

## Referee Comment

- *Also, it is good to be aware of what the time of collapse is really telling you about the model formulation, and to look at other informative aspects of the solution. For example, you mention apparent 'dissipation' of the solution. You could measure that dissipation quantitatively by diagnosing a conserved quantity like energy or potential enstrophy, for example, and look at how their conservation depends on resolution and order of accuracy.*

## Response to Referee:

Your suggestion is very well-founded. Therefore, we have incorporated a new figure into the manuscript depicting the time series of total energy, total potential enstrophy and total zonal angular momentum. The following addition has been made to the manuscript:

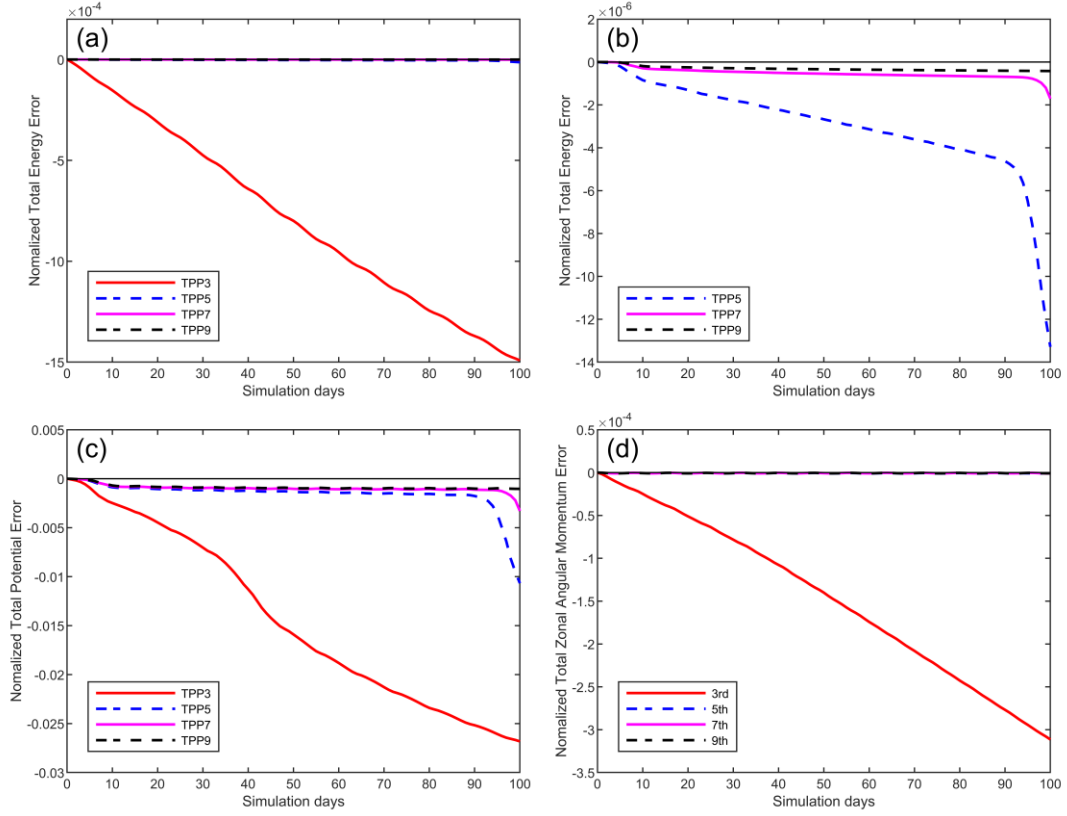
We measure the conservation errors by defining the normalized error  $\epsilon_r$  of the variable  $\eta$  as  $\epsilon_r = \frac{I_g(\eta^n) - I_g(\eta^0)}{I_g(\eta^0)}$ , where  $\eta^0$  and  $\eta^n$  stand for  $\eta$  value at initial time and time slot  $n$ , respectively.

The global integral is defined as:

$$I_g(\eta^n) = \sum_{p=1}^{n_p} \sum_{j=1}^{n_y} \sum_{i=1}^{n_x} \sqrt{G_{i,j,p}} \bar{\eta}_{i,j,p}$$

where  $\bar{\eta}_{i,j,p}$  represents the average value of  $\eta$  in cell  $(i, j, p)$

A 100-day simulation of the Rossby-Haurwitz wave was conducted using a C90 grid ( $1^\circ$  resolution). The total energy simulated with the TPP3, TPP5, TPP7, and TPP9 schemes underwent dissipation to varying degrees. By day 100, the normalized total energy errors reached  $-1.49 \times 10^{-3}$ ,  $-1.33 \times 10^{-5}$ ,  $-1.71 \times 10^{-6}$ ,  $-4.20 \times 10^{-7}$ , respectively, indicating significantly stronger dissipation for the TPP3 scheme compared to the other higher-order schemes Figure 1 (a). Figure 1 (b) presents a scaled view of the energy evolution for TPP5, TPP7, and TPP9, clearly demonstrating that increasing the reconstruction order progressively reduces energy dissipation. Furthermore, following the RH wave collapse, a significant drop in total energy was observed for the TPP5 scheme (after approximately 90 days) and the TPP7 scheme (after approximately 95 days).



**Figure 1** Time series of normalized conservation errors for the Rossby-Haurwitz wave simulation on the C90 grid over days 0 to 100, with LMARS scheme as Riemann solver. (a) Normalized total energy error for TPP3, TPP5, TPP7 and TPP9. (b) The total energy normalized error for TPP5, TPP7 and TPP9. (c) Normalized potential enstrophy error for TPP3, TPP5, TPP7 and TPP9. (d) Normalized total zonal angular momentum error for TPP3, TPP5, TPP7 and TPP9.

Analysis of the normalized total potential enstrophy error (Figure 1 (c)) and the normalized zonal angular momentum error (Figure 1 (d)) over time yields conclusions consistent with those for total energy. Specifically, the TPP3 scheme exhibited substantially higher dissipation than the higher-order schemes, confirming that employing higher-order reconstruction schemes effectively minimizes dissipation. Notably, significant dissipation surges occurred in these quantities following the RH wave collapse.