Referee Report

The manuscript develops a stylised stochastic framework for thermokarst–lake dynamics at landscape scale. Formation and abrupt drainage are represented as Poisson processes with intensities scaled by regional area fractions (two variants). Individual lake areas evolve via Geometric Brownian Motion (GBM). An area-fraction cap can limit further growth. Parameters are inferred from annual lake-area time series; three qualitative regimes (complete drainage, oscillation, quasi-stabilisation) are demonstrated. The modelling separation between birth/death (Poisson) and multiplicative size dynamics is mathematically attractive and invites analysis and principled calibration.

Major comments

(1) There may be an issue with the GBM discretisation and Brownian scaling. Please implement the standard GBM increment for time step Δt:

$$\log \frac{a_i(t)}{a_i(t-\Delta t)} = (\mu - \tfrac{1}{2}\sigma^2)\,\Delta t + \sigma\sqrt{\Delta t}\,Z, \quad Z \sim \mathcal{N}(0,1).$$

The text describes sampling the Normal with standard deviation equal to dt (the time step). This should be $\sqrt{dt}$. If steps remain annual, state $\Delta t = 1$ year explicitly and correct the noise scaling.

(2) Linear area-fraction scaling implies constant hazards per unit area and no explicit dependence on lake size, clustering, or covariates. A modest generalisation with state-dependent intensities $\lambda_f(\cdot), \lambda_d(\cdot)$ (e.g. log-link GLM or Cox hazards) would capture simple nonlinear feedbacks while remaining identifiable.

(3) Specify the boundary condition at zero area (absorbing vs. truncation) and whether very small areas are killed. This choice affects the conceptual split between gradual (GBM) and abrupt (Poisson) drainage under annual sampling.

(4) Beyond merging, lakes evolve independently. Introducing weak correlation (shared random environment / spatial frailties) or lightly correlated GBM shocks could capture hydrologic connectivity without materially increasing complexity.

(5) The current merging rule can produce implausibly large single lakes and is computationally heavy. A stochastic, geometry-consistent alternative (continuum percolation / Boolean union-of-sets with polydisperse footprints), optionally with post-merge fission, would improve realism and cluster statistics.

(6) The birth–growth–death structure invites analysis:

- a size-structured Fokker–Planck (McKendrick–von Foerster + diffusion) for the area density $p(a,t)$ with integral birth/death terms;
- conditions for stationarity or self-similarity (lognormal-type tails) and closed-form moment dynamics;

- expectation dynamics for the total water fraction under an A_{lim} ceiling.

Even partial moment equations would substantively strengthen the mathematical core.

(7) With annual data and gaps, the drift μ is weakly identified relative to the volatility σ. Consider a composite-likelihood or state-space formulation with uncertainty bands; Bayesian pooling (across neighbouring cells) can stabilise μ and the hazards.

(8) Simple contemporaneous or one-year-lag correlations have low power and can miss nonlinearity. Try distributed-lag specifications, information-criteria-based lag selection, partial correlations (conditioning on antecedent water fraction), or spline thresholds before concluding there is no relationship.

(9) State the units of λ_f, λ_d (e.g. events per year per area) and specify how rates scale under aggregation/disaggregation so ESM tiles can ingest them consistently.

(10) Abrupt drainage = (near) complete loss within a year via the Poisson process; gradual drainage = negative GBM drift. With annual sampling, large partial losses (e.g. 60–90%) can be ambiguous; a competing-risks view with size-dependent abrupt-drainage hazard would help.

(11) Provide a short Δt sensitivity (e.g. semiannual with rescaled noise) to test robustness of annual stepping.

(12) Complement the qualitative regimes with simple statistics (cluster count, Gini coefficient of lake areas, distribution quantiles) that distinguish regimes numerically.

Minor comments

(13) Bring the Variant 1/2 definitions forward and summarise implications in a small table. Clarify whether A_{lim} is a hard cap (projection) or a soft ceiling (e.g. logistic drift modulation).

(14) Clearly state the initial area assigned to newly formed lakes.

(15) Collect symbols/units in one table (including whether μ, σ are per-year) to aid reproducibility.

Overall recommendation

The framework is promising for ESM-facing parameterisation. I recommend: (i) correcting the GBM noise scaling, (ii) formalising state-dependent event rates, (iii) adding basic analytical results (moments / Fokker–Planck), and (iv) replacing or augmenting the merging rule with a stochastic, geometrically consistent alternative. These steps would materially strengthen the rigour of the developed approach.