  
103 ify the topographic landscape of possible Anthropocene trajectories. Way to do so include,  
104 mitigation strategies, such as halting deforestation and changing agricultural practices that  
105 contribute to CO<sub>2</sub> emission; transformation strategies, such as shifting from fossil fuel-based  
106 economies to ones based on renewable energy, and; restoration strategies, such as restoration

107 of degraded ecosystems and CO<sub>2</sub> capture technologies.

108 As previously discussed, the proposed Landau-Ginzburg model allows for getting the  
109 evolution equation of the ES, the so-called Anthropocene equation, and to associate the  
110 sharp rise of the physical parameters that characterise the ES to the great acceleration of  
111 the human activities (Bertolami and Francisco 2018), which became conspicuous from the  
112 second half of the 20th century and onwards (Steffen et al. 2015a).

113 However, as will be seen below, the original model did not exhibit explicit features that  
114 resemble resilience. This is the main purpose of the present work. As the model is based  
115 on thermodynamical arguments, one must seek for physical properties that would lead to  
116 a more resilient behaviour of the ES. In the context of the model, resilience is regarded  
117 as the resistance the ES shows in changing from one equilibrium state to another. At the  
118 present transient period, the Anthropocene, it has been hypothesised that the ES is moving  
119 away from the Holocene equilibrium state to a new state, potentially a Hothouse Earth state  
120 (Steffen et al. 2018) (Fig. 1). As we shall see, our results show that resilience is associated  
121 to the existence of metastable states and explicit dissipation of energy that prevent the ES  
122 to runaway towards the Hothouse Earth state.

123 A pleasing feature of the proposed description is that it allows for drawing trajectories of  
124 the ES in the phase space of model's variables. By considering that the PBs and the ensued  
125 temperature display dynamics that are affected by PBs self-interactions which are shown to  
126 be different from zero (Barbosa et al. 2020), two well defined and distinct sets of trajectories  
127 were identified upon assumptions about the evolution of the PB: a linear growth of the  
128 human activities,  $H(T) = H_0 t$ , where  $H_0$  is an arbitrary constant, from which follows that  
129 all ES trajectories starting at the Holocene are led to Hothouse Earth state (Steffen et al.  
130 2018) (Fig. 1) with a necessarily higher temperature than the Holocene average temperature  
131 (Bertolami and Francisco 2019); if instead, the increase of the human activities impact on  
132 the ES obey a discrete logistic map (Jakobson 1981, Kingsland 1995, May 1976), trajectories  
133 can display bifurcations or chaotic behaviour (Bernardini et al. 2025). Of course, as human  
134 activities are bounded by the finiteness of resources, the logistic map might be a more  
135 accurate description of its behaviour, although it is not quite clear what is the time span  
136 elapsed between successive steps of the logistic map. In any case, it is relevant to keep in  
137 mind that a too fast increase might give origin to trajectory bifurcations or even chaotic  
138 behaviour, which, of course, precludes predictions and control measures on the evolution of

139 the ES.

140 In this work we extend the previous studies of the ES model carried out in Refs. (Bar-  
141 bosa et al. 2020, Bernardini et al. 2025, Bertolami and Francisco 2018, 2019) on various  
142 aspects. Previously, we aimed to show the inevitability of the Hothouse Earth state given  
143 the disestablishing nature of the human activities and the interplay among the PBs. Here,  
144 we consider the dynamic features arising from the self-interactions of the 9 identified PBs,  
145 here generically denoted as  $h_i$ ,  $i = 1, \dots, 9$ , and show the specific conditions to implement  
146 resilience in the the eleven dimensional space  $(\psi, h_i, F(\psi, h_i))$ . Resilience can be regarded  
147 as a set of measures that prevent or delay the evolution of the ES towards a Hothouse Earth  
148 state and ensuring that this state is as close as possible to the Holocene state<sup>1</sup>. This can be  
149 implemented by creating metastable states to avoid a runaway situation due to a barrier that  
150 arises as higher-order terms into the Helmholtz free energy are introduced (cf. discussion  
151 below). A further requirement is dynamic friction, that is friction introduced via a kinetic  
152 energy-type term, to restrict the change of state in the phase space. This is a fairly natural  
153 condition as any realistic system dissipates energy. The specific conditions for the ES to  
154 acquire effective resilience features will be discussed below. Trajectories of the ES without  
155 and with resilience are depicted in Figs. 1 and 2 respectively (cf. a detailed discussion  
156 below).

157 This paper is organised as follows: in section II we review the cardinal aspects of the  
158 LGT of the ES and discuss the most relevant features of the dynamical system emerging  
159 from the model; in section III, we discuss the implementation of the resilience features in  
160 the model and connect them to properties that any model of the ES should have. Finally, in  
161 section IV we present our conclusions and discuss how our work can be extended to address  
162 several issues concerning features and transformation of the global social-ecological system.

## 163 II. A THERMODYNAMICAL MODEL FOR THE EARTH SYSTEM

164 We first review the main features of the proposed model for the ES (Bertolami and  
165 Francisco 2018) and discuss in the next section the conditions to extend it in order to  
166 explicitly exhibit resilient properties.

The proposal of Ref. (Bertolami and Francisco 2018) is to regard transitions of the ES as  
phase transitions which can be described by the LGT through an order parameter,  $\psi$ , and

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<sup>1</sup> Notice that prior the Anthropocene, the equilibrium states of the ES correspond to cooler (glaciation) and hotter (Hothouse Earth) equilibrium states with respect to the Holocene. However, at the Anthropocene, human activities lead inevitably the ES towards a Hothouse Earth state due to the massive emission of greenhouse gases. This materialises in the minus sign of the linear term in Eq. (1) below.

natural parameters (astronomical, geophysical, internal). In the Anthropocene, the natural forces average out to zero and the system is driven by the strength of the human activities, collectively denoted by  $H$ . In this approach, the thermodynamic description of the system is obtained through the Helmholtz free energy,  $F$ , which can be written as an analytic function of an order parameter,  $\psi$ , which is chosen to be the reduced temperature relative to Holocene average temperature,  $\langle T_H \rangle$ ,  $\psi := (T - \langle T_H \rangle) / \langle T_H \rangle$ . Thus, in the Anthropocene, disregarding the spatial variation of  $\psi$ , one can write (Bertolami and Francisco 2018, 2019):

$$F(\psi, H) = F_0 + a\psi^2 + b\psi^4 - \gamma H\psi, \quad (1)$$

167 where  $F_0$ ,  $a$ ,  $b$  and  $\gamma$  are constants. The linear term in  $\psi$  corresponds to the human activities,  
 168 which at the Anthropocene can match the quadratic and quartic contributions due to natural  
 169 causes (astronomic, geological internal).

The strength of the human activities are probed by their impact via the PBs (Rockström et al. 2009, Steffen et al. 2015b),  $h_i$ ,  $i = 1, 2, \dots, 9$  with respect to their Holocene values. Given that the PB can interact among themselves, the most general expression for  $H$  is given by (Bertolami and Francisco 2019):

$$H = \sum_{i=1}^9 h_i + \sum_{i,j=1}^9 g_{ij} h_i h_j + \sum_{i,j,k=1}^9 \alpha_{ijk} h_i h_j h_k + \dots, \quad (2)$$

170 where  $[g_{ij}]$  is a non-degenerate,  $\det[g_{ij}] \neq 0$   $9 \times 9$  matrix. Similar conditions should be  
 171 imposed on the coefficients  $\alpha_{ijk}$  and  $\beta_{ijkl}$  of the higher-order interaction terms. In principle,  
 172 these interactions terms are sub-dominating, however, their importance has to be established  
 173 empirically. As pointed out in Ref. (Bertolami and Francisco 2019), the interaction terms  
 174 may lead to new equilibrium states and suggest some mitigation strategies depending on  
 175 their sign and strength in the matrix entries (Bertolami and Francisco 2019). This will be  
 176 explicitly discussed in the next section. In Ref. (Barbosa et al. 2020), it was shown that the  
 177 interaction term between the climate change variable CO<sub>2</sub> concentration), say,  $h_1$ , and the  
 178 oceans acidity, say,  $h_2$ , was non-vanishing and contributed to about 10% of the value of the  
 179 individual contributions themselves.

180 In order to introduce resilience features into the model, that is, resistance to change from  
 181 one equilibrium state into another, we have to consider, contrary to previous works (Barbosa  
 182 et al. 2020, Bernardini et al. 2025, Bertolami and Francisco 2018, 2019), that the PBs are  
 183 dynamical variables that are not only passively changed due to human activities, but that

184 can be actively altered so to boost the resilience features of the ES. This allows us to project  
 185 how the ES would behave depending on its initial state and subsequent trajectory in the  
 186 phase space of the model, specified through the variables  $(\psi, \dot{\psi}, h_i, \dot{h}_i)$ . Thus, for a given set  
 187 of initial conditions, corresponding to a state  $(\psi(0), \dot{\psi}(0), h_i(0), \dot{h}_i(0))$  in the phase space, one  
 188 can, in principle, obtain the trajectories, *orbits*, in the phase space after solving the initial  
 189 value problem through the evolution equations of the system. The equations of motion are  
 190 obtained through the Lagrangian or equivalently through the Hamiltonian formalism. The  
 191 latter, yielding to first order differential equations, is more suitable to establish a dynamical  
 192 system in its canonical form.

193 The Lagrangian function must include, besides the potential, which is given by the free  
 194 energy, a set of kinetic energy terms for the canonical coordinates. The simplest possible ki-  
 195 netic term is a quadratic term proportional to the squared first derivative of each coordinate.  
 196 Thus, we can write the following Lagrangian:

$$\mathcal{L}(q, \dot{q}) = \frac{\mu}{2} \dot{\psi}^2 + \frac{\nu}{2} \sum_{i=1}^9 \dot{h}_i^2 - F_0 - a\psi^2 - b\psi^4 + \gamma H\psi, \quad (3)$$

197 where  $\mu$  and  $\nu$  are arbitrary constants and the dots stand for time derivatives. The constant  
 198  $\nu$  is assumed to be the same for all PB variables.

Aiming to get the Hamiltonian function, we evince the relevant canonical conjugate mo-  
 menta associated to  $\psi$  and to a generic PB variable,  $h_i$ :

$$p_\psi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \mu \dot{\psi}, \quad (4)$$

$$p_{h_i} = \frac{\partial \mathcal{L}}{\partial \dot{h}_i} = \nu \dot{h}_i, \quad (5)$$

from which follows the Hamiltonian function

$$\mathcal{H}(\psi, p) = \frac{p_\psi^2}{2\mu} + \sum_{i=1}^9 \frac{p_{h_i}^2}{2\nu} + F_0 + a\psi^2 + b\psi^4 - \gamma H\psi, \quad (6)$$

and Hamilton's equations,

$$\dot{\psi} = \frac{\partial \mathcal{H}}{\partial p_\psi}, \quad \dot{p}_\psi = -\frac{\partial \mathcal{H}}{\partial \psi}, \quad (7)$$

$$\dot{h}_i = \frac{\partial \mathcal{H}}{\partial p_{h_i}}, \quad \dot{p}_{h_i} = -\frac{\partial \mathcal{H}}{\partial h_i}. \quad (8)$$

The equations of motion read, considering for while just the contribution from the lowest  
 order terms in Eq. (2):

$$\mu \ddot{\psi} = -2a\psi - 4b\psi^3 + \gamma H \quad (9)$$

and

$$\nu \ddot{h}_i = \gamma \psi. \quad (10)$$

To exemplify the behaviour of variables  $\psi$  and  $h_i$ , let us obtain the resulting solutions for the simple case considered in Ref. (Bertolami and Francisco 2019). For  $b \simeq 0$ , we can neglect the cubic term in the equation of motion for  $\psi$  to get the equation of an harmonic oscillator under the action of an external force,  $H(t)$ . This yields for the simple case of an initial linear time evolution,

$$H(t) = H_0 t, \quad (11)$$

for an equilibrium initial state,  $\dot{\psi}(0) = 0$ , the analytical solution:

$$\psi(t) = \psi_0 \cos(\omega t) + \alpha t, \quad (12)$$

199 where  $\omega = \sqrt{2a/\mu}$  is an angular frequency,  $\alpha = \gamma H_0/2a$  and  $\psi_0$  is an arbitrary constant  
200 fixed by the initial conditions.

The solution for the impact on the PB,  $h_i(t)$ , which initially behaves collectively as Eq. (11), that is  $\sum_{i=1}^9 h_i(t \simeq 0) = H_0$ , quickly evolves to a cubic growth in time:

$$h_i(t) = A \cos(\omega t) + B t^3 + \alpha_i t, \quad (13)$$

201 where  $A = -\gamma \psi_0/\nu \omega^2$ ,  $B = \alpha \gamma/6\nu$ , for an arbitrary  $\alpha_i$ .

202 These solutions show that if the temperature  $\psi$  grows from an initial linear collective  
203 behaviour of the PBs,  $H = H_0 t$ , then it quickly drives the  $h_i$ s towards a cubic growth.  
204 Clearly, this model shows no resilience features as depicted in Fig. 1, where one clearly sees  
205 that from the Holocene, Anthropocene trajectories inevitably evolve towards a Hothouse  
206 Earth state.

207 In what follows we shall consider the introduction into the free energy function of a  
208 cubic term for  $\psi$  and higher than linear order terms for the PBs as these will allow for  
209 metastable states to arise, thus leading to bounded solutions for  $\psi$  and the PBs. Metastable  
210 states correspond to potential intermediate energy states between the Holocene state and  
211 the Hothouse Earth least energy state. In the LGT, metastable states can be considered and  
212 studied through cubic terms in the Helmholtz free energy. The conditions for the appearance  
213 of metastable states were already discussed in a completely different context, namely in a  
214 proposal to classify rocky planets (Bertolami and Francisco 2022), using the ideas developed

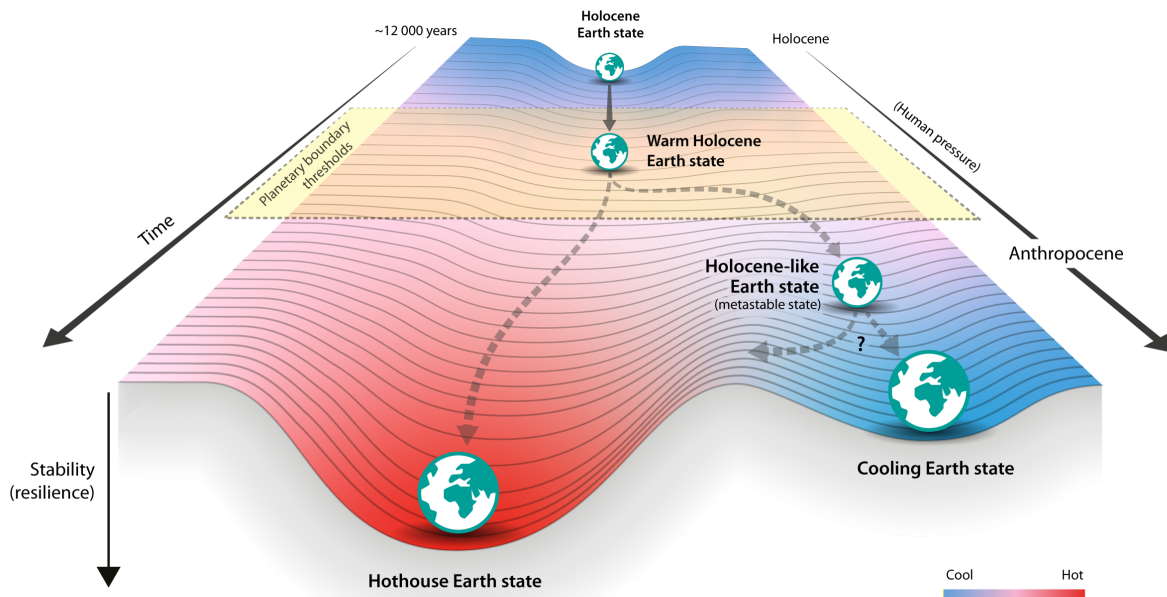


FIG. 1: A schematic illustration of the evolution of the Earth System with a start from the Neolithic revolution ( 12.000 years ago). Leading up to its current state (i.e. "warm Holocene Earth state") 7 of 9 planetary boundaries have been transgressed. A continuation on this pathway suggests that the Earth system may end up in a Hothouse Earth state (Steffen et al. 2018) (left pathway). However, explicit dissipation of energy, and policies and actions geared at building resilience of a metastable "Holocene-like Earth state" (see also Fig. 2) could provide an opportunity to build a trajectory toward a future "cooling Earth state" (right pathway).

215 in Refs. (Barbosa et al. 2020, Bertolami and Francisco 2018, 2019) to describe the ES. In  
 216 concrete terms, cubic terms might arise from PB interactions that have a strong dependence  
 217 on the temperature.

218 Before concluding this discussion it is worth stressing once again that the behaviour of  
 219 the ES depends crucially on the assumptions about the evolution of the PB. Indeed, as  
 220 pointed out in the introduction, the supposition that human activities grow linearly as in  
 221 Eq. (11) implies, as exemplified above, that ES trajectories lead to a potential "Hothouse  
 222 Earth" state (Bertolami and Francisco 2019) as discussed by Ref. (Steffen et al. 2018).  
 223 However, if the human activities impact on the ES behaves as a discrete logistic map <sup>2</sup>, as  
 224 suggested in Ref. (Bernardini et al. 2025), then evolution will depend the rate of growth of  
 225 human activities as solutions admit regular trajectories as well as trajectories that present  
 226 bifurcations and even chaotic behaviour. In the next section we shall consider the features  
 227 that must be introduced in the Helmholtz free energy and the conditions they must satisfy

<sup>2</sup> This means that the evolution of the PB,  $h_i$ , ( $i \in \overline{1, 2, \dots, 9}$ ) is considered to be discrete and obey the equation  $h_{i(j+1)} = rh_{i(j)}(1 - \alpha h_{i(j)})$ , where  $j$  denotes the number of "generations",  $r$  is the rate of growth and  $\alpha$  a constant.

228 in order to avoid the ES evolves towards the Hothouse Earth state.

### 229 III. SETTING UP THE PHYSICAL PRINCIPLES OF RESILIENCE

As mentioned above, **in this model** resilience features are associated to bounded trajectories in the Anthropocene and these ask for the existence of metastable states. In the LGT the metastable states arise by intruding cubic terms on the free energy. As pointed out in Ref. (Bertolami and Francisco 2022), the introduction of a cubic term allows for a richer variety of equilibrium states. Indeed, consider the free energy:

$$F(\psi, H) = F_0 + a\psi^2 - c|\psi|^3 + b\psi^4 - \gamma H\psi, \quad (14)$$

230 where we assume that constants  $b$ ,  $c$  and  $\gamma$  are positive, while constant  $a$  can be negative.

The existence of extrema is given by two conditions. The first one reads:

$$\frac{\partial F(\psi, H)}{\partial \psi} = 0 = 2a\psi - 3c\psi^2 + 4b\psi^3 - \gamma H. \quad (15)$$

231 The resulting cubic equation admits at least one real solution, say,  $\psi_M$ , meaning that there  
232 are at least two metastable states,  $\psi_M$  and  $-\psi_M$ . Clearly,  $\psi_M \neq 0$  as far as  $H \neq 0$ .

233 However, the unboundedness of the evolution of the variables  $(\psi, h_i)$  is due to the un-  
234 boundedness of the PBs. Recent assessment of the PBs has shown that 6 out of the 9 PBs  
235 have gone beyond their Holocene values where they were at equilibrium, a state usually  
236 referred to as Safe Operating Space (SOS).

The motion in the eleven-dimensional configuration space,  $(\psi, h_i, F(\psi, h_i))$ , is quite complex, so in order to simplify the analysis we consider one single generic PB,  $h_i$ , and assume that the remaining ones are unchanged<sup>3</sup>. The free energy can be written explicitly in terms of the high order contributions in  $H$  depicted in Eq. (2). We consider the essential set of terms in order to carry out the minimisation procedure, that is:

$$F(\psi, H) = \hat{F}_0 + a\psi^2 - c|\psi|^3 + b\psi^4 - \gamma(h_i + g_i h_i^2 + b_i h_i^3)\psi, \quad (16)$$

237 where we have aggregated all contributions to the quadratic and cubic terms in  $h_i$ , bf a  
238 generic PB, within the constants  $g_i$  and  $b_i$ . To ensure boundedness it is necessary that  $g_i$  is  
239 negative and that  $b_i$  is positive.

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<sup>3</sup> Notice that the analysis of two-variables case is quite relevant as the Kolmogorov-Arnold representation theorem establishes that any continuous function of several variables can be constructed out of a finite sum of two-variable functions.

Thus, from Eq. (16), one gets the condition:

$$\frac{\partial F(\psi, h_i)}{\partial h_i} = 1 + 2g_i h_i + 3b_i h_i^2 = 0, \quad (17)$$

240 which admits real non-vanishing solutions,  $h_{iM}$ . as far as  $g_i^2 > 3b_i$  for  $b_i \neq 0$  or  $h_{iM} = \frac{1}{-2g_i}$   
 241 if  $b_i = 0$ .

The general conditions to ensure that the extremum  $(\psi_M, h_{iM})$  corresponds to a minimum and hence to a metastable state are given by:

$$\frac{\partial^2 F(\psi_M, h_{iM})}{\partial \psi^2} \frac{\partial^2 F(\psi_M, h_{iM})}{\partial h_i^2} - \left( \frac{\partial^2 F(\psi_M, h_{iM})}{\partial \psi \partial h_i} \right)^2 > 0. \quad (18)$$

and

$$\frac{\partial^2 F(\psi_M, h_{iM})}{\partial \psi^2} > 0, \quad (19)$$

which yield the following relationships:

$$g_i < -3b_i h_{iM} \quad (20)$$

and

$$2a - 6c|\psi_M| + 12b\psi_M^2 > 0. \quad (21)$$

242 Satisfying these conditions imply the ES can settle in a the metastable state,  $(\psi_M, h_{iM})$ ,  
 243 that is, the system shows resilience and does not runaway towards the "Hothouse Earth"  
 244 state as depicted in Fig. 2 as far as  $3b_i < g_i^2 < 9b_i^2 h_{iM}$ . .

245 Notice that the conditions for the existence of a metastable state can be met if  $g_i < 0$   
 246 even if coefficients  $b_i$  vanish. This is quite welcome as these coefficients are associated to  
 247 higher-order interaction terms, which from phenomenological considerations, are presumably  
 248 small. On the other hand, a non-vanishing and negative contribution from the quadratic  
 249 term  $h_i^2$  is absolutely necessary. Actually, the concrete case studied in Ref. (Barbosa et al.  
 250 2020) shows that this is indeed the case. Furthermore, condition Eq. (21) can be satisfied  
 251 if  $a < 0$ .

252 Another feature associated to resilience is the "inertia" that the ES shows in changing  
 253 from a given state to another. This feature can be identified with the ubiquitous dissipation  
 254 of energy present in any physical system. Most often dynamical dissipation processes can be  
 255 described through velocity-proportional frictional forces which imply that just part of the  
 256 free energy of a system is turned into kinetic energy, that is, motion of the system. In the

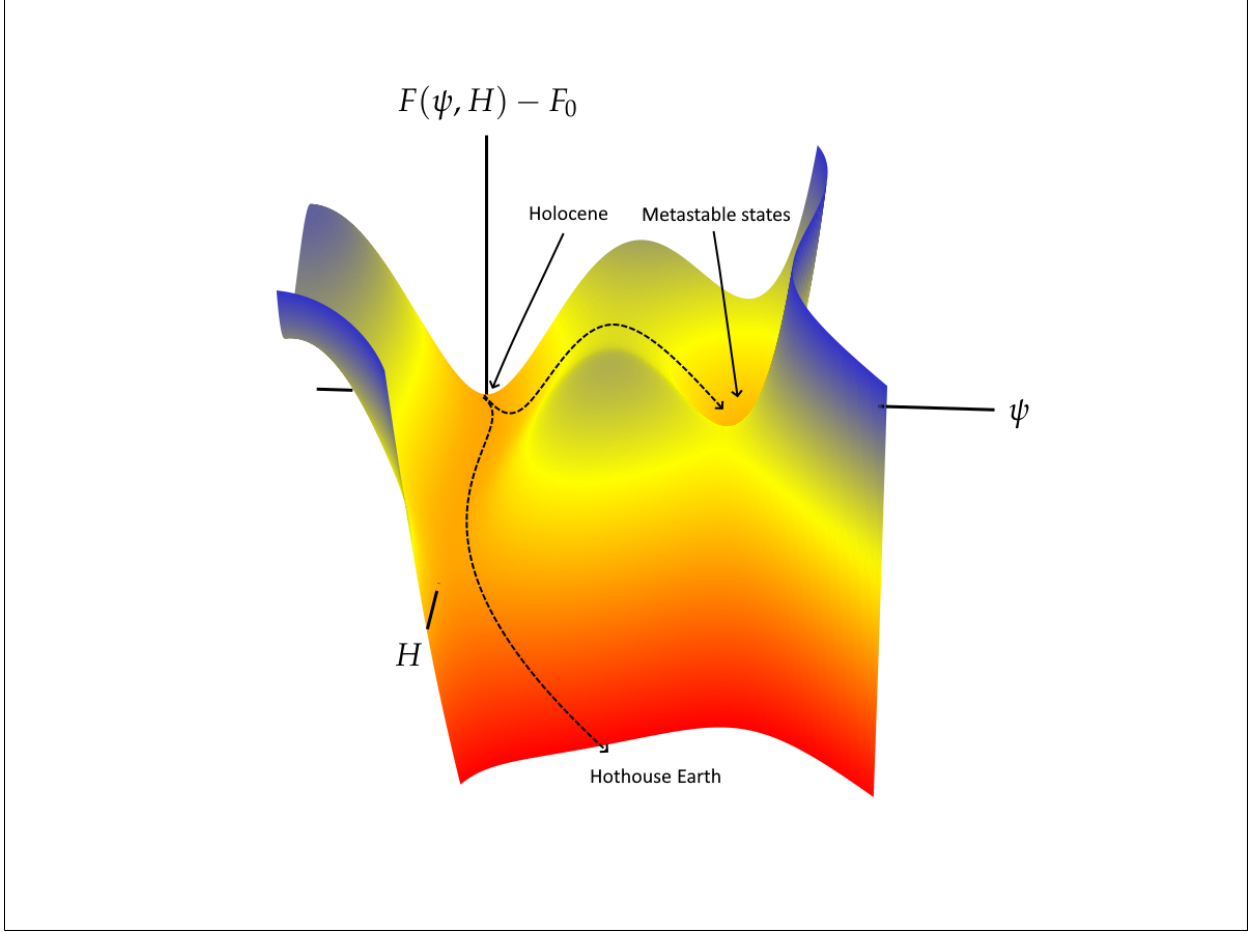


FIG. 2: Free energy in function of the temperature, planetary boundaries ( $H$ ) and resilience features (metastable state).

257 Lagrangian/Hamiltonian formalism for a particle, the effect of these forces can be accounted  
 258 through the Rayleigh dissipation function,  $R = -\kappa p^2/m$ , where  $\kappa$  is a constant,  $p$  is the  
 259 canonical conjugate momentum and  $m$  the mass of the particle.

260 For the ES, introducing dissipation through the Rayleigh function implies that the left  
 261 hand side of the equations of motion (9) and (10) acquire extra terms  $-\kappa_\psi \dot{\psi}$  and  $-\kappa_{h_i} \dot{h}_i$ ,  
 262 respectively. The effect of these terms is to reduce the amplitude of the motion of the ES  
 263 once it goes from one state to another, thus acting as a resistance of the system to the  
 264 change of its state. This can be clearly associated to resilience.

265 These considerations are sufficient for setting the physical conditions for the resilience  
 266 of the ES. As we have seen, a metastable state corresponding to the solution  $(\psi_M, h_{iM})$  of  
 267 equations (15), (18), and (19), whose free energy (16) coefficients satisfy the conditions (20)  
 268 and (21) together with the unavoidable dynamic friction /energy dissipation that exists in

269 any system are the physical properties that endow the ES for having a resilient behaviour.  
270 For sure, further research is needed in order to establish which PBs are more suitable for  
271 setting up the conditions obtained above. This means that the PB properties concerning  
272 their dependence on the temperature and strength of their self-interaction and with other  
273 PBs must be further studied.

#### 274 IV. CONCLUSIONS

275 In this work we have considered the physical principles to ascertain the conditions of re-  
276 silience in a LGT model of the ES. In order to implement resilience features we have endowed  
277 and considered modifications of the free energy so to ensure the existence of metastable  
278 states. Furthermore, we have modelled the ES capability to remain in an equilibrium state  
279 by arguing that it can be suitably prevented to runaway towards a potential Hothouse Earth  
280 state by the presence of metastable states whose existence conditions were explicitly shown  
281 and the unavoidable dissipation of energy during the evolution of the relevant variables.

282 Indeed, we have shown that, thanks to the PBs interactions, a metastable state  $(\psi_M, h_{iM})$   
283 can exist if the conditions, Eqs. (20) and (21), for the coefficients of the free energy, Eq.  
284 (16), are satisfied. As pointed out in the above discussion, these conditions can be satisfied  
285 even if coefficients  $b_i$  vanish as far as  $g_i < 0$ .

286 Based on the observational data, it is possible to infer that the metastable state found  
287 above might correspond either to an actual state that the ES is close to reach or to a state  
288 that can be reached by policy and actions (i.e. mitigation, transformation and restoration  
289 strategies) to drive the ES away from the Anthropocene traps it seems to be currently  
290 entangled in (see. Ref. (Søgaard Jørgensen et al. 2023) for a description of the 14 major  
291 Anthropocene traps).

292 A recent assessment has shown that 7 out of the 9 PBs have been crossed (Kitzmann  
293 et al. 2025) meaning that the evolution of most of the PBs is uncontrolled. Moreover it is  
294 unclear if the ES has already reached a point of no return, but it is evident that urgent  
295 measures to reverse the current development are needed. In fact, no single set of measures  
296 seems to be sufficient to halt the evolution of the PBs beyond the safe operating state. Two  
297 of the PBs that deserve particular attention are climate change and biosphere integrity.  
298 Both are deemed “core” because their essential role in the ES. The climate system reflects  
299 the distribution and balance of energy at the Earth’s surface, while the controls material

300 and energy flows, helping to strengthen the systems’s resilience against both rapid and long-  
301 term changes. This calls for a concerted action involving stewardship measures (Bertolami  
302 2022, Steffen et al. 2011, 2015a), bringing into the economy (internalising) the workings of  
303 the ES (see eg. Ref. (Bertolami 2024)) and making them become part of revised economic  
304 paradigms (Bertolami and Gonçalves 2024, 2025, Sureth et al. 2023), mitigation strategies  
305 that may include technological carbon sequestration (see e.g. (Bertolami 2025, Bertolami  
306 and de Matos 2024) and refs. therein), and storage as means to curb climate overshoot, to  
307 avoid irreversible changes to the ES that will compromise the navigation space for the future  
308 generations. **Given that tipping in some major ecosystems constituting the Earth System is**  
309 **already being observed, such as in Pacific coral reef systems, and that other tipping elements**  
310 **including the Amazon rainforest and the polar ice sheets may be approaching the threshold**  
311 **of self-reinforcing tipping dynamics (Lenton et al. 2025), a critical question arises as to**  
312 **whether irreversible damage to the Earth System has already occurred, or is imminent.** The  
313 answer comes only through the understanding of the mechanisms of resilience and how their  
314 boosting, through the PB interactions, can be effective. We hope that our work can provide  
315 a modest help in this respect.

## 316 **Acknowledgements**

317 O.B. would like to thank the kind hospitality extended to him during his stay at the  
318 Stockholm Resilience Centre in May 2024 where the initial discussions that led to this work  
319 took place. He is also grateful for the fruitful discussions on various matters with Lan  
320 Wang-Erlandsson, Peter SØgaard Jørgensen, Dieter Gerten, Ricardo Elísio and Uno Svedin.

## 321 **V. COMPETING INTERESTS**

322 The authors declare that they have no conflict of interest.

## 323 **VI. AUTHOR CONTRIBUTIONS**

324 OB and MN conceptualised the study. OB performed the formula calculations. OB and  
325 MN wrote and edited the paper.

326 **VII. FINANCIAL SUPPORT**

327 The work of Magnus Nyström was supported by the Swedish Research Council grant  
328 (number 2020-04586).

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