TECHNICAL NOTE: ANALYSIS OF CONCENTRATION-DISCHARGE HYSTERESIS LOOPS USING SELF-ORGANIZING MAPS

RESPONSE TO REFEREES

Referees' comments appear in black; authors' responses are in red. Verbatim excerpts from the manuscript are indented and shown in black.

AUTHORS' RESPONSE TO REFEREE 1

Referee 1 Comment 1: Overall, this manuscript is well written and supported with literature. You demonstrate strong scientific rigor, present informative figures, and provide a supportive narrative.

Response to R1C1: First and foremost, we sincerely appreciate your positive assessment of our manuscript. Your comments were both encouraging and helpful. We have carefully addressed your suggestions in the revised version, which we believe has improved the overall clarity and quality of the work.

Changes to the manuscript: As described in subsequent responses.

Referee 1 Comment 2: Line 26: You mention that event-scale concentration has been employed for decades, but the oldest citation is only 5 years old (*Malutta et al., 2020*). Can you add some older/original literature in this first sentence to support your claim? Perhaps *Williams (1989), Hamshaw et al. (2018), Bettel et al. (2025)* since you mention in Line 225 that hysteresis loops were first recognized in these articles.

Response to R1C2: The citations originally included in this line are review papers that provide comprehensive discussions on concentration–discharge relationships, including its historical development. However, we agree it is valuable to cite some of the original, foundational studies to better support our claim.

Changes to the manuscript: We will update the manuscript to include the following references:

> *Heidel, S. G. (1956). The progressive lag of sediment concentration with flood waves. Eos, Transactions American Geophysical Union, 37(1), 56–66.*
>
> *Williams, G. P. (1989). Sediment concentration versus water discharge during single hydrologic events in rivers. Journal of Hydrology, 111(1), 89–106.*
>
> *Evans, C., & Davies, T. D. (1998). Causes of concentration/discharge hysteresis and its potential as a tool for analysis of episode hydrochemistry. Water Resources Research, 34(1), 129–137.*

Referee 1 Comment 3: Section 2.2 – 2.4: I think these sections would be easier visualized with a workflow diagram that links to Figure 3. You could include a visualization for the process of training the model and finding the BMU. In the same workflow diagram, you can include a visualization for the process of using the DTW. Then, those can have an arrow pointing to the "SOM training" in Figure 3. Additionally, if possible, including the topological preservation and quantization accuracy into the diagram would help create a complete "picture" of the process. Although these three sections are written well, it is hard to visualize the order of the process. Additionally, Figure 3 in its current state is too general to provide a specific picture of the training process.

Response to R1C3: Thank you for your suggestion. We have restructured section 2, expanded Figure 3, and added new diagrams to provide a clearer picture of the use of SOM in hysteresis analysis. Changes made to the manuscript aim to better illustrate the general workflow, the SOM training process, and the hyperparameter tuning workflow, including a better description of the role of topographic and quantization errors. We have also added a new section to the Supplementary Information to better explain the DTW algorithm. We believe that these new diagrams, together with further revisions in the text prompted by Referee's 3 comments (see comment 2 from Referee 3 and its response) will enhance significantly the clarity of section 2.

Changes to the manuscript:

1. Fig. 3 was expanded to include the SOM evaluation as part of the training phase and to link the SOM training and hyperparameter tuning steps to the newly added figures 4 and 5.
2. A new figure (Fig. 4 in the revised manuscript) was added to describe the SOM training algorithm in detail.
3. A new figure (Fig. 5 in the revised manuscript) was added to illustrate the process of hyperparameter tuning, including the elbow method and Pareto analysis, and the role of topographic and quantization error in model selection. Together, Figs. 3, 4, and 5 will help to better show the training and refinement of the SOM for hysteresis analysis.
4. A new section (Section S1) was added in the Supplementary Information to explain the dynamic time warping algorithm (DTW) and how it compares to Euclidean distance.
5. In addition to the new figures, we reworked section 2 and adjusted section 3 to more clearly describe the proposed workflow in alignment with the added figures and suggestions from Referee 3 (see more details in our response to comment 2 from Referee 3).

Referee 1 Comment 4: Line 228: I recommend simply adding a parenthesis such as (as seen on the left rows in Figure 4) after the sentence to immediately direct your viewers eyes to the single-line, clockwise, and counterclockwise.

Response to R1C4: Accepted.

Changes to the manuscript: We will modify the sentence as follows (Note that former Fig. 4 corresponds to Fig. 6 in the revised manuscript):

> *However, our analysis suggests that sediment hysteresis loop diversity can largely be explained by the 17 loop types identified in Fig. 6, encompassing single-line (first row in Fig. 6), clockwise (second row in Fig. 6), and counterclockwise (third row in Fig. 6) topologies*

Referee 1 Comment 5: Figure 4 caption: I would also recommend adding additional information on the single-line, Figure 8, clockwise, and counterclockwise topologies. You have a lot of detail in Figure 1-3, and I would suggest continuing that format.

Response to R1C5: Accepted

Changes to the manuscript: We will modify the caption as follows (Note that former Fig. 4 corresponds to Fig. 6 in the revised manuscript):

> *"Figure 6. Loop types considered in the training dataset. Except for Figure-L loops—introduced by Hamshaw et al. (2018)—all loop types were originally described by Williams (1989). While past studies often lump together all subclasses within the broader categories of Single Line, Clockwise, and Counterclockwise loops, our dataset introduces a more granular classification that captures multiple shape variations within each type, such as differences in concavity, which reflect the relative spread of the sedigraph with respect to the hydrograph, or Figure-L loops, which indicates strong decoupling between discharge and sediment concentration. Readers interested in the hydrological interpretation of these loop types are encouraged to consult the original studies."*

Referee 1 Comment 6: Line 256: It seems like Line 256 should be appended to line 255.

Response to R1C6: After reworking sections 2 and 3 following Referee's 3 suggestions (see comment 2 from Referee 3 and its response), former Line 256 is no longer included in the manuscript.

Changes to the manuscript: NA

Referee 1 Comment 7: Line 259: Would the highly distorted maps be defined by topological error (e.g. referring to section 2.3) and not the "topographic" error that is listed? If so, topographic is used throughout the paper (Line 169 and Line 325, Line 327, etc.) and would need corrected.

*Response to R1C7***:** The terms *topological preservation* and *topographic error* refer to related but distinct concepts. Topological preservation describes the abstract ability of a SOM to maintain relative distances between data samples (in our case loops) when projecting them from high-dimensional space to a two-dimensional map. Topographic error, by contrast, is a specific metric used to quantify the degree of topological preservation achieved by a trained SOM. Therefore, the correct term in this context is topographic error.

Changes to the manuscript: We have revised the manuscript to ensure consistent and accurate use of these terms and to avoid potential confusion.

Referee 1 Comment 8: Line 262: A citation should be provided for the Pareto-optimal analysis.

Response to R1C8: Accepted

Changes to manuscript: The following reference will be added where the Pareto-optimal analysis is thoroughly described

> Koppa, A.; Gebremichael, M.; Yeh, W. W.-G. Multivariate Calibration of Large Scale Hydrologic Models: The Necessity and Value of a Pareto Optimal Approach. *Advances in Water Resources* 2019, *130*, 129–146.

Also, the newly added Figure 5 illustrates the Pareto-optimal analysis.

Referee 1 Comment 9: Figure 8 caption: Please provide an explanation of what is shown in (a), (b), (c), (d), specifically.

Response to R1C9: Accepted

Changes to manuscript**:** We will modify the caption as follows (Former Fig. 8 corresponds to Fig. 10 in the revised manuscript):

> *"Figure 10. Mapping of all samples from the curated loop dataset to their respective BMUs on the trained SOM. For visual clarity, loop types and their mappings are organized into four distinct panels: (a) single-line loops; (b) clockwise loops including Async, Sync, Concave, and Figure-L; (c) counterclockwise loops including Async, Sync, Concave, and Figure-L; and (d) Figure-eight and Line+loops. Acronyms used in the figure include: SP – Sync-Peak, AP – Async-Peak, CU – Concave-Up, CD – Concave-Down, ccw – Counterclockwise, cw – Clockwise, fL – Figure-L, f8 – Figure-eight."*

AUTHORS' RESPONSE TO REFEREE 2

General comments

Referee 2 Comment 1: I have no expertise in watershed hydrology, nor on hysteresis loops. I read the paper from the perspective of a Kohonen algorithm practitioner.

My overall assessment is that it is a very good paper from this point of view. As far as I understand, the application problem is very well posed and explained. The reasons for using Kohonen's algorithm are perfectly introduced and justified. The algorithm itself is very well defined, as well as its practical implementation. The methodology and the different steps are clear and reproducible. Finally, the results are, according to the authors, quite satisfactory.

Response to R2C1: We sincerely appreciate this generous feedback from the Referee. It's encouraging to receive such positive remarks.

Changes to manuscript: Although referee 2 did not specifically request changes to Sections 2 and 3, we have revised these sections in response to suggestions from referees 1 and 3, who requested further clarification regarding the SOM algorithm and its implementation (see comment 3 from Referee 1 and comments 2 and 3 from Referee 3 and their responses) with respect to hysteresis analysis. Importantly, the results remain unchanged.

Specific comments

Referee 2 Comment 2: The authors present hysteresis loops and the different methods used to classify them (which makes it possible to characterize hydrological events). They explain why traditional clustering methods are not satisfactory, in particular because we may not see what the important factors are.

The authors then discuss methods based on the calculation of certain indices, but show that these indices do not always make it possible to distinguish between very different events

To improve the analysis of loops, the authors propose to use the Kohonen algorithm (SOM) for its well-known clustering and visualization of results on two-dimensional maps.

The presentation of Kohonen's algorithm is very well done, very educational. The authors carefully explain the role of each hyperparameter. They offer to choose a distance adapted to the nature of their data. They introduce two criteria for choosing an "optimal" map, which must make a compromise between topographic error and quantization error, both of which must be minimized, although they vary in opposite directions.

The implementation of the algorithm is carefully detailed. The authors explain the different steps, pre-processing, filtering by a moving median, standardization of the lengths of the data vectors (representing loops), normalization (to keep only the shape).

The Kohonen map is trained on a balanced and labeled database (although the label is not included in the input data) where all the observed loop shapes are represented.

The authors present the choices (map size, parameters) they have chosen.

However, they remain inaccurate with regard to the decay function of the alpha parameter and the size. It would be desirable to propose and explain decay functions in accordance with the theory.

Then, once the map has been calibrated from the labeled data, the authors use it to classify the hysteresis loops from several observation stations by projecting them onto the map. They carefully study the clusters obtained and determine the significant explanatory variables through statistical analyses. They compare the conclusions obtained with the conclusions derived from the study of the indices presented in the introduction.

To conclude and allow the use of their methodology, the authors provide a program in Python.

Technical corrections: There is nothing to report, the manuscript is very neat and really very well written.

*Response to R2C2***:** In response to your comment on the decay function, the revised section 2 provides further details on the training process, including the decay function for the learning rate (α) and neighborhood radius (σ).

Changes to manuscript: The following text and figure will be included in Section 2.5.1 in the revised manuscript to better describe the role of the decay function and our particular choice:

> *"Once the BMU is identified, the learning parameters for the current iteration—learning rate* (α) *and neighborhood radius* (σ)*—are computed using a decay function. This decay function gradually reduces* α *and* σ*, resulting in a transition during training from an initial ordering and placement phase where prototypes broadly align with the spatial structure of the input data, to a fine-tuning phase where prototypes are refined to better represent the input samples (Samarasinghe, 2016).*
>
> *Common choices of decay functions include hyperbolic, exponential, and linear (Kohonen, 2013). In our implementation, we adopted an exponential decay rate which varies from initial to final values defined during the map initialization (Fig. 4)."*

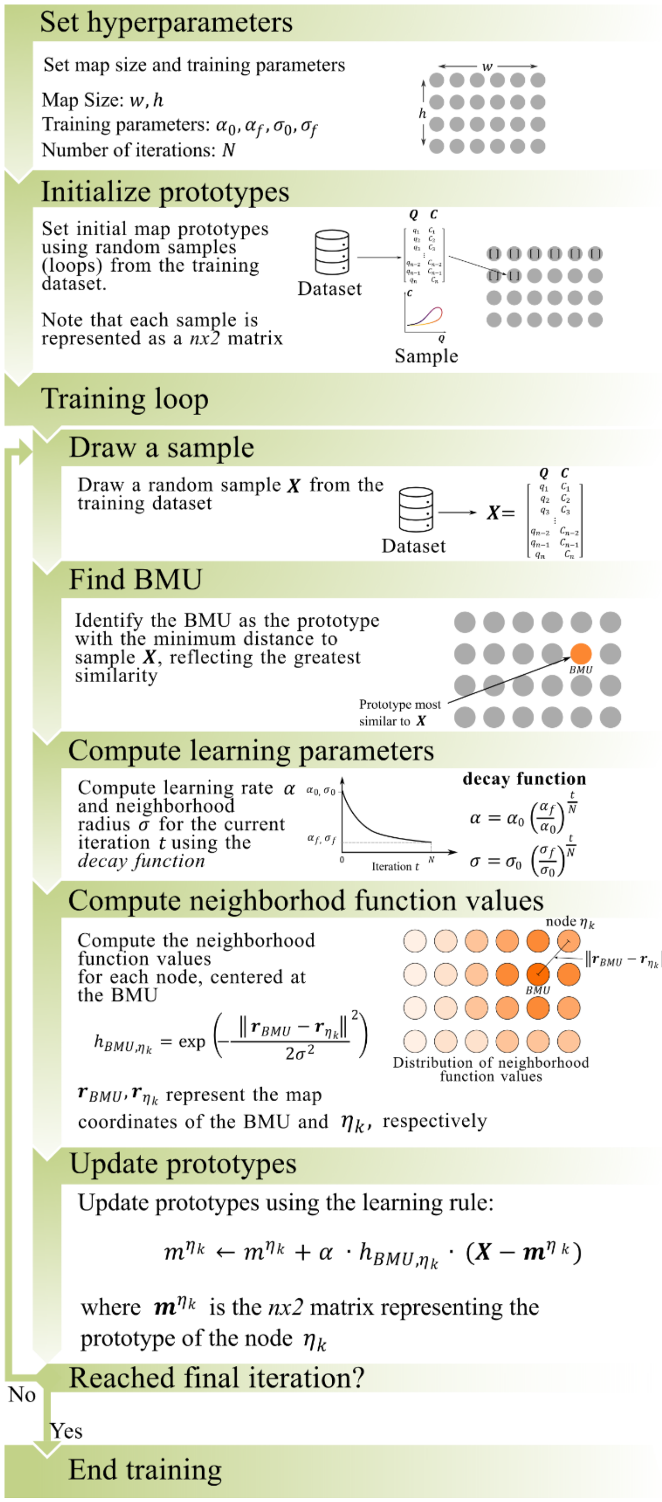


Figure 4. Workflow for training an SOM. α: *learning rate,* σ: *neighborhood radius, subscripts* 0, f *indicate initial (first iteration) and final (last iteration) values, respectively.* Q: *Discharge,* C: *Concentration,* n: *length of the sequence of* (Q, C) *data pairs representing a loop (section 2.3). Other symbols are defined in the figure.*

AUTHORS' RESPONSE TO REFEREE 3

Referee 3 Comment 1: The study by Ramirez et al "Analysis of concentration-discharge hysteresis loops using Self-Organizing Maps" describes a novel method to analyse hysteresis patterns in high-frequency concentration-discharge data.

The paper is generally very well written and I particularly like the innovative analyses and combination of methods to characterize and assess hysteresis loops and to assign event characteristics and catchment properties using an unsupervised machine learning algorithm. Also the figures are very nicely presented.

The presented method can provide a major step forward in the field of hysteresis analysis. However, despite having worked with hysteresis analysis in the hydrological and water quality context extensively, I have major difficulties understanding the content. I acknowledge that this is due to the fact that I have no background on SOM and Kohonen's algorithm, but given that this study is going to be published in HESS, I think a large part of the audience is likely to have a background in watershed hydrology. Therefore, I stress that the authors should significantly improve on the explanation of the methods and results. I hope the authors find my comments below useful in improving the manuscript.

Response to R3C1: We appreciate your positive comments and constructive feedback. Based on your suggestions, we have made several revisions to improve the clarity of the methods and results, especially for readers without prior experience using the SOM algorithm. Specifically, we have reworked and expanded section 2 and 3 of the manuscript to better explain the SOM method within the context of hysteresis analysis. We've also prepared two additional figures to explain our workflow and will also provide additional clarification of the SOM algorithm herein and in the updated manuscript. We believe that these changes will make the manuscript much more accessible to the broader hydrology community. We expand upon our proposed changes to the manuscript below.

Changes to manuscript: As detailed below

General and major comments

Referee 3 Comment 2: The general workflow is hard to grasp and it is unclear what your python package can accomplish and what the user needs to do get all this to work - particularly because this is meant to be a technical note. Therefore, I think the paper would benefit from a figure/flowchart explaining the required steps and purpose of the steps from downloading/acquiring a dataset, via the curation of the dataset, training, refinement, application, to the resulting map. This would be like a summary map linking and expanding on figures 3, 4 and 5. It might make sense to move the current chapter 2.5 to the beginning of section 2 and expand with the points I highlighted.

Response to R3C2: We appreciate the Referee's suggestion to clarify the overall workflow and the functionality of the HySOM Python package. In response, we have revised Sections 2 and 3 to improve clarity and accessibility, particularly for readers unfamiliar with SOM. We believe it may be too cumbersome for readers if we were to fully include the necessary information regarding the training and refinement of the SOM for hysteresis analysis in Fig. 3. Thus, we have added two new figures (Figures 4 and 5) to detail the SOM training process and hyperparameter tuning.

Figure 3 provides a general overview of the workflow, while the newly added figures 4 and 5 present the methodological specifics, including the SOM training algorithm (Fig. 4) and hyperparameter tuning (Fig, 5), using the elbow method and Pareto analysis. We note that Figs. 3, 4, and 5 are meant to be agnostic of water quality parameters, therefore we reserve information regarding curation of the dataset for sediment hysteresis analysis in Section 3.

Additionally, we have added a section to the Supplementary Information (S1) to describe the Dynamic Time Warping (DTW) algorithm so readers can better understand the concept of a distance function and how it is used to quantify similarities between loops.

In addition to incorporating the updated Fig. 3 and new Figs. 4 and 5 in the revised manuscript, we have reworked sections 2 and adjust section 3 to more clearly describe the proposed workflow in alignment with the added figures. We have also expanded Section 7 (Code and Data Availability) to include further details on the functionality, inputs, and outputs of the HySOM package. These visual and textual enhancements aim to improve transparency, reproducibility, and overall clarity of the proposed methodology.

Changes to manuscript:

1. Figure 3 was expanded to present a general workflow for: 1) Generating a C-Q SOM, and 2) Analyzing a C-Q dataset using the trained SOM. This figure highlights required user inputs, outputs after training, and types of analyses supported:

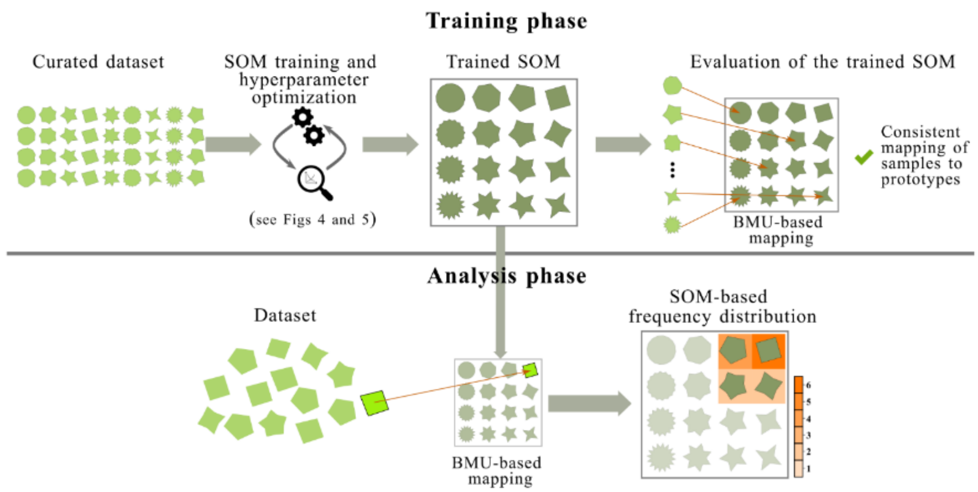


Figure 3. Workflow to generate an SOM for C-Q hysteresis loops (Training phase) and apply the SOM for C-Q hysteresis analyses in watersheds (Analysis phase). Here, we illustrate the generation of the SOM using different shapes, which are analogous to the hysteresis loop types that might be found for a dissolved or particulate constituent in a watershed. In the bottom panel (Analysis phase), we demonstrate how hysteresis loops from a new dataset get mapped to the trained SOM, where the shade of orange represents the frequency with which the shape occurs in the dataset. The HySOM python package (see section 7) allows users to implement this workflow.

2. A new figure (Figure 4) was added to illustrate the SOM training algorithm in detail *(*see figure in our response to comment 2 from Referee 2*).*

3. A new figure (Figure 5) was added to depict the hyperparameter tuning process, including the role of the elbow method and Pareto analysis:

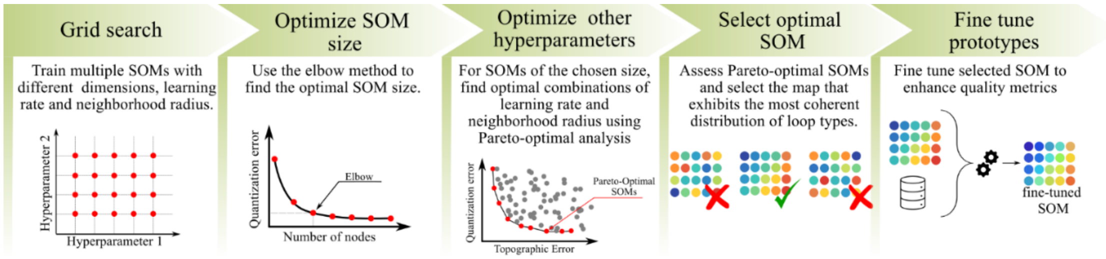


Figure 5. Workflow for fine-tuning SOM hyperparameters using a combination of quantitative metrics and qualitative assessment. The process begins with training multiple SOMs across a grid of map sizes, learning rates, and neighborhood radii. Quantization error is evaluated to identify the optimal map size using the elbow method. A subset of high-quality maps—selected from the Pareto frontier of topographic and quantization errors—is then examined in detail. SOM selection is based on visual inspection, prioritizing maps that exhibit coherent transitions between similar loop types and clear separation between contrasting ones. Finally, retraining of the selected SOM may enhance quantization accuracy.

4. A new section was added to the Supplementary information (S1) to explain the dynamic time warping algorithm and how it compares to Euclidean distance (see revised SI).
5. Section 2 was restructured (see revised manuscript):
   - Original Section 2.5 and Figure 3 were moved to the beginning of Section 2, as recommended.
   - A new subsection (2.3) was introduced to explain how hysteresis loops are represented within the SOM framework, clarifying terms such as “sample” and “n-length sequence.”
   - Text was adjusted to align with the revised and newly added figures.
6. Section 3 was adjusted to align with the edits made to section 2 (see revised manuscript).
7. Section 7 (Code and Data Availability) was expanded to include (see revised manuscript):
   - A description of the HySOM package’s functionality, inputs, and outputs.
   - Clarification that the package supports both sediment transport hysteresis analysis using the General T-Q SOM and training of new SOMs for other constituents.
   - A note that detailed documentation is available in the GitHub repository.

Referee 3 Comment 3: For someone not familiar with SOM, I feel section 2 is written very abstract and is of very limited usefulness. First, I think it is extremely important that you stick to one definition and don't use a different word to describe the same SOM property. Perhaps this is the case, but I am doubtful, e.g. is a prototype and a sample the same (this is unclear in l.108)? Second, it would help if you could link the SOM properties (such as 'prototype', 'BMU', 'samples', 'n-length sequence', 'number of nodes', 'random samples', 'distance function' (distance between what?), 'topological preservation', 'quantization accuracy', 'topographic error', 'radius of influence') to actual properties of the C-Q data analysis. I assume that the SOM algorithm properties you mention must be associated to some kind of 'metrics' that are derived from the C-Q data (such as duration, time steps, magnitude, difference, event and hysteresis properties). In my opinion it would improve the understanding of the methods tremendously if you could highlight such links wherever possible. For instance, would the number of nodes be similar to the 'sensitivity' of defining individual C-Q-events or to the number of different hysteresis classes, or...?

Response to R3C3: We thank the Referee for these suggestions. As described in our previous response, we have reworked section 2 and adjusted section 3 with the goal of better describing the SOM algorithm in the context of hysteresis analysis and clarify key concepts like 'prototype,' 'BMU,' 'samples,' 'n-length sequence,' 'number of nodes,' 'random samples,' 'distance function,' etc. Furthermore, we will ensure that all SOM terminology is placed in the context of hysteresis.

For convenience, we have defined several of these terms in this response. An input *sample* is the data for a (measured) hysteresis loop, represented as a sequence of paired discharge and concentration values. A *prototype* is a hysteresis loop belonging to a node in the trained SOM that represents a cluster of similar samples, derived from the SOM algorithm. In other words, the *prototype* can be used to represent a subset of similar hysteresis loops in a dataset. There is a nuanced difference between *prototype* and *BMU* in that a *prototype* is a characteristic representation of C-Q hysteresis loops, while the *BMU* is the specific node in the SOM whose prototype best matches an individual storm event's hysteresis data. *n-length sequence* is a sequence of n C-Q pairs representing a given hysteresis loop (we recommend resampling the loops such that each event and prototype have a consistent dimension). *Number of nodes* refers to the size of the SOM, i.e., the number of distinct loop types needed to represent the hysteresis dataset. *Random samples* refers to a C-Q hysteresis loop that is randomly selected from the training dataset. *Distance function* refers to the measure of similarity (or dissimilarity) between two hysteresis loops (such as hysteresis data from a watershed and a prototype on an SOM). *Quantization error* measures how closely the hysteresis prototypes approximate the input hysteresis samples. *Topographic error* measures how well the SOM maintains neighborhood relationships from the input space, indicating whether similar prototypes are positioned near each other on the map. Finally, *neighborhood radius* refers to a hyperparameter which controls the transition between hysteresis prototypes. For example, a narrower radius may improve the quantization error of prototypes, but risks *topological fragmentation*, which can result in dissimilar hysteresis loops being placed close together.

Changes to the manuscript:

1. *A new subsection (2.3) was introduced to explain how hysteresis loops are represented within the SOM framework, clarifying the terms, "sample" and "n-length sequence":*

> ***"2.3 Representing hysteresis loops for SOM***
> *In Fig. 3, samples (the light green shapes) and prototypes (the dark green shapes) are depicted as geometric shapes for illustrative purposes. However, the SOM algorithm requires that input data be represented as numeric arrays. In conventional SOM applications, each sample is represented as an n-dimensional vector and each resulting prototype is a vector of the same dimension. For C-Q hysteresis analysis, we propose representing each loop as a sequence of paired discharge and concentration values, forming a matrix with dimensions $\mathrm{n} \times 2$, where n indicates the number of data pairs used to represent the loop. n therefore may be equal to the number of entries in the C-Q time series, however because hydrologic events usually vary in duration, we recommend resampling the data such that hysteresis samples and prototypes each share a consistent dimension. We expand on this more in section 3.*
> *Alternative formats for samples and prototypes are also possible. For instance, Hamshaw et al. (2018) encoded loops as greyscale images with a resolution of 28×28 pixels for use in a*

classification algorithm. However, in our implementation, representing loops as sequences of (Q, C) pairs yielded better results through the SOM algorithm."

2. A new figure (Fig 4) was added to illustrate the SOM training algorithm, and the corresponding description is included in section "*2.5.1 SOM training and hyperparameter optimization*". These additions will clarify concepts like 'prototype', 'BMU', 'random samples' and radius of influence.
3. Section *"2.3 Quality assessment"* was replaced by the new section "*2.5.2 Hyperparameter optimization"* which better fits into the general workflow described in the updated Fig. 3. This clarifies the concepts of topographic and quantization errors and their role in SOM training phase.
4. A new figure (Fig 5) was added to illustrate the workflow for hyperparameter tuning, further clarifying the roles of topographic and quantization errors and pareto analysis.
5. We have added a new section to the Supplementary Information that explains the distance function used in our SOM implementation (*Dynamic Time Warping, DTW*), which quantifies the difference between hysteresis loops and prototypes. This section describes how DTW operates in the context of hysteresis loops and its role in measuring loop similarity.

Specific comments

Referee 3 Comment 4: l. 20: "while preserving the continuum of loop variability lost in classification schemes" this is unclear to me and could be explained in a little more detail here.

Response to R3C4: Thank you for pointing this out. We'll revise the sentence in the abstract to better convey the intended meaning. Further explanation is provided in the Introduction.

Changes to manuscript: The sentence will be revised to: "…*while preserving the gradual transitions between loop types that are often lost in classification schemes*"

Referee 3 Comment 5: l. 138: would it be possible to link/explain C-Q data characteristics with the (some) terms of equation 1 (basically similar to my second major comment above)?

Response to R3C5: Equation 1 represents the SOM learning rule, where each hysteresis loop is denoted as $\boldsymbol{X}$ and the prototypes are denoted by $\boldsymbol{m}_t^i$ ($\boldsymbol{m}^{\eta_k}$ in the revised notation as shown Fig.4). Both $\boldsymbol{X}$ and $\boldsymbol{m}_t^i$ are $n \times 2$ matrices representing a sequence of paired discharge and concentration values. This representation of hysteresis loops is better explained in section 2. Also, the factors in equation (1) are now better explained in the (new) Fig. 4 which details the training process. Additionally, we now provide some more context for how components of SOM can directly be linked to hysteresis.

Changes to manuscript: Figure 4 (newly added) provides more detail on how equation 1 is applied. Furthermore, its accompanying text describes the training process. For example:

> "*Finally, the prototype m^{η_k} of each node η_k is updated using the learning rule (Eq. 1), which adjusts the prototypes toward the input sample. In the context of hysteresis, each prototype hysteresis loop is iteratively refined to resemble C-Q hysteresis loops in the training data. This learning rule ensures that both the BMU and its neighboring nodes are moved closer to the current sample, allowing nearby prototypes to become more similar to the input loop, which preserves the continuity of patterns across the map.*"

Referee 3 Comment 6: l. 146-148 ff: between what exactly are these quantization errors calculated? It means you have to define a 'true' classification manually and map the SOM against this 'subjective' classification that compromizes your aim of "using SOM to discriminate and characterize loop types commonly seen in sediment transport literature"?

Response to R3C6: As defined in section 2.3 (2.5.2 in the revised manuscript) "*[quantization error] is defined as the average distance between each sample and its BMU*". Hence, calculating quantization error doesn't rely on any manual or subjective classification since this is an unsupervised algorithm. We believe the proposed revisions, including the newly added section S1, which clarifies the distance function, and Fig 5 which illustrates the role of topographic and quantization errors, will improve clarity for readers.

Changes to manuscript: In addition to Figure 5 and section S1, we will define quantization and topographic errors as follows in section 2.5.2:

> *"Topographic error quantifies the degree to which the SOM preserves the topological structure of the input space, that is, how well it maintains neighborhood relationships between loop types. A low topographic error indicates that similar loop prototypes are positioned close together, while dissimilar ones are placed farther apart. In the context of hysteresis, this means that loops with similar amplitude, rotational direction and shape should be placed close together. This metric is defined as the fraction of samples for which the Best Matching Unit (BMU) and the second BMU (i.e., the prototype with the second smallest distance to the input vector) are not adjacent on the map (Kiviluoto, 1996; Pölzlbauer, 2004). Lower values are preferred.*
>
> *Quantization error, on the other hand, measures how closely the hysteresis prototypes approximate the input hysteresis samples. It is defined here as the average DTW distance between each sample and its BMU (Pölzlbauer, 2004). Lower quantization errors indicate improved accuracy."*

Referee 3 Comment 7: l. 175-176: similarity between two samples - is one sample Q and the other sample C?

Response to R3C7: Each sample refers to a full hysteresis loop, i.e., a matrix of Q-C data and not Q or C individually. This is now better explained in Section 2:

Changes to manuscript: The following will be included at the start of section 2:

> "*In the context of hysteresis, the input samples to the SOM algorithm are paired sequences of discharge and concentration data, each of them representing a hysteresis event*"

This will be further clarified by adding the following subsection:

> "*2.3 Representing hysteresis loops for SOM*
>
> *In Fig. 3, samples (the light green shapes) and prototypes (the dark green shapes) are depicted as geometric shapes for illustrative purposes. However, the SOM algorithm requires that input data be represented as numeric arrays. In conventional SOM applications, each sample is represented as an n-dimensional vector and each resulting prototype is a vector of the same dimension. For C-Q hysteresis analysis, we propose representing each loop as a sequence of paired discharge and concentration values, forming a matrix with dimensions n×2, where n indicates the number of data*

pairs used to represent the loop. n therefore may be equal to the number of entries in the C-Q time series, however because hydrologic events usually vary in duration, we recommend resampling the data such that hysteresis samples and prototypes each share a consistent dimension. We expand on this more in section 3.

Alternative formats for samples and prototypes are also possible. For instance, Hamshaw et al. (2018) encoded loops as greyscale images with a resolution of 28×28 pixels for use in a classification algorithm. However, in our implementation, representing loops as sequences of (Q, C) pairs yielded better results through the SOM algorithm."

Referee 3 Comment 8: l. 191ff: the difference between the first (training on a curated dataset?) and second (application to 'any dataset'?) phase of the SOM algorithm is not clear to me. Does it mean you need to split time series at one location/gauge into training and application? Or can you train at one location and then apply it to another location? Please explain the requirements and limitations in a bit more detail.

Response to R3C8: The first phase of our workflow (shown in Fig. 3) involves training and generating a SOM for a given constituent. The end goal of this phase is to create a generalizable SOM that can be applied broadly to that constituent. Hence, the dataset used to train the SOM should capture the *full range of loop types* for a given constituent, which enables broader application of the SOM without the need for retraining. This underscores the importance of curating a representative dataset.

In our proof-of-concept, we developed—and released through our Python package—a SOM trained on a curated Turbidity–Discharge dataset. According to the sediment transport literature, this dataset encompasses the most commonly observed hysteresis loop shapes. The resulting map, referred to as the *General T–Q SOM*, is proposed as a standard tool for characterizing primary loop types in sediment hysteresis analysis.

The second phase of our workflow involves using the SOM derived from phase one to classify loop types in new datasets. For example, one might input a dataset consisting of a sequence of T-Q loops, and the algorithm would then identify the Best Matching Unit for each hysteresis loop in the dataset. Therefore, one could use our *General T-Q SOM* to create a frequency distribution for their own watershed.

So—to your question—training of the SOM was based on data from many locations, and because the training dataset includes most of the known hysteresis loop types (barring irregular loops), the *General T-Q SOM* can be used to classify sediment transport loops from any watershed, without retraining. A limitation of this, however, is that rare loop types may not be well captured on the map. In which case, as stated in the discussion, one could either retrain our *General T-Q SOM* or train a new map (via phase 1 of the workflow). We discuss strategies to incorporate "*less-common loop patterns*" in Section 5.2 (Future use of the SOM algorithm for C-Q analyses).

Changes to the manuscript:

The new section 2.2 provides:

A better description of the training phase:

"The Training phase involves constructing an SOM to represent the spectrum of hysteresis loop types associated with a specific dissolved or particulate constituent.
[…]
The output of the first phase is a trained SOM composed of coherently arranged prototypes reflecting the range of loop types in the training dataset. For instance, along with this technical note, we released a trained SOM for sediment transport hysteresis analysis—referred to as the General T–Q SOM—which is publicly available (section 7) for use in sediment hysteresis analysis studies. "

A description of the role of the curated dataset in this training phase:

"To ensure the trained SOM adequately captures the diversity of loop patterns, we recommend curating a training dataset that includes a similar number of representative examples of all known loop types for the constituent under investigation. This curated dataset serves three key purposes: (i) to represent the known hysteresis loops for a given constituent, thereby ensuring broad applicability of the resulting SOM; (ii) to guarantee representation of less frequent patterns; and (iii) to provide a reference for evaluating the SOM's ability to distinguish loops in line with conceptual classifications (last step of the Training phase, Fig. 3)"

And a better description of the analysis phase:

"The Analysis phase consists of using the trained SOM to classify loop types in new datasets, provided they correspond to the same constituent of interest. First, each loop in the dataset is mapped to its most similar loop prototype, known as the Best Matching Unit (BMU), based on a distance function as described in section 2.4. This classification enables systematic analysis of hysteresis patterns across watersheds. For example, by quantifying the number of samples assigned to each prototype, researchers can characterize the frequency distribution of loop types and investigate their associations with hydrological variables, thereby providing valuable insights into the underlying controlling mechanisms, as demonstrated in Section 4."

Finally, we expanded our discussion to highlight alternative methods to the curated dataset for training the SOM:

"While we chose to curate a dataset containing representative sediment hysteresis patterns for training the General T-Q SOM, alternative methods to train hysteresis SOMs are worth exploring. For example, training on large, uncurated datasets can serve as a powerful exploratory tool, leveraging SOM's visualization strengths to uncover dominant or previously unnoticed loop patterns. Note, however, that less frequent patterns may not be properly captured in the trained SOM due to the learning algorithm's limited exposure to rare types during training (Douzas and Bacao, 2017). Further research is encouraged to develop workflows that better identify and incorporate rare loop types with hydrological relevance"

Referee 3 Comment 9: l. l.194-201: 'curating a dataset with all known loop types' - this sounds to me like a major limitation of the method. Does it mean that you first need to analyse C-Q time series for 'old-style' loop types? Then, if C-Q relationships are very homogeneous in a catchment, it might be impossible to have a time series with different loop types - is the method then not applicable? For instance, different watersheds can cause quite different loop types for the same constituents - this limits the transferability of the method?

Response to R3C9: We would like to respectfully disagree with the statement that, "curating a dataset with all known loop types," is a major limitation of the method. The curated dataset serves three key purposes: (i) to represent the known hysteresis loops for a given constituent, thereby ensuring broad applicability of the resulting SOM; (ii) to guarantee representation of less frequent patterns; and (iii) to provide a reference for evaluating the SOM's ability to distinguish loops in line with conceptual classifications. We viewed the third point as especially important for this technical note because SOM has never been applied to water quality hysteresis loops (to our knowledge) and therefore its efficacy for characterizing hysteresis loops needed to be tested. The curated dataset facilitates this process, because we can see broadly what types of loops we included in the training dataset, and thus evaluate how well the *General T-Q SOM* could replicate those.

Our process to curate the dataset with all known loop types is detailed in section 3.1.1. Importantly, the curated dataset consisted of many loop types *across many catchments* to evenly represent the currently known sediment hysteresis loop types. This helped to mitigate potential bias in the training dataset, since for example, we found that infrequent loops weren't well-captured in the SOM when we tried training the SOM for an individual watershed. Since curating the training dataset ensured even representation of less frequent patterns, we argue that our *General T-Q SOM* can be applied to all types of watersheds, including those that might have more homogenous C-Q relationships, as well as those with quite heterogeneous C-Q relationships.

With this being said, we do acknowledge that curating a dataset does have the limitation that rare loop types may not readily be *represented by prototypes on the map*. In a subsequent comment, you mention correctly that another approach for training an SOM for a given constituent would involve compiling a large dataset with a "sufficiently long / high number of time series, which implies that all possible loop types exist therein." This perhaps is one way to overcome limitations related to identification of rare loop types (and we hope that researchers will test this in the future). However, this method, too, has limitations. Specifically, compiling such a dataset would perhaps be an even larger (and longer) undertaking than what we have done in this manuscript. Second – we found that even with the curated dataset, the SOM algorithm occasionally has difficultly with representing loop types such as figure eight loops (this is a point that we mention in the discussion). Thus, if a loop occurs infrequently, or is of an increasingly complex nature, it is possible that the algorithm will have difficulty representing it on the map, which would remain unnoticed with the uncurated dataset. Finally, we should mention that *it is possible* to identify rare loop types via the *General T-Q SOM*. As we mention in the discussion, the quantization error can be used to "flag" loops that deviate highly from the loops represented on the *General T-Q SOM*, thereby serving as means to identify complex and rare loop types.

Regarding your question, "if C-Q relationships are very homogeneous in a catchment, it might be impossible to have a time series with different loop types - is the method then not applicable?" You are correct, though this applies primarily when training an SOM on a single catchment. In that case, the algorithm may only be exposed to a limited range of loop types, reducing its ability to generalize to other catchments. This limitation was one of the key reasons we opted to curate the dataset. That said, there is potential value in comparing individually trained SOMs across catchments, and we encourage future research to explore this direction. We'll make this explicit in the revised discussion.

Changes to manuscript: The following paragraph will be added in the discussion:

> "*Similarly, training an SOM on loops from a single watershed may reveal site-specific dynamics and deviations from the General T–Q SOM, offering valuable insights into local hysteresis behavior. However, such watershed-specific SOMs limit comparability across sites, as their prototypes are tailored to a single watershed. Additionally, if the number of available samples is small, the SOM may not be adequately trained, reducing its ability to capture meaningful structure in the hysteresis data.*"

Referee 3 Comment 10: l. 210: This figure is very (!) useful and I would strongly suggest to: (1) expand it to explain additional properties of the SOM algorithm that you introduced earlier and link it to the actual data properties, (2) refer to the figure earlier in the methods.

Response to R3C10: Thank you. We have accepted your suggestions as explained previously.

Changes to manuscript: As explained earlier, Figure 3 has been moved to the beginning of section 2.

Referee 3 Comment 11: l. 233: compiling the 'curated dataset' through manual delineation looks like a major effort. First, you need to identify relevant events in the time series, then you need to derive loops and classify them... These steps usually require many subjective decisions (when is an event an event, which class types to use, which class is the resulting hysteresis loop in). How did you do this?

Response to R3C11: Regarding your statement, "First, you need to identify relevant events in the time series, then you need to derive loops and classify them." Yes, it is necessary to first identify the relevant events (which concurrently derives the hysteresis loops). This process would be requisite for anyone doing hysteresis analysis.

Classification of hysteresis loops is technically not required by the SOM algorithm, but again, we did this because we wanted to ensure that the known hysteresis loops for a constituent were represented. We should acknowledge that while curation of such a dataset is time-intensive, the process usually is done only once per constituent. Once trained, the SOM can be reused across locations without retraining, and it is also unnecessary to manually classify the loops from new watersheds. We will state this explicitly in the revised section 3.1.1.

Furthermore, we'll expand Section 3.1.1 to clarify our event selection process and why subjective decisions of this workflow are a minor concern for our study.

Changes to manuscript: The following will be included in section 3.1.1:

> *"Additionally, while dataset curation is a time-intensive task, requiring the extraction of multiple hydrologic events from several watersheds, it is typically performed only once for each constituent's SOM. Once trained, the SOM can be readily shared and reused by other researchers without the need for retraining."*

And to clarify the event delineation process:

> "*Event delineation was supported by a custom-built application designed to label hydrologic time series.*
>
> *Watershed selection and event delineation followed the criteria outlined below:*

- *Only rainfall-dominated hydrologic events were included; heavily regulated rivers and snowmelt-dominated watersheds were excluded.*
- *Multi-peak events were split into single-peak events if turbidity receded fully before the onset of the subsequent peak.*
- *Irregular loops (such as complex loops) or loop shapes that did not conform to the typologies illustrated in Fig. 6 were excluded.*
- *We then manually classified the loop shape to one of the typologies illustrated in Fig. 6 according to visual comparison until an even number of event types was identified.*

We should note that while manual classification during data curation can be somewhat subjective, this is a minor concern since the SOM is not constrained to enforce strict boundaries between loop types and does not see the assigned labels. For example, whether a narrow clockwise loop is labeled as a single line or a clockwise Sync-Peak loop may be debatable—but including it in training allows the SOM to learn the gradual transition between these shapes, independent of the assigned label"

Referee 3 Comment 12: l.257: I thought the number of nodes would have to be already defined in the previous step by arranging the manually delineation hysteresis loops into the grid you show in Figure S2? Is Figure S2 an output of your SOM, is it required as an input/during training of SOM? Or is it purely for information purpose?

Response to R3C12: The number of nodes is a hyperparameter that is optimized during training. It is not predefined. We believe that this will be clarified with the addition of Fig. 5. The reference to former Figure S2 in this section will be removed as it follows from the creation of the *General T-Q SOM*. Only the reference to figure S1 will be retained here. Figure S2 will be referenced only in the discussion section as it does leverage the arrangement of prototypes to show the distribution of loops.

Changes to manuscript:

- We added figure 5 to clarify the hyperparameter selection process (including the number of nodes).
- We removed reference to Figure S2 in section 3 to avoid any confusion.

Referee 3 Comment 13: l. 258-259: some additional methods and variables are mentioned here, such as "elbow method" or "number of epochs" - you didn't mention it in 2.2 Training process section.

Response to R3C13: The role of the elbow method in the proposed workflow is now better described with the addition of figure 5 and section "2.5.2 Hyperparameter optimization." *Epoch* is a common concept in machine learning which refers to one complete pass of the entire training dataset through the learning algorithm. We will briefly describe this for readers not familiar with machine learning.

Changes to the manuscript:

- We added figure 5 to describe the role of the elbow method in the hyperparameter tuning process.
- We added the following in section "*2.5.2 Hyperparameter optimization*" to describe the role of the elbow method in the hyperparameter tuning process:
  "The optimal map size is identified by examining the relationship between quantization error and number of nodes. While a decreasing trend in quantization error is expected as the number of nodes

increases, the elbow method (Nainggolan et al., 2019) helps pinpoint the optimal number of nodes for a map."

- We define the concept of *epoch* in section 2:
  "The final hyperparameter is the number of epochs—training iterations where the entire dataset is processed once—defining the length of the training process"

Referee 3 Comment 14: l. 262: to me, this sounds like you calibrate the SOM map to the 'manual' delineation of loop types you conducted during dataset curation. This does not sound like 'unsupervised learning' (see also l. 282) and the 'subjective' classification you critizised in the introduction, is driving the properties of the SOM?

Response to R3C14: SOM training is fully unsupervised as stated in the introduction and section 2. The manual labels are not used at any point by the algorithm during the training process or in the application. We created labels for each of the loops in the curated training dataset, however, this was merely to ensure that we were representing the known types of hysteresis loops in the training data. As mentioned, the curated dataset serves three key purposes: (i) to represent the known hysteresis loops for a given constituent, thereby ensuring broad applicability of the resulting SOM; (ii) to guarantee representation of less frequent patterns; and (iii) to provide a reference for evaluating the SOM's ability to distinguish loops in line with conceptual classifications.

While manual classification during data curation can be somewhat subjective, this is a minor concern since the SOM is not constrained to enforce strict boundaries between loop types and does not see the labels. For example, whether a narrow clockwise loop is labeled as a single line or a clockwise loop may be debatable—but including it in training allows the SOM to learn the gradual transition between these shapes, independent of the assigned label.

Changes to manuscript: We believe the revisions to Section 2, as previously described, will help clarify these points.

Referee 3 Comment 15: l.283-l.287 here you finally mention that the whole manual delineation/classification is only needed to accomplish the curation of the dataset. Why is this needed? Couldn't I simply take a sufficiently long / high number of time series which implies that all possible loop types exist therein? - ok later you mention that you don't want an uneven distribution of loop classes - but isn't forcing a similar distribution introducing a bias? What would happen if you don't use this dataset curation (perhaps this can be elaborated on in the discussion).

Response to R3C15: As stated before, the curated dataset serves three key purposes: (i) to represent the known hysteresis loops for a given constituent, thereby ensuring broad applicability of the resulting SOM; (ii) to guarantee representation of less frequent patterns; and (iii) to provide a reference for evaluating the SOM's ability to distinguish loops in line with conceptual classifications.

Training an SOM on a large, uncurated dataset is certainly possible, and may be a good future use of SOM. We state this explicitly in the revised discussion. However, for the reasons outlined above, we found dataset curation to be the most suitable approach for our study.

We should also mention that a balanced training dataset doesn't impose a distribution on the resulting SOM. While it imposes a (uniform) distribution on the training dataset, this is done to ensure that both common and uncommon loop types are represented during training. However, the actual frequency of loop types in any watershed is revealed later when mapping new data to the trained SOM.

Changes to manuscript:

The justification to employ a curated dataset is now more clearly stated in section 2:

> *"This curated dataset serves three key purposes: (i) to represent the known hysteresis loops for a given constituent, thereby ensuring broad applicability of the resulting SOM; (ii) to guarantee representation of less frequent patterns; and (iii) to provide a reference for evaluating the SOM's ability to distinguish loops in line with conceptual classifications (last step of the Training phase, Fig. 3). More details on the training and evaluation procedures are provided in section 2.5."*

Alternatives to using a curated dataset are included in the discussion:

> *"While we chose to curate a dataset containing representative sediment hysteresis patterns for training the General T-Q SOM, alternative methods to train hysteresis SOMs are worth exploring. For example, training on large, uncurated datasets can serve as a powerful exploratory tool, leveraging SOM's visualization strengths to uncover dominant or previously unnoticed loop patterns. Note, however, that less frequent patterns may not be properly captured in the trained SOM due to the learning algorithm's limited exposure to rare types during training (Douzas and Bacao, 2017). Further research is encouraged to develop workflows that better identify and incorporate rare loop types with hydrological relevance"*

Referee 3 Comment 16: l. 295 suggest to add data sources of the catchment characteristics and how these were derived/extracted.

Response to R3C16: Accepted

Changes to manuscript:

We will add the following information in section 3.2:

> *"Watershed boundaries were obtained from the USGS StreamStats online application, watershed slope was calculated using the USGS 3DEP (10m) digital elevation model, urban area was extracted from the 2021 National Land Cover Database and soil texture was extracted from the Probabilistic Remapping of SSURGO (POLARIS) database (Chaney et al., 2019)."*

Referee 3 Comment 17: l. 300-307: The different methods for the three watersheds are confusing. Why additional variables only for 03289000 and not for the others? Why Zuecco-indices for the two others and not for 03289000? Suggest to explain how 'old-water to event-water' was calculated.

Response to R3C17: We will clarify in Section 3.2 that the three watersheds were selected to illustrate two distinct types of analysis: (1) loop type frequency (gages 07364130 and 03254480), and (2) associations with hydrologic variables (gage 03289000).

The additional variables for 03289000 come from a previous study (cited in the manuscript), and we chose not to detail their derivation here since it's already covered there and not central to this paper's focus. We also excluded hysteresis indices for 03289000 as they didn't add value to the analysis.

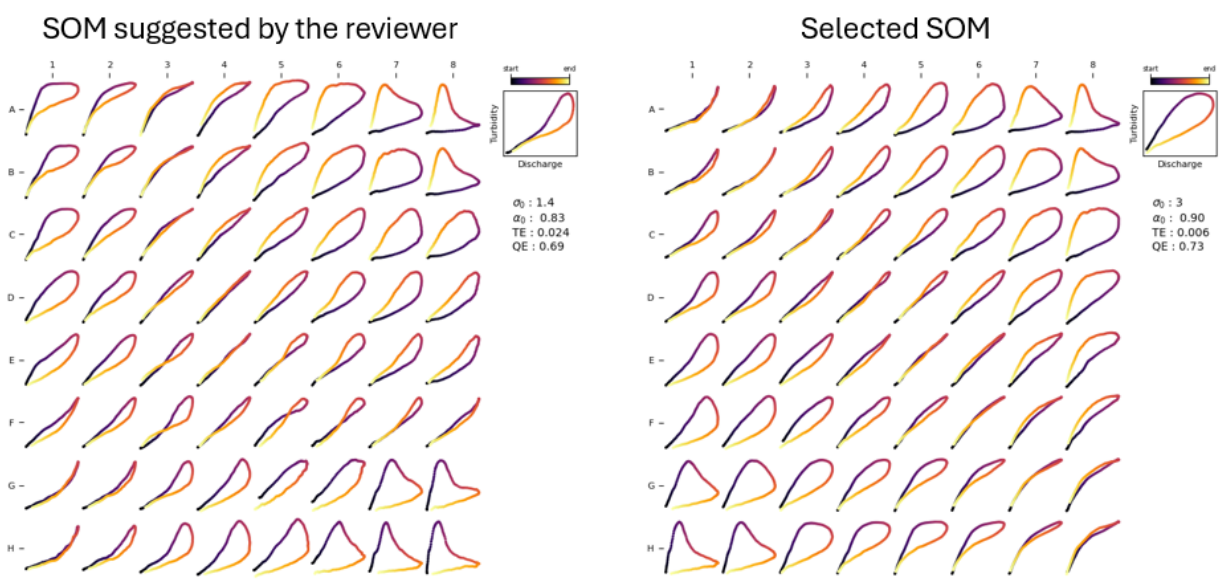
Changes to manuscript: The following was added in section 3.2:

> "*These watersheds were selected to illustrate two primary analyses for characterizing hysteresis patterns: (1) exploring the frequency distribution of loop types within a watershed (gages 07364130 and 03254480), and (2) identifying associations between loop types and hydrologic variables (gage 03289000).*"

Referee 3 Comment 18: l. 329-333: why not selecting 0.02/0.7 which would seem to have a lower euclidean distance error than the one you chose.

Response to R3C18: The 0.02/0.7 SOM was indeed one of the Pareto-optimal maps we reviewed (see figure below). As explained in Section 3.1.2, we selected the final SOM based on visual evaluation, favoring smoother transitions and clearer separation between contrasting loop types. While both maps performed similarly in terms of QE and TE (both were located along the pareto frontier), the chosen SOM offered a more intuitive layout for understanding loop variability.

Note for example how the transition from clockwise to counterclockwise loops follows a clear diagonal (lower-left to upper-right), while in the discarded SOM this transition is more scattered and harder to interpret. Both maps capture similar loop types, but we prioritized the one that offers a more intuitive layout for understanding loop variability.



Changes to the manuscript: We will add this figure and a short explanation to the SI (Section S3) so readers can better grasp the visual evaluation of Pareto-optimal SOMs.

Referee 3 Comment 19: l. 340ff: Given my difficulties with the methods, I am also confused by chapters 4.1.2 and 4.1.3. Chapter 4.1.2 is based on the curated dataset (=manual classification as far as I understand your earlier explanation) - but it also shows a 'trained SOM' (l.341). Then in 4.1.3, (l.370) you mention that "manual classification was not seen by the model as part of the training process" - This is confusing.

*Response to R3C19***:** The General T–Q SOM was trained using the curated dataset, but as an unsupervised algorithm, it never sees or uses the manual labels during training. The curated dataset simply ensures a balanced representation of loop types and allows us to evaluate how well the SOM distinguishes between them. We believe that section 4 will be much easier to understand with the proposed figures and revisions to section 2.

Changes to the manuscript: As described in previous responses, we have reworked section 2 and adjusted section 3 to better explain methods. We believe that those changes will make the results much clearer.

Minor comments

Referee 3 Comment 20: l. 74-79 the information given in the caption partly duplicates information given in the main text. Suggest to streamline this.

Response to R3C20: Thank you for the suggestion. We prefer self-explanatory figures for readers who may not refer to the main text.

Changes to manuscript: NA

Referee 3 Comment 21: l. 228: suggest to write "figure eight" to avoid confusion with Figure 8.

Response to R3C21: Accepted.

Changes to manuscript: We now use the term *figure-eight.*

Referee 3 Comment 22: l. 313: something is missing here "a two dimensional..." array? matrix?

Response to R3C22: The right word is *index.*

Changes to manuscript: For clarity, this sentence now reads:

> "To quantify these associations, we applied a correlation-based approach using multiple linear regression. The BMU coordinates of each loop served as predictor variables and the response variable was a rank-normalized hydrologic metric"

Referee 3 Comment 23: l.400: reference and explain a, b, c and d in the caption.

Response to R3C23: Accepted.

Changes to manuscript: The new caption will reads:

> *"Figure 10. Mapping of all samples from the curated loop dataset to their respective BMUs on the trained SOM. For visual clarity, loop types and their mappings are organized into four distinct panels: (a) single-line loops; (b) clockwise loops including Async, Sync, Concave, and Figure-L; (c) counterclockwise loops including Async, Sync, Concave, and Figure-L; and (d) Figure-eight and Line+loops. Acronyms used in the figure include: SP – Sync-Peak, AP – Async-Peak, CU – Concave-Up, CD – Concave-Down, ccw – Counterclockwise, cw – Clockwise, fL – Figure-L, f8 – Figure-eight."*

Referee 3 Comment 24: l.432: suggest not to name this technical note a "report"

Response to R3C24: Accepted.

Changes to manuscript: the term "technical note" is now used.

Referee 3 Comment 25: l.475: the three arrows are hard to distinguish. It might make sense plotting them in three different colors and since they seem to overlap with a transparency value

Response to R3C25: Thanks for the suggestion. The arrows don't overlap. Antecedent discharge (Q_{5D}) has a near-zero coefficient, so its arrow is simply too short to be visible.

Changes to manuscript: We'll make this clear in the text:

> "*In contrast, antecedent discharge shows negligible association, reflected in a near-zero coefficient and short arrow*."