

--- Beginning of message ---

Dear Francisco,

Thank you for the kind and careful assessment of our work, for collecting the important points for revision and for leaving flexibility in the style of presentation.

While introducing new subsections would have been beneficial, we could not identify natural break points until now. In terms of topics discussed and their organisation, we believe that they align with the goal of our manuscript; we also hope that this goal and its implications have become even clearer as a result of our present revision.

We hereby reflect on your point 3 and provide responses to the reviewers below.

> 3. Discussion of Lovejoy (2014) notions of micro/macro climate as a "data-driven" approach for defining climate, as suggested by reviewer #3;

While discussing many different notions of climate unrelated to the ensemble approach does not fit into our manuscript, we acknowledge that we had not given sufficient credit to the importance of the issue, namely scaling behaviour in time series, raised and discussed by works of Lovejoy. We prefer to cite his review paper from 2023 in this context, as actually suggested by reviewer #3, and we now more carefully discuss this issue along with its potential implications and treatments in conjunction with citing Franzke et al. (2020).

All changes to the text are highlighted in the PDF document provided for this purpose. A minor change in the second paragraph of Section 2 and an explanatory addition in Appendix D are results of remarks made by B. Kaszás who was kind to read through the manuscript today. Further improvements to the formulation have also been introduced.

We hope that we have finally reached a satisfactory version and are grateful to you and the reviewers for insisting on a number of important points.

With best regards,
Gábor Drótos
on behalf of both authors

Response to Report #1

We thank Michael Ghil for pointing out that he found interesting thoughts in the article. It is now classified for consideration as a perspective piece, and we agree that it is a more appropriate classification.

Response to the Comment on highlight status by Referee #1

We believe that the article is rather complicated than "confused & confusing", and this is so because the issue at hand is complicated itself. We did try our best and hope that the latest additional clarification of the goal of our article makes both the main conclusions and the side remarks easier to comprehend; this is also facilitated by a minor explanatory addition in Appendix D. Unfortunately, there are many essential aspects and many possible pitfalls in the story, each of different nature, and we cannot omit their individual assessments. We prefer to point this out irrespective of a possible highlight status.

Response to Report #2

We thank Gianmarco Del Sarto for formulating his opinion that "the rest of the paper very interesting and potentially valuable" and pointing out that he "found the discussion of

the snapshot/pullback viewpoint, convergence time scales, and ensemble initialization particularly valuable". We have further clarified the goal of our article in Section 1 (see the highlighted text in the track-changes file), and explicitly describe corresponding novelty with respect to Del Sarto and Flandoli (2024) in Footnote 2.

In our perception, some differences in language use may have led to the confusion of the Referee. On the one hand, we did not claim that we had invented a new mathematical tool for describing or defining climate; our approach is not new in this sense, we just point out that already existing mathematical tools are useful. On the other hand, while we acknowledge that both the pullback ingredient and the time scale separation ingredient have been present in the literature (in fact, even both together in Del Sarto and Flandoli, 2024), the discussion of time scales never took, to our knowledge, an *ab initio* point of view; in particular, it never reached down to equations of motion that can be regarded as fundamental ones in describing the dynamics of the Earth system. As we hope to explain now clearly in the manuscript, we are aiming to fill this particular but, in our view, important gap. As a side remark, we find this gap especially important because of the way the Earth system is modeled in cutting-edge research that actually provides realistic model output in terms of a wide range of performance metrics.

We hope that our modifications to the manuscript now settle the novelty issue raised by the Reviewer.

We also thank the Reviewer for drawing attention to typos and linguistic issues. The only point where we kept our original version is the subsection title on page 9: we do not find the suggestions of the Reviewer grammatically correct, at variance with our original version.

As for the minor remark about page 9, lines 2-3, indeed, we are just referring to the fact that "not what happens shortly after initialization" is looked for about climate. In particular, the first few decades after initialization are too short, the evolution of an ensemble in this time frame does not provide an objective characterisation of all relevant "weather outcomes"; this is the guiding principle of our article.

Response to Report #3

We thank the anonymous Reviewer for formulating their really kind and encouraging assessment. We first try to clarify a few points below, then we comment on the last major paragraph of the Report which we find very important and which has led us in the manuscript to a more appropriate discussion of possible pitfalls of the approach we are presenting.

> What I think is relevant is to focus on the notions of pullback attractors and the importance of time scales of the forcing.

Time scales of forcing, in fact, do not directly enter into our discussion. While they may have a determining role in time scales of convergence which we are concerned about, our approach and proposal is practically "blind" to them. Also, the tale could have been told for an autonomous dynamical system as well --- explicit time dependence is a further complication in the context of climate. We hope that the new formulation of the goal of the manuscript, in the introductory section, is clearer in these aspects.

> Introducing the question of initialization of climate models is one example which should be removed.

This part is essential because ensemble simulations are the only tools for applying our approach in practice, and because our scheme illustrates that there is a relatively simple and robust way for implementing this application.

> To my opinion, it is useful to bring back the history in the main text [...] I would also suggest to go back to the beginning of the 20th century

These are not essential for our claims and would make the manuscript even longer and, supposedly, more difficult to read.

> try to avoid diverting toward other aspects of biases, drifts, the use of Plasim...

These aspects may have a decisive role about whether or how our approach can be applied or the corresponding results interpreted. The use of PlaSim is now restricted to an appendix where we believe it is practical.

Now we turn to the last major paragraph of the Report.

As we explain to the Editor above, while we find the notions of micro-climate and macro-climate not directly relevant to our manuscript, we acknowledge that we had not addressed the issue of scaling appropriately in the previous round of review. It is much more important than our earlier manuscript version may have suggested, and, along with citing Lovejoy (2023), we now present a more careful discussion and set out necessary future work in the direction. We now also mention this issue in our concluding section.

Regarding Vannitsem et al. (2021), we have now recognised that we had not paid sufficient attention to the issue of multiple basins of attractions either. We have now corrected this along with citing Vannitsem et al. (2021).

Please refer to the highlighted text in Sections 3 and 4.1 in the track-changes file to identify our modifications. We hope to not having overlooked further important points by now, so that our manuscript finally reaches an acceptable and hopefully useful content.

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