Referee report for "On defining climate by means of an ensemble" by G. Drótos and T. Bódai

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1 Summary

The main aim of this paper is to provide a definition of climate, as highlighted by the title and the abstract. Accordingly, this will be the central concept on which this feedback will focus.

In Section 2, the authors begin by considering an intermediate-complexity climate model as an illustrative example. Then, climate is introduced as the probability distribution obtained from the time evolution of ensemble trajectories. The idea is that this ensemble approximates what the authors call the "natural probability density," which is the abstract target of their definition.

In Section 3, the authors point out that two aspects must be taken into account: (i) the time evolution of the ensemble should be long enough to allow convergence to occur, and (ii) the previous point implies that, when the study is performed on a time interval much shorter than the one needed for convergence, one must take care of the slow variables. If the separation between the slow and fast time scales is large, then the definition of climate still holds, provided that it is considered as a conditional definition, i.e. depending on the fixed values assumed by the slow variables.

Furthermore, the authors identify two main problems that can undermine the definition of climate if either of the following conditions is not satisfied: (a) there is a large separation between the slow and fast time scales, and (b) the slow variables are initialized far away from a regime transition.

In Section 4, the authors review how the limitations to the definition of climate presented in Section 3 appear in real examples.

Finally, in Section 5.1, the definition of climate is recalled, and an initialization scheme is proposed to decide whether the uniqueness of the probability density holds and whether the slow variables affect the targeted probability density.

2 Evaluation

In my view, the paper does not introduce a sufficiently new or significant contribution to the community. For example, consider the closing definition of climate proposed by the authors (lines 358–372): "an operational definition of climate [...] might rely on a decadal-scale convergence of an ensemble [...] to a (practically) unique but time-dependent probability density: this density could be identified with climate." My question is: what is the novelty here? While I appreciate that the authors emphasize the difficulties that can arise if conditions (a) or (b) are not satisfied, they do not seem to offer a way of overcoming these issues; instead, they incorporate (a) and (b) as assumptions in their definition of climate.

To phrase my concern in a different framework: assume that the intermediate-complexity model presented by the authors is given by a non-autonomous stochastic differential equation

$$dX_t = b(X_t, t)\, dt + d\eta_t,$$

where $(X_t)_t$ is a stochastic process in $\mathbb{R}^d$, and $(\eta_t)_t$ models the noise. The authors appear to be saying—if I have understood correctly—that if the time-varying family of invariant measures $(\mu_t)_t$[1] of the SDE is unique, then that is "the climate." My point is that this is already a well-established concept in the literature. I would therefore encourage the authors to state more explicitly what the novel contribution of their paper is.

In addition, echoing a point already raised by another referee, I would suggest that the manuscript be revised for greater precision in the formulation of its arguments and statements. I acknowledge that this comment may be influenced by my own mathematical background, but I believe that improving the clarity of exposition would benefit a broad readership.

3 Typos

The paper is essentially free of typographical errors. I list below the few that I noticed:

- Line 12: replace "in" with "if".
- Line 32: should read "characteristic of".
- Lines 499–501: the sentence "The conditional definition motivated [...] in this situation" has a singular subject ("conditional definition") but a plural verb ("become").

[1] I refer, for instance, to [DSF24, Section II.E] for the rigorous definition of the family $(\mu_t)_t$.

References

[DSF24] G. Del Sarto and F. Flandoli. A non-autonomous framework for climate change and extreme weather events increase in a stochastic energy balance model. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 34(9):093122, 09 2024.