Overview

This supplement provides the model fitting results for the Auckland Volcanic Field (AVF) with and without the most recent eruption at Rangitoto, (S1), the derivations for each of the non-trivial normalisation factors, Z (S2), and the AVF vent locations and volumes (S3).

S1 Model goodness of fit for the Auckland Volcanic Field

Table S1: Model loglikelihoods, Akaike Information Criterion (AIC), and fitted parameters with and [without] Rangitoto (most recent AVF eruption)

Spatial Intensity Model	Log-Likelihood	AIC	Fitted Parameters
Isotropic Gaussian (Eq. 1 & 2)	-322.4 [-311.3]	646.8 [624.6]	h = 2.367 km [h = 1.829 km]
Disk-based depletion (Eq. 5 & 6)	-318.6 [-310.4]	643.2 [626.8]	h = 1.538 km, α = 6.683, l = 0.0053 km [h = 1.710 km, α = 14.246, l = 0.156 km]
Inverse-volume weighted (Eq. 7 & 8)	-321.4 [-311.0]	646.8 [626]	h = 2.395 km, α = 0.248 [h = 1.846 km, α = 0.127]
Distance-multiplied (Eq. 9 & 10)	-324.0 [-312.9]	652 [629.8]	h = 11.65 km, α = 0.456 [h = 2.928 km, α = 0.158]

S2 Derivations of the normalisation factor, Z

Disk-based depletion

$$\boldsymbol{K = \frac{1}{Zn2\pi h^2} exp\left(\frac{-r^2}{2h^2}\right) \mathbf{exp}\,(-\alpha I|r < R|}$$

where Z is a constant to be determined. Since the kernel term must integrate to $1/n$

$$Z = \int_0^{2\pi}\int_0^{\infty} rKdrd\theta = \frac{1}{h^2}\left(e^{-\alpha}\int_0^R r\ exp\left(-\frac{r^2}{2h^2}\right)dr + \int_R^{\infty} r\ exp\left(-\frac{r^2}{2h^2}\right)dr\right)$$

Integrating directly

$$Z = \frac{1}{h^2}\left(e^{-\alpha}\left[-h^2\mathrm{exp}\ \left(-\frac{r^2}{2h^2}\right)\right]_0^R + \left[-h^2\mathrm{exp}\ \left(-\frac{r^2}{2h^2}\right)\right]_R^{\infty}\right) = e^{-\alpha}\left(1 - exp\left(-\frac{R^2}{2h^2}\right)\right) + exp\left(-\frac{R^2}{2h^2}\right).$$

Distance-weighted kernel

$$K = \frac{r}{Zn2\pi h^2} exp\left(\frac{-r^2}{2h^2 v^{2\alpha}}\right)$$

where Z is a constant to be determined. Since the kernel term must integrate to $1/n$

$$Z = \int_0^{2\pi}\int_0^{\infty} rKdrd\theta = \frac{2\pi v^{2\alpha}}{2\pi X}\int_0^{\infty} r^2 exp\left(-\frac{r^2}{2X}\right)dr$$

Where $X = (hv^{\alpha})^2$. Integrating by parts

$$\int_0^{\infty} r^2 exp\left(-\frac{r^2}{2X}\right)dr = \left[-rX\ exp\left(-\frac{r^2}{2X}\right)\right]_0^{\infty} + \int_0^{\infty} X\ exp\left(-\frac{r^2}{2X}\right)dr$$

The first term is zero by l'Hôpital's rule. The second term is

$$\int_0^{\infty} X\ exp\left(-\frac{r^2}{2X}\right)dr = X\sqrt{2\pi X}\int_0^{\infty}\frac{1}{\sqrt{2\pi X}} exp\left(-\frac{r^2}{2X}\right)dr$$

And recognizing the last integral as half of a Gaussian pdf, we arrive at

$$Z = \frac{v^{2\alpha}}{X} X\sqrt{\frac{\pi X}{2}} = \frac{\sqrt{\pi} h v^{3\alpha}}{\sqrt{2}}\ .$$

S3 Auckland Volcanic Field data

Vent ID	X (km)	Y (km)	Magmatic volume (10^6 m^3)	Tephra volume (10^6 m^3)
1	9.9	20.7	11.37	2.1145
2	9.2	22.1	27.762	0.4326
3	18.1	22.1	2.197	0.6163
4	7.9	26.6	2.623	0.7685
5	8.2	27.3	5.873	2.0637
6	9.3	29.5	46.664	10.0554
7	12.4	24.7	0	0.4419
8	12.2	24.5	2.223	3.8738
9	13.3	24.4	2.611	0.6143
10	5.1	17.6	19.917	4.7157
11	6.5	15.2	13.058	2.068
12	16.4	15.8	7.145	2.7727
13	16.1	13.1	18.13	5.8981
14	15.4	12.7	2.638	5.1406
15	15.6	11.1	27.665	0.6015
16	17.7	11.5	2.478	0.8654
17	9.4	8	10.989	1.1463
18	7.1	6.2	9.681	1.3117
19	8.1	7.4	32.761	3.5062
20	7.9	7.1	3.263	0.6554
21	15.8	4	5.307	1.4853
22	17.7	5.1	7.154	0.9011
23	15.5	7.7	0.076	0.0262
24	14.2	7	7.238	2.5505
25	16.9	4.6	23.714	4.0868
26	7.2	9.5	15.492	1.338
27	21.1	17.4	15.397	4.7226
28	18.4	20.2	3.038	1.0795
29	12.7	7.4	4.891	0.5084
30	13.1	14.4	9.188	3.5514

31	20.8	12.4	11.076	3.5172
32	20.7	11.9	0.366	0.1229
33	20.5	11.2	10.616	2.4323
34	20.1	10.5	1.602	1.1016
35	10.1	10.2	2.012	0.6509
36	10.5	11	2.028	0.3736
37	10.4	16.4	31.2	22.5083
38	8.2	16	254.736	8.5558
39	10.6	13.3	66.308	4.506
40	9.2	18.5	0.856	0.1542
41	9	19	28.067	0.12
42	10.3	18.4	83.833	8.9165
43	11	18.8	5.475	1.7997
44	12.9	19.1	1.207	0.749
45	13.3	20.1	6.703	1.8873
46	16.4	17.6	1.659	0.144
47	16.2	17.3	79.264	5.4914
48	20.5	24	3.243	2.2964
49	17.4	28.9	657.299	64.7299
50	13.2	10.2	0.322	0.092
51	15.6	6.2	0.242	0.0688