Evaluation of Extreme Sea-Levels and Flood Return Period using Tidal Day Maxima at Coastal Locations in the United Kingdom

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Abstract. Tidal storm surges can result in significant inundation and damage if sea defences are insufficiently robust. Coastal planners need to know the risk of flooding so that sea defences and coastal developments can be specified and located appropriately. Since the original work on extreme value statistics by Gumbel & Lieblein (1954), several alternatives have been proposed for evaluating the risk of tidal inundation, with the Skew Surge Joint Probability Method (SSJPM) gaining popularity. However, SSJPM is complex and cannot always be applied generally. Guided by the search for a general method having wide application and amenable to automation, this paper re-examines the original approach of Gumbel & Lieblein and proposes a simple modification for combined peak selection and declustering; it is termed here TMAX, since it selects one maximum per tide. In comparison with the Gumbel & Lieblein (1954) method using annual maxima (later termed AMAX), the TMAX method offers more efficient use of extreme data events, and in addition simpler handling of missing data. The results of the TMAX method are compared with those of a recent UK study using the SSJPM method at the same 39 United Kingdom coastal locations. The broadly applicable TMAX method has the potential to offer more widespread calculation of flood return period, thereby improving strategies regarding coastal management and resilience.

1. Introduction

Coastal planners and developers require accurate estimates of coastal flood risk to design and site sea defences, coastal buildings, harbours, nuclear power stations, and other critical infrastructure. Coastal floods generally occur when three factors converge; a significant storm surge, a spring tide and the time of high tide occurs at or near the peak of the storm surge. It is predicted the frequency of such events will double by 2050 due to rising sea levels (Vitousek et al., 2017). This challenge is especially critical for low-lying coastal nations, including parts of the United Kingdom (UK), (Williams et al., 2016).

The required height of sea defences can vary quite rapidly along the coastline. This is because the coastal seabed topography can locally magnify or diminish the effects of tidal and surge effects. Tide gauge data provides a statistical snapshot relevant to its location; whether it is sited in an exposed coastal location or in a sheltered harbour, the recorded information can serve as a vital local source for estimating flood probability and return period. Tide gauge data is traditionally used for tidal harmonic analysis and for the establishment of tidal levels such as the Highest Astronomical Tide

(HAT), see Doodson & Warburg 1941); more recently it finds use in the determination of boundary conditions for tidal modelling and in their numerical validation (Zhang et al., 2003). However, extreme sea-level analysis can leverage the same tide-gauge data to predict the flood return probability. It is important here to distinguish between HAT and extreme flood height. HAT assumes average conditions for the meteorological component, whereas the conditions leading to extreme flooding include the effects of the meteorological component. Throughout this paper, the term tide refers to the total fluctuating sea level, while the astronomical tide refers to the deterministic predicted component caused by tidal forces generated by the earth-solar and lunar-earth orbits. The difference between the two is termed the residual. The residual, especially when large, may be caused by a storm surge but this is not necessarily the case. Similarly, although the residual may often result from meteorological effects and may appear as noise it may contain a deterministic component, and the terms noise and storm surge are generally best avoided in this context. The term 'ESLs' is used throughout to represent the expression 'estimates of the exceedance probabilities of Extreme Sea-Levels'. Several statistical methods have been developed to assess Extreme Sea Levels (ESL) from measurements of sea level, these are now briefly reviewed.

The original work by Gumbel & Lieblein in 1954 describes a method for the conversion of selected extreme maxima into ESLs, with the majority of cases discussed being based upon the analysis of annual maxima; this later became known as the AMAX method. Since AMAX uses annual maxima, significant events that rank second or lower each year are not utilised. However, in his original paper, Gumbel & Lieblein also examined some extreme events without using annualized data, such as the breaking point of yarn, and the breakdown voltage of electrical capacitors. Subsequently an approach based upon threshold rather than grouping has become known as the "Peaks over Threshold" (POT) method, see Coles (2001), while an approach based upon selection of the largest, r, peaks within each time block has become known as the "r-largest" method (Smith 1986; Tawn 1988).

A different approach by Pugh and Vassie (1978) splits the total sea level into two components which sum to the total tide: these are the deterministic astronomical tide component and the residual component. Each component is converted into separate probability distributions which are then combined by convolution to produce a joint probability distribution function (PDF). The method is known as the joint probability method (JPM). Its main advantage lies in its efficient use of source data; all values of the residual contribute to the final probability distribution function (PDF), even if they are not at or near high tide. However, there are two notable drawbacks to the JPM. First, the conversion of the PDF into design risk is challenging, as it can depend on sampling period (Middleton & Thompson 1986; Tawn & Vassie, 1989). Second, the timing of actual high tide is often shifted in relation to the underlying predicted astronomical tide; this introduces correlation between two components, undermining the convolution (Tawn 1992). Nevertheless the JPM is widely used (McInnes et al., 2013), despite criticism on the above grounds (e.g. from Batstone et al., 2013).

The skew surge joint probability method, SSJPM, aims to overcome these shortcomings of the JPM, by replacing the residual (i.e. the instantaneous difference of measured and predicted water level) in JPM with the height difference between the maxima of the measured and predicted water level for each tidal cycle. This new height difference, referred to as the "skew surge height," is claimed to have minimal correlation with tidal height at most locations and is therefore considered an ideal parameter for characterizing surge statistics (Williams et al., 2016). To accurately extrapolate the tail of the probability curve, an extreme value probability distribution, such as the generalized Pareto distribution (GPD), is used. The method was used in the UK Environment Agency Study, 2011, EA (2011) as described by Batstone et al. (2013), and referred to onwards as EA2011, which used source data from the UK National Tide Gauge Network (UKNTGN) locations. However, Batstone et al. reported: *"The ESL's values derived from the SSJPM were compared with the AMAX time series of sea-level.", "At approximately one-quarter of the UKNTGN sites, it became clear that the GPD fitted on the skew surge distribution was leading to a seemingly implausible representation of the most extreme sea levels."..''Adjustments to the GPD shape parameter were performed by averaging the value with four immediate neighbours weighted by the length of data at each site"*. The problem was attributed to correlation between skew surge and predicted tide. Williams et al., 2016, reported that when "seasonal relationships between tides and the storm season were removed, then skew surge and associated HW are completely independent at 68 of our 77 study sites." Thus, even after removing seasonal effects, skew surge and associated HW were not completely independent at a minority, but significant number, of locations. Here is the dilemma of the SSJPM method. It works well when the skew surge is independent of the tide but how can we know in advance when this is the case? As we have seen, in EA2011, approximately one-quarter in that study had to be examined and re-calculated using other adjustments.

The above situation indicates that a generally applicable means of delivering quality ESLs from tide gauge data would be a most useful tool if, even if the results were somewhat less accurate than the very best results of the SSJPM, i.e. where skew tides and the tidal heights are substantially independent. This paper pursues such a goal by re-examining the work of Gumbel & Lieblein (1954) and incorporating a relatively simple modification. The results of this approach, called here TMAX, are compared with the EA2011 study which used the SSJPM method. Unlike in EA2011, the TMAX method did not require manual intervention, and used an identical algorithm for each location. Note that the primary purpose of this study is to examine the suitability of the TMAX technique, by comparing its results with those of the AMAX and SSJPM methods using the results of EA2011. Note that, although it would be useful to make comparisons at as many locations world-wide as possible, the author has not been able to locate the necessary suitable data sources for carrying out such a study. Therefore this study is limited to the UK locations in the EA2011. Its purpose is to examine the proof of concept for the described TMAX method, rather than being an attempt to redefine UK ESL flood defence heights.

2. Background

W.E. Fuller (1914) claimed that, on a purely empirical basis, the size of floods increases proportionately to the logarithm of time. Some 40 years later Gumbel & Lieblein, in their classic 1954 paper "Statistical Theory of Extreme Values and Some Practical Applications", theoretically justified Fuller's claim, provided the extreme values are derived from a stationary series, i.e. one whose mean did not drift uniformly with time, and have a probability distribution of an exponential type. This latter condition, later known as a Gumbel/Fisher-Tippett Type I distribution, applies but is not limited to exponential, normal, chi-squared, logistical and log-normal distributions. (See also Leadbetter 1983, Tawn 1988). Gumbel gave many practical examples ranging from floods, radioactive decay, human life expectation, electrical capacitor breakdown, the strength of yarn, and the stock market share value. In Gumbel's original description (see his Eq.(2.17), a number, N, of observed peak values (generally annual), were ranked in ascending order with each ranked value i, being converted into the cumulative probability of a value not exceeding the ranked value, F_i by the formula.

$$F_i = i / (N+1) \quad (1)$$

The probability F_i is related to the return period T (generally in years) by Gumbel's Eq.(2.8), i.e.

$$T_i = 1/(1 - F_i) \quad (2)$$

Gumbel showed that for a Type I distribution, the probability of the value being below a given value can be written as

$$F = \exp(- \exp(- y)) \quad (3)$$

The reduced variate, y exhibits a linear relationship with observed extreme value x, via a scale factor α and location factor, μ being given by

$$y = \alpha (x - \mu) \quad (4)$$

Therefore, in a plot of observed extreme height x, versus reduced variate y, each extreme value falls approximately in a straight line. In Gumbel's time, each ranked value was physically plotted on probability paper, where the horizontal scale had been marked out according to Eq.(3), in terms of either return period or reduced variate or both, depending upon the manufacturer of the paper. A straight line was fitted to the points, and extrapolation of the line gave the probability of a value not exceeding a given value x, without explicitly requiring the calculation of the scale factor, α or location factor, μ.
Returning now to Eq.(1), Gringorten (1963) proposed a widely accepted correction to improve the plotting accuracy as

$$F_i = (i - 0.44) / (N + 0.12) \quad (5)$$

3. TMAX Method

The method described in this study named TMAX, differs from the method described by Gumbel in three main ways. Firstly, the ascending order of maxima used by Gumbel is replaced by a descending rank of maxima. Secondly, only one maximum is detected within each tide. Finally, only a subset of the total number of peak values are selected and used to fit the straight line in the probability plot. These differences are now considered in more detail.

3.1 Descending Rank

There are advantages in reversing the rank order as compared with Gumbel's original scheme, see Harris (1996), and this reversal is used here. Since the largest most extreme value is known it can readily be indexed as 1 while successively smaller values are indexed with rising integers (2,3, etc). We consider here F(x) to be the probability of the tide exceeding a height value x (i.e. to be the flood probability), rather than the opposite as in Gumbel's original formulation. Adding a prime to those equations of Gumbel i.e. F in Eq.s (2 & 3) and noting that F '= 1 - F, Eq.(2) & Eq.(3) now become

$$T = 1 / (1 - F') = 1 / F \qquad (6)$$

$$F = 1 - \exp(- \exp(- y)) \qquad (7)$$

where y is the reduced variate. Making y the subject of Eq.(7) gives

$$y = - \log_e(- \log_e(1 - F)) \qquad (8)$$

We note that F is very much less than 1, hence the right hand logarithm can be represented by -F, and since F and T are inversely related we obtain y = log(T), confirming Fuller's original claim that maximum flood height varies with the logarithm of time. Writing Gumbel's descending rank, as i', we write the ascending rank, i, as i = N + 1 - i ' . Substituting this into Eq.(1) we obtain

$$F_i = 1 - F_i' = 1 - i' / (N+1) = i/(N+1) \qquad (9)$$

and applying Gringorten's Correction we obtain

$$F_i = 1 - F_i' = 1 - (i' - 0.44)/(N + 0.12) = (i - 0.44)/(N + 0.12) \qquad (10)$$

Since (10) is identical in form to Eq.(5), the reversal of rank order does not affect Gumbel's original plotting formulae, nor Gringorten’s correction to it. The Gringorten formula, rather than Eq.(1), was used in all of the relevant calculations from hereon. The design risk D(x), i.e. the probability that a given value of x will be exceeded during a design life consisting of n durations (usually years), is given by.

$$D(x) = 1 - (1- F(x))^n \quad (11)$$

3.2 Tide Peak Detection Algorithm

Most but not all of the examples given by Gumbel use the annual maxima. Gumbel stated in his conclusion, "If the number of observed extremes N is not excessive, do not group the observations." Therefore, although extrema are generally grouped into annual time blocks, it is not necessary to do so. In the TMAX method described, maxima are not explicitly grouped. Only one peak is detected per tide (semi-diurnal or diurnal) with the largest peaks in the tide gauge record being identified as follows. The data is initially scanned and the mean height value, msl, is approximately found. The data is again re-scanned, and when the height value transitions above msl a search flag is set and the date and time of the upwards transition is stored. Once the search flag is set, the highest value is determined until the sequence transitions downwards below the msl. At this point the search flag is unset and the date and height of the highest value found during the period when the tide is above msl. Provided that the date-time of the downwards transition minus the date-time of the upwards transition is less than half of a tidal day the event is stored in a list as a maximum. This latter condition avoids counting transitions when the data has large gaps. Furthermore, assuming the storm surge duration is less than that of the tide, it also prevents the selection of multiple peaks from within the same storm event. Therefore, in addition to providing a determination of relevant peak value, it is also a form of declustering (see below). This list produced may include smaller peaks associated with noise and smaller semi-diurnal tides, but their existence in the list is irrelevant as only a small number n of the most significant are ultimately used as is now described.

3.3 Subset Selection

Clearly, for a duration of tidal data of many decades, there will be many thousands of peak values. However, using all of the peak values on a probability plot is not advisable because the plot of height against the logarithm of return period sometimes deviates from a straight line at lower values of height; Fig. 1 shows this curvature and how it adversely affects the straight line fit. The curvature is probably related to shallow water effects during the neap of a spring-neap cycle. We use a fit of only the n largest values, since these fall, more or less, in a straight line as shown in Fig. 1, where n is calculated from an average number per year, n_A multiplied by the tide gauge record duration in years n_y, i.e. $n = n_A \times n_y$. The optimization of n_A is discussed in Sect. 4.1 and a value of 5 per year seems appropriate.. The reader may consider here some similarities with r-largest, AMAX and Peaks Over Threshold (POT) methods. In r-largest, a fixed number of the greatest extreme events are selected for each time unit, whereas in TMAX the selection process applies to all of the data. In AMAX, one annual maxima is used, so TMAX could be viewed as being similar to AMAX but with a time unit of one or half a tidal day, depending upon whether the tide is diurnal or semidiurnal. However, TMAX incorporates further selection, using only the n largest values, whereas AMAX does not. The greatest similarity is with POT, where peaks are selected over the entire dataset, but in POT selection uses a certain threshold value, as opposed to in TMAX, where a given number of the largest values are selected.

This is more much convenient than employing a threshold, since the number selected corresponds directly to the rank number which is already known; whereas using a threshold definition involves tidal range and datum, which are different for each location. Furthermore, in TMAX, the algorithm for peak detection (as described above), provides a level of isolation between successively selected peaks to select only one significant maximum per tidal day time block. This declustering avoids selection of multiple maxima from within the same extreme event "cluster", since such maxima would not be statistically "Independent and Identically Distributed" (IID). The key assumption here is that surges are of a sufficiently short duration that none will persist for more than one tidal day. Indeed on the list of maxima produced, none were found to occur on adjacent dates. One other key difference is that POT uses the generalised Pareto distribution (GPD) for fitting, whereas in TMAX, a simple least square regression linear fit is used.

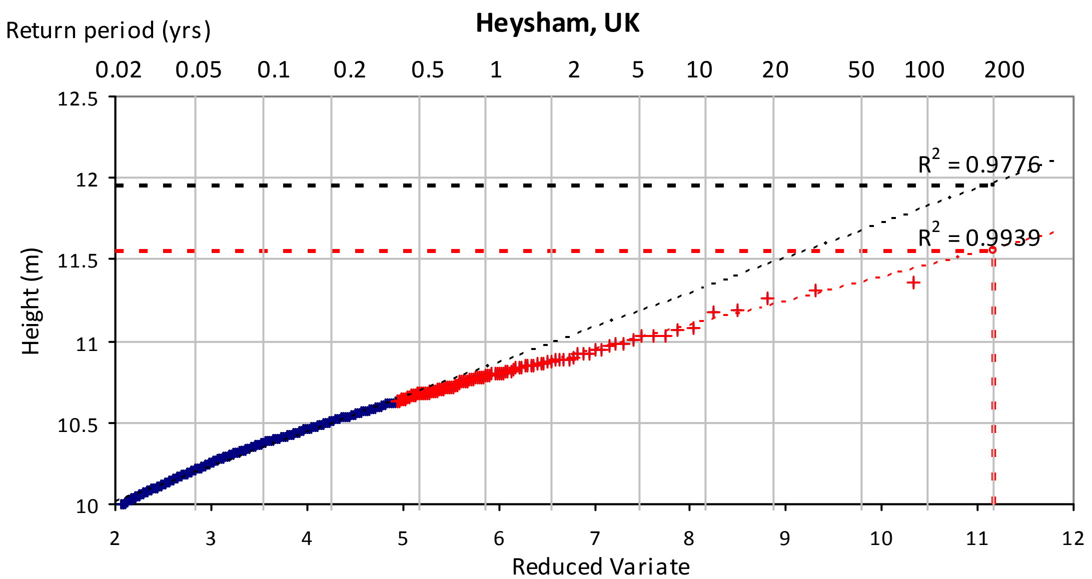

Figure 1. Probability extreme tide plot showing its curvature in a 54-year record. Setting a threshold of 50 greatest extremes tides per year (2439 total), the LSQ intercept is 11.95m ± 0.02m R2= 0.9776 (black). At 5 greatest extreme tides per year (242 points) the straighter red section gives an improved LSQ intercept of 11.6m (red) ± 0.009m R2=0.9939. Location Heysham UK See Sect. 4.1 & Figs. 2&3.

But what should be the value of N be in TMAX? If TMAX is considered as a modified form of AMAX, but with a tidal day rather than the annual time window, then N becomes the data duration D_D divided by tidal day length, T_D (in the same units), and the number of tidal days in a year is given by $1/T_D$, hence the usual ranking Eq.(9) becomes

$$T_i = N / i \quad \text{in tidal days} \qquad (12)$$

where $N=D_D/T_D$. Multiplying by T_D (in years) to convert to years, eliminates N becoming

$$T_i = D_D / i \quad \text{in years}$$

where 'i' is 1 based and corresponds to the rank having a maximum value of 'n' as defined earlier in this section. Incorporating Gringorten's Correction Eq.(10) in Eq.(12) leads to

$$T_i = (N + 0.12) / (i - 0.44) \quad \text{in tidal days} \qquad (13)$$

However, Eq.(14) can again be converted to years by multiplying by T_D giving

$$T_i = (D_D + 0.12\, T_D)/(i - 0.44) \quad \text{years} \qquad (14)$$

In practice, on TMAX, eliminating Gringorten's correction and using Eq.(12) rather than Eq.(13) was found on average to increase the ESL values by a few centimetres, and increased the 95% confidence limits slightly by a few millimetres; on AMAX the results were similar, except the 95% confidence limits were also increases by centimetres. Accordingly, this study utilised Eq.(13) with N specified in tidal days, the results being converted to years for display.

3.4 Straight Line Fit

As discussed above, the ordinary least squares (LSQ) fit, also known as linear regression, was used to determine the line fit (Morrison 2021). This is despite the existence of many other more complex methods including the method of maximum likelihood estimation (MME), method of moments (MOM), method of L-moments (MLM), method of probability-weighted moments (PWM), and the generalized least-squares methods GLSM/V. See Harris (1996), Coles (2000), Hong et al (2013), van Zyl & Schall (2012). Consideration of these other fitting methods in conjunction with TMAX is viewed as a potential refinement for future research. With both AMAX and TMAX methods, the expected value of a predicted new point (i.e. a flood) is the mean y value of the intersection of the extrapolated regression line with the ordinate corresponding to the required return period. Morrison gives expressions for the mean slope m_m, the mean intercept c_m, the variance of the expected mean σ_m and the variance of the new predicted mean σ_p, where n represents the number of data points, x_m, and y_m are the mean x and y values respectively, as

$$SS_{xy} = \Sigma(x_i - x_m)(y_i - y_m)$$

$$SS_{xx} = \Sigma(x_i - x_m)^2$$

$$m_m = SS_{xy} / SS_{xx}$$

$$c_m = ((\Sigma x_i)^2 \Sigma y_i - \Sigma x_i y_i \, \Sigma x_i)/(n \Sigma x_i^2 - (\Sigma x_i)^2)$$

$$\sigma_m = s^2_{y,x} \left(1/n + (x_p - x_m)^2 / SS_{xx} \right) \qquad (15)$$

$$\sigma_p = s^2_{y,x} \left(1 + 1/n + (x_p - x_m)^2/SS_{xx}\right) \qquad (16)$$

$$s^2_{y,x} = \left(1/(n-2) \right) \Sigma(y_i - y_m)^2$$

$$sd = \sigma_p^{1/2} \qquad (17)$$

Given the standard deviation, sd, the mean value of a predicted value y at a given value xp is given by

$$y = m_m x_p + c_m \pm t_{\alpha/2,n-2} \, . \, sd$$

where 95% confidence interval, $t\alpha/2,n-2$ signifies the t distribution and is commonly taken as approximately 2. The variance of the expected mean σ_m Eq.(15) is always less than the variance of the new predicted value σ_p Eq.(16), since the variance in the expected mean value is subject only to the distribution of data points, and tends to zero as the number of points, n increases, whereas the variance in a new estimated point is subjected also to variation in the process under examination, and does not tend to zero as n increases.

3.5 Missing Data

Tide gauge records often contain temporal data gaps due to faulty instrumentation, or damage to the gauge installation. Using the TMAX method, files containing missing tidal data can be accommodated by simply reducing the value of N Eq.(13) to correspond to the number of tidal days of valid data. By contrast, it is not so straightforward in the AMAX or r-largest methods to accommodate missing data, since either synthetic data must be provided to fill the gaps, as was discussed by Gumbel, or whole years of data must be rejected if a significant portion of the annual data content is missing.

4. Comparative Study

The AMAX and TMAX methods were applied to calculate the ESLs for 39 of the 41 UK stations used in the UK study EA2011; the difference in number of ports arising because two, Hilbre Island and Exmouth were not included in the available BODC download. After downloading the tide gauge data, it was first concatenated for each port to form a single data file for each port; these were then imported and manually inspected. Most of the data before 1993 had been recorded at hourly intervals whereas subsequently, data was at 15-minute intervals. Although a spline fit had been considered as a strategy to artificially fill-in hourly results at 15 minute intervals it was considered advantageous to use the data directly, especially bearing in mind that the spline fit could result in new high tide values which had not actually been recorded. Bearing in mind that the TMAX method is dependant upon the height of the recorded peaks rather than their exact timing, the advantage of using much longer records would easily outweigh the potential small inaccuracies induced due to the change in sample rate. In addition a down-sampling experiment was carried out, by degrading the 15-min periods to hourly sampling. It changed the 200 year ESL by <1cm and the 95% confidence level by 1~2mm. Therefore, to obtain the longest data duration possible, both 15-minute and one-hourly data were used within a single file if necessary. Each file was examined for tide gauge malfunctions and faulty data, and any data suspected of being faulty was deleted; this included isolated extreme maxima of non-contiguous data, gauge drifts and gauge slips. The maximum 5 values in each file were re-checked to verify the peaks represented a surge and not intermittent tide gauge faults

4.1 Optimising the average number of peaks per year.

As discussed above, selecting only the highest peaks avoids the graph curvature of Fig 1, improving its accuracy. Therefore, although larger sample values are generally associated with a decrease in variance, in this case the opposite may be true. This section investigates how, using the TMAX method, the variance of the mean, σ_m, and variance of a predicted new point, σ_p , as determined using Eqs.(10) & (11), varies with n_A, the average number of extremes selected per year. Figure 2 shows how σ_p attains a minimum at a value of n_a of around five extremes per year. This concurs with the value of five per year found by Robson and Reed (1999) for extreme river flow studies and used by Smith (1986), although this may be coincidental. Nevertheless, since we obtain a minimum variance at the value of $n_a = 5$, this was adopted for all subsequent results.

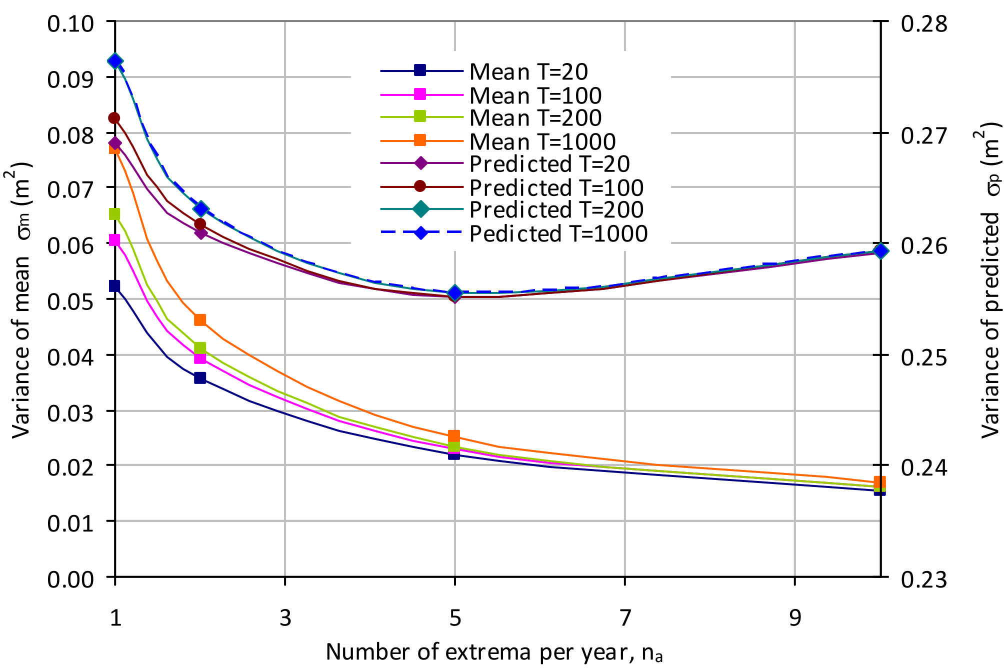

Figure 2. The mean-variance σ_m and of the predicted new point variance σ_p plotted against the number of extrema per year n_a for the TMAX method as determined from the tidal data

4.2 Analysis

For compatibility with EA2011, all data was sea-level rise de-trended, using the constant value and date origin adopted in that study of 2mm per year, with zero being applied on 1 January 2008. For further comparison purposes, the records were truncated to have a maximum date of 1 Jan 2009, which corresponded to the data used in EA2011, see Table A1.3 . The analysis for dates to 1 Jan 2009 was run using both AMAX and TMAX methods, which are referred to here as AMAX2009 and TMAX 2009. During the analysis the peak selection algorithm as described above was used. The number of valid days of data was determined and only those in the top rank corresponding to a total number, n, corresponding to n_a of 5 per year were used. The peaks were ranked and plotted using Gringorten's formula, where the total number of samples was calculated on the basis of one per tidal day for each tidal day of valid data. The LSQ regression line analysis was then used to give an intercept for the required return periods. The entire analysis was then repeated using the complete data set to 1 May 2018, being references as AMAX2018 and TMAX2018. The periods from tidal days to years were converted as required. The results were derived solely from the tide gauge data and have not been re-processed in any other way. These are now described. Comparisons with the results from EA2011 are indicated where appropriate.

4.3 Results

Table 1 compares the results of TMAX2009 with EA2011 with ESLs in metres above ODN relative to MSL2008 for all 39 ports. The results are shown for each of the return periods of 20, 100, 200, and 1000 years. The column headed SD shows standard deviation for theTMAX2009 ESLs as obtained from the Least Squares Fit using Eq.(17). The column headed "Difference" shows the values returned from TMAX2009 minus those from EA2011. The lower two rows show the mean values and their standard deviations. the mean differences were of the order of centimetres while the standard deviations are of the order of 10 to 20 centimetres, with both rising for the longer return periods.. Figure 3 shows the estimated ESL values for a 100-year return period at 39 UK sites on a map as derived from TMAX2018 rather than from TMAX2009, utilising all of the tidal data examined. The results are discussed in the error analysis in the following section.

Table 1. ESLs (m) above ODN relative to MSL2008.
Results of TMAX2009, Standard Deviation of LSQ fit, Values of EA2011, and Difference TMAX2009-EA2011

	TMAX2009				SD				EA2011				Difference			
Return Period (Yrs)	20	100	200	1000	20	100	200	1000	20	100	200	1000	20	100	200	1000
Aberdeen	3.0	3.1	3.2	3.3	0.1	0.1	0.1	0.1	3.0	3.1	3.2	3.3	0.0	0.0	0.0	0.1
Avonmouth	8.7	8.9	9.0	9.3	0.2	0.2	0.2	0.2	8.7	9.0	9.1	9.4	0.0	-0.1	-0.1	-0.1
Barmouth	3.9	4.1	4.2	4.3	0.1	0.1	0.1	0.2	3.9	4.1	4.2	4.4	0.0	-0.1	-0.1	-0.1
Bournemouth	1.6	1.8	1.8	2.0	0.1	0.1	0.1	0.1	1.7	1.8	1.9	2.0	-0.1	0.0	0.0	0.0
Cromer	3.5	3.8	3.9	4.1	0.2	0.2	0.2	0.2	3.7	4.1	4.3	4.7	-0.2	-0.3	-0.4	-0.6
Devonport	3.2	3.3	3.4	3.5	0.1	0.1	0.1	0.1	3.2	3.4	3.5	3.6	-0.1	-0.1	-0.1	-0.1
Dover	4.3	4.5	4.6	4.8	0.1	0.2	0.2	0.2	4.2	4.5	4.6	4.8	0.0	0.0	0.0	0.0
Felixstowe Pier	3.0	3.3	3.3	3.6	0.1	0.2	0.2	0.2	3.3	3.7	3.9	4.4	-0.3	-0.5	-0.6	-0.8
Fishguard	3.3	3.5	3.5	3.7	0.1	0.1	0.1	0.1	3.4	3.5	3.6	3.7	0.0	-0.1	-0.1	-0.1
Heysham	6.4	6.7	6.8	7.1	0.2	0.2	0.2	0.2	6.4	6.7	6.8	7.1	0.0	0.0	0.0	0.0
Hinkley Point	7.4	7.5	7.6	7.7	0.1	0.1	0.1	0.1	7.5	7.7	7.8	8.1	-0.1	-0.2	-0.3	-0.3
Holyhead	3.7	3.8	3.9	4.1	0.1	0.1	0.1	0.1	3.7	3.8	3.9	4.0	0.0	0.0	0.0	0.0
Ilfracombe	5.7	5.9	5.9	6.1	0.1	0.1	0.1	0.1	5.7	5.9	5.9	6.1	0.0	0.0	0.0	0.0
Immingham	4.6	4.8	4.9	5.1	0.1	0.1	0.1	0.2	4.6	4.9	5.0	5.3	-0.1	-0.1	-0.1	-0.1
Kinlochbervie	3.8	4.0	4.1	4.4	0.2	0.2	0.2	0.2	3.6	3.8	3.9	4.2	0.2	0.2	0.2	0.2
Leith	3.7	3.8	3.9	4.0	0.1	0.1	0.1	0.1	3.7	3.9	4.0	4.2	0.0	-0.1	-0.1	-0.2
Lerwick	1.8	1.9	2.0	2.1	0.1	0.1	0.1	0.1	1.7	1.8	1.9	2.0	0.1	0.1	0.1	0.2
Llandudno	5.1	5.3	5.4	5.6	0.1	0.2	0.2	0.2	5.1	5.3	5.4	5.6	0.0	0.0	0.0	0.0
Lowestoft	2.7	3.0	3.2	3.6	0.2	0.2	0.2	0.3	2.7	3.1	3.3	3.8	0.0	0.0	-0.1	-0.2
Milford Haven	4.6	4.8	4.9	5.0	0.1	0.1	0.1	0.1	4.5	4.7	4.8	5.0	0.1	0.1	0.1	0.1
Millport	3.3	3.6	3.7	4.0	0.2	0.2	0.2	0.2	3.2	3.5	3.7	4.0	0.1	0.0	0.0	0.0
Moray Firth	3.2	3.4	3.4	3.5	0.1	0.1	0.1	0.1	3.1	3.3	3.4	3.5	0.1	0.1	0.1	0.0
Mumbles	5.8	6.0	6.1	6.3	0.1	0.1	0.1	0.2	5.8	6.1	6.2	6.4	0.0	0.0	0.0	-0.1
Newhaven	4.1	4.3	4.3	4.5	0.1	0.1	0.1	0.1	4.2	4.4	4.5	4.6	-0.1	-0.1	-0.1	-0.1
Newlyn	3.4	3.5	3.5	3.7	0.1	0.1	0.1	0.1	3.3	3.5	3.5	3.6	0.0	0.0	0.0	0.0
Newport	8.0	8.2	8.3	8.5	0.2	0.2	0.2	0.2	8.0	8.3	8.4	8.7	0.0	-0.1	-0.1	-0.2
North Shields	3.5	3.7	3.8	4.0	0.1	0.1	0.1	0.1	3.6	3.8	3.9	4.1	0.0	0.0	-0.1	-0.1
Portpatrick	3.2	3.4	3.5	3.7	0.1	0.1	0.1	0.1	3.2	3.4	3.5	3.6	0.0	0.1	0.1	0.1
Portsmouth	2.9	3.0	3.1	3.2	0.1	0.1	0.1	0.1	2.9	3.1	3.1	3.3	0.0	0.0	0.0	-0.1
Port Ellen	1.7	2.0	2.1	2.3	0.2	0.2	0.2	0.2	1.9	2.1	2.2	2.4	-0.2	-0.2	-0.2	-0.1
Port Erin	3.7	3.9	4.0	4.2	0.1	0.2	0.2	0.2	3.7	3.8	3.9	4.0	0.0	0.1	0.1	0.2
Sheerness	4.0	4.2	4.3	4.5	0.1	0.1	0.1	0.2	4.1	4.5	4.6	5.1	-0.1	-0.2	-0.3	-0.5
Stornoway	3.2	3.4	3.5	3.6	0.1	0.1	0.1	0.1	3.2	3.3	3.4	3.5	0.0	0.1	0.1	0.1
Tobermory	3.4	3.6	3.7	3.9	0.1	0.2	0.2	0.2	3.5	3.7	3.9	4.2	0.0	-0.1	-0.1	-0.2
Ullapool	3.7	3.9	3.9	4.1	0.1	0.1	0.1	0.1	3.6	3.8	3.8	4.0	0.1	0.1	0.1	0.2
Weymouth	2.1	2.2	2.3	2.4	0.1	0.1	0.1	0.1	2.1	2.2	2.3	2.4	0.1	0.0	0.0	0.0
Whitby	3.6	3.7	3.8	3.9	0.1	0.1	0.1	0.1	3.8	4.0	4.1	4.4	-0.2	-0.3	-0.3	-0.5
Wick	2.8	2.9	3.0	3.1	0.1	0.1	0.1	0.1	2.7	2.8	2.9	3.0	0.1	0.1	0.1	0.1
Workington	5.7	6.0	6.1	6.3	0.2	0.2	0.2	0.2	5.6	5.8	5.9	6.2	0.1	0.1	0.2	0.2
mean (m)					0.1	0.1	0.1	0.1					0.0	0.0	0.0	-0.1
stdev (m)													0.1	0.1	0.2	0.2

Figure 3. 100 Year ESL ODN (m) at 39 UK locations From TMAX2018 relative to MSL 2008

4.4 Error Analysis

Table 2 shows the standard deviation for each given return period and were calculated from the best fit to the source "n" maxima, using Eq.(10) and those of Sect 3.4. The standard deviation is calculated from the expected variance in a new forecast point σ_p Eq.(16) rather than the smaller variance in the mean σ_m Eq.(15), as indicated in Sect. 3.5. It can be seen that there is a significant difference between the standard deviation for the two methods, AMAX and TMAX, with TMAX significantly outperforming AMAX, reflecting the larger number of points involved in the TMAX method. On average the standard deviation for TMAX were approximately 60% of those of AMAX which suggest a useful margin of improvement in accuracy. The level of agreement between AMAX and TMAX with the results of EA2011 are now described.

Table 2: Mean of Standard Deviation for AMAX and TMAX based upon variance in predicted level, $\sigma_p^{1/2}$ of the LSQ fit.

Return Period (years)	**20**	**100**	**200**	**1000**
AMAX2009	0.19	0.22	0.23	0.27
AMAX2018	0.20	0.22	0.23	0.26
TMAX2009	0.12	0.13	0.14	0.15
TMAX2018	0.12	0.13	0.14	0.14

*** means of confidence bounds in Table A6.1 EA2011**

Figure 4 shows the differences between the ESLs from TMAX and AMAX with EA2011 as histograms. Table 3 shows the means differences, and standard deviations of the differences. With AMAX the mean of the differences were generally slightly positive, indicating marginally higher ESLs as compared to EA2011, whereas with TMAX the ESLs were generally lower, indicating a slight reduction in sea-defence requirements. For AMAX the mean differences were slightly smaller than TMAX, indicating marginally better agreement, perhaps reflecting the use of AMAX in EA2011 at those locations where SSJPM was not used. However, the standard deviation of the difference were much greater than the mean difference, for all return periods listed. The standard deviation of the difference ranged from 0.09m for the 20 year return period with TMAX2009/2018 to 0.25 m for the 1000 years return period with AMAX. The Students "t" test was used establish the significance of the mean difference values, (α=0.05, 39 pairs, df=38 tails=2), giving a critical value at a 95% confidence level of t_{crit} = 2.02. Out of sixteen comparisons made, fifteen determined "t" to be considerably smaller than this critical value, the exception being AMAX09 at a 1000 year return period where only a marginal significance was found. Thus, according to the paired t-test the values of the mean difference is not significant. Across all studies for TMAX2009, TMAX2018, AMAX2009 and AMAX2018 and their comparison with EA2011 there were no consistent positive outliers exceeding two standard deviations above the mean. However there were consistently significant negative outliers, with over two standard deviations below the mean; these were Cromer, Felixstowe and Sheerness. Significantly, these ports had been singled out for discussion of levels in the EA2011 study. The port of Felixstowe consistently had return levels which are showed the largest significant difference from those of EA2011. Further study using TMAX of tidal records for the port of Harwich, at a distance of only 3 nautical miles from Felixstowe, gave ESL's showing a considerably improved level of agreement with the results of EA2011 for Felixstowe, although the implication of this are not clear.

Table 3. Standard Error in Mean and Mean Difference between AMAX, TMAX and EA2011.

	Return Period (years)	**20**	**100**	**200**	**1000**
AMAX2009	Mean Difference (m)	0.01	0.01	0.01	-0.01
	Std Dev(m)	0.10	0.15	0.18	0.25
AMAX2018	Mean Difference (m)	0.02	0.04	0.04	0.05
	Std Dev(m)	0.11	0.16	0.18	0.25
TMAX2009	Mean Difference (m)	-0.01	-0.04	-0.05	-0.08
	Std Dev	0.09	0.13	0.15	0.22
TMAX2018	Mean Difference (m)	-0.01	-0.03	-0.04	-0.07
	Std Dev(m)	0.09	0.13	0.15	0.22

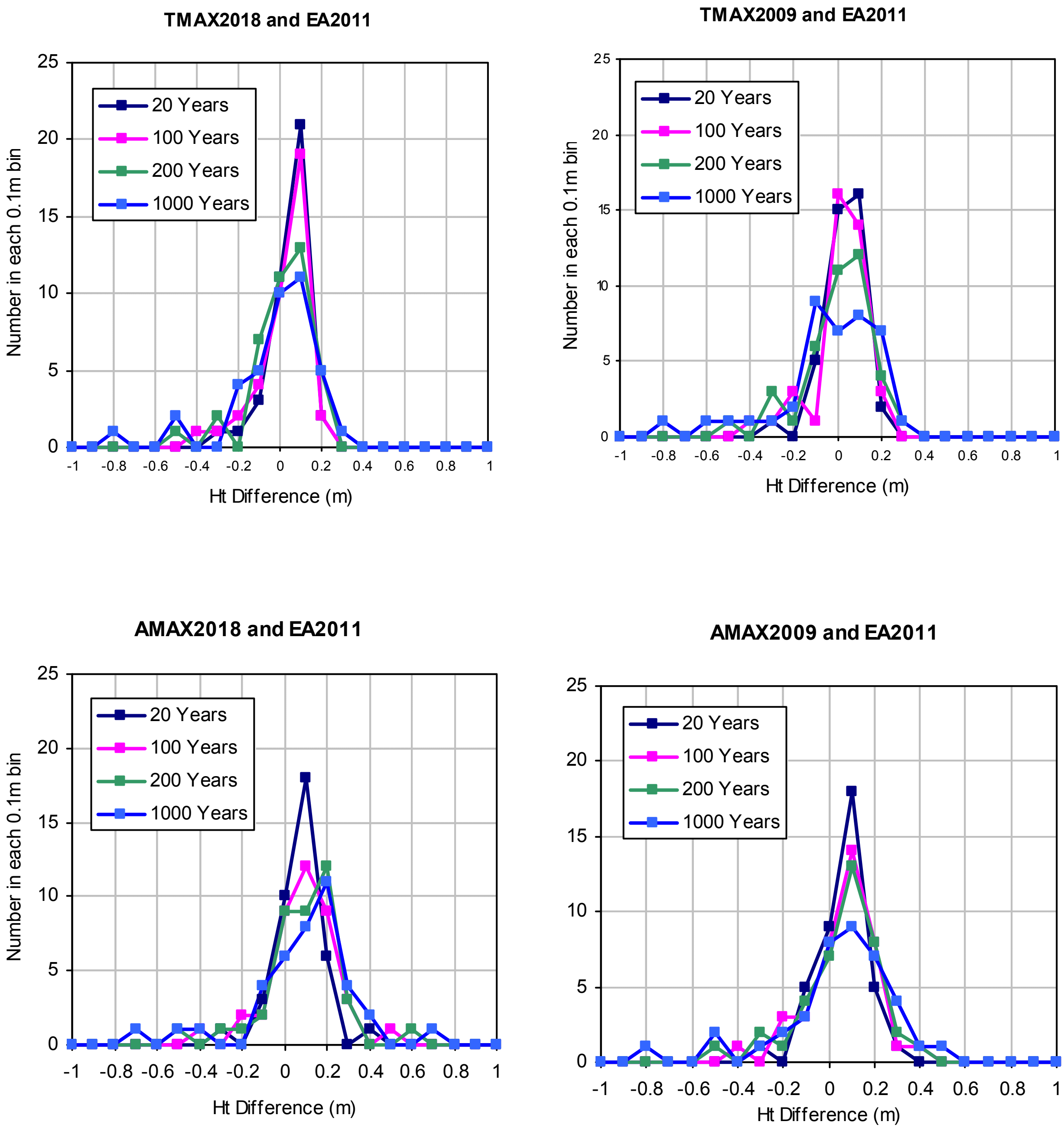

Figure 4. Differences between TMAX, AMAX and EA2011 using data to 2009 and 2018

5. Conclusion

The paper describes a method named here as "TMAX", which is designed to derive extreme sea level statistics (ESLs) from tide gauge data, based upon a modification of Gumbel and Lieblein's method of 1954. When compared with the AMAX method, TMAX confidence bounds are approximately 40% smaller than those for AMAX, suggesting a significant margin of improvement in accuracy. Further, it is shown that TMAX generates estimates of ESLs which are in broad agreement when compared with those published by UK Environment Agency 2011 (EA 2011) at all 39 locations, and good agreement at 36 locations, with agreement at 3 location in the southern North Sea, in particular at the port of Felixstowe being outliers. Overall mean differences between the two methods were of the order of centimetres and standard deviation of the order of decimetres. However, the TMAX method is much simpler than the SSJPM used in EA2011. Unlike in the EA2011 study, where one quarter of sites were singled out for special attention, the same algorithm was used at each location without "manual intervention". Also, unlike the SSJPM, the TMAX assumes a Gumbel Type 1 statistic and does not split the tide into two components, i.e. the predicted tide and skew tide. Hence it does require a) harmonic tidal analysis, b) the use of a probability density function, or c) the fit of a generalized Pareto distribution (GPD). TMAX therefore does not suffer from any errors induced in this process, which may, in a minority of cases, significantly influence the accuracy of the result. A further advantage of the TMAX method is that missing data, quite common in tide gauge records, is handled in a simple, efficient and elegant way. The study indicates that the TMAX method shows promise as a general screening method for extreme sea level analysis. It is hoped the approach may facilitate the generation of ESLs from tide gauge data, thereby informing and improving strategies for coastal management and resilience.

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Competing Interests

The author declares that he has no conflict of interest.

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