I think this study has value, for example in pointing out the lack of subseasonal variance in the model, and trying to dispel the continuing myth about PPL being an upper limit.

Dear Prof Adam Scaife,

Thank you very much for your constructive comments on our manuscript. We greatly appreciate the time you have taken to review it. We value your recognition of the study's contribution. Below, we address major and minor points in detail, outlining our responses and proposed revisions to the manuscript.

- **Major points:**

L146-147, L188, L281, Fig.6: I don't think we can make the general statement that seasonal precipitation is only modulated by sub-seasonal components. This is one of the points of the reviewer and I think it has some weight. Similarly, it is not clear that you can just dismiss the first term on the right of eqn12. On this, I think reviewer 1 also has a point. In fact there is evidence this term could be large, for example for the NAO (see Keeley et al GRL 2009 for an illustration of where it is the interannual variability per se and not the shorter timescales that dominates). I therefore don't think L188 is justified in general.

We agree that our statement regarding the modulation of seasonal precipitation by sub-seasonal components may have been overly general and not applicable universally. Our intention was to emphasize this mechanism in the context of tropical and subtropical precipitation (e.g., during the boreal summer monsoon), where sub-seasonal events like synoptic systems (e.g. Yoon and Chen (2005)) and intra-seasonal oscillations (e.g. Goswami et al., 2006; Webster eet al. 1998) play a dominant role in building the seasonal mean.

Unlike variables such as temperature or surface pressure, which vary smoothly in time, rainfall is inherently discrete and typically occurs in pulses (rain or no-rain) concentrated within preferred time bands (i.e., sub-seasonal bands). Furthermore, the amplitude of these events is often much larger than that of the annual cycle. As a result, variations in sub-seasonal rainfall can significantly modify the annual cycle or seasonal anomaly.

To illustrate this, here we use daily 1°×1° IMD rainfall over a grid point in central India (20°N, 80°E), a homogeneous monsoon region. The amplitude of the climatological mean annual cycle (1901–2018) is about 20 mm/day (upper panel). The daily rainfall and corresponding annual cycle for a particular year (here, 2002; lower panel) show strong temporal fluctuations with large amplitudes (blue). The smooth annual cycle is reconstructed using the mean and the first three harmonics. The difference between the climatological mean annual cycle (black curve) and the annual cycle for 2002 represents the seasonal summer monsoon rainfall anomaly (a deficit monsoon year).

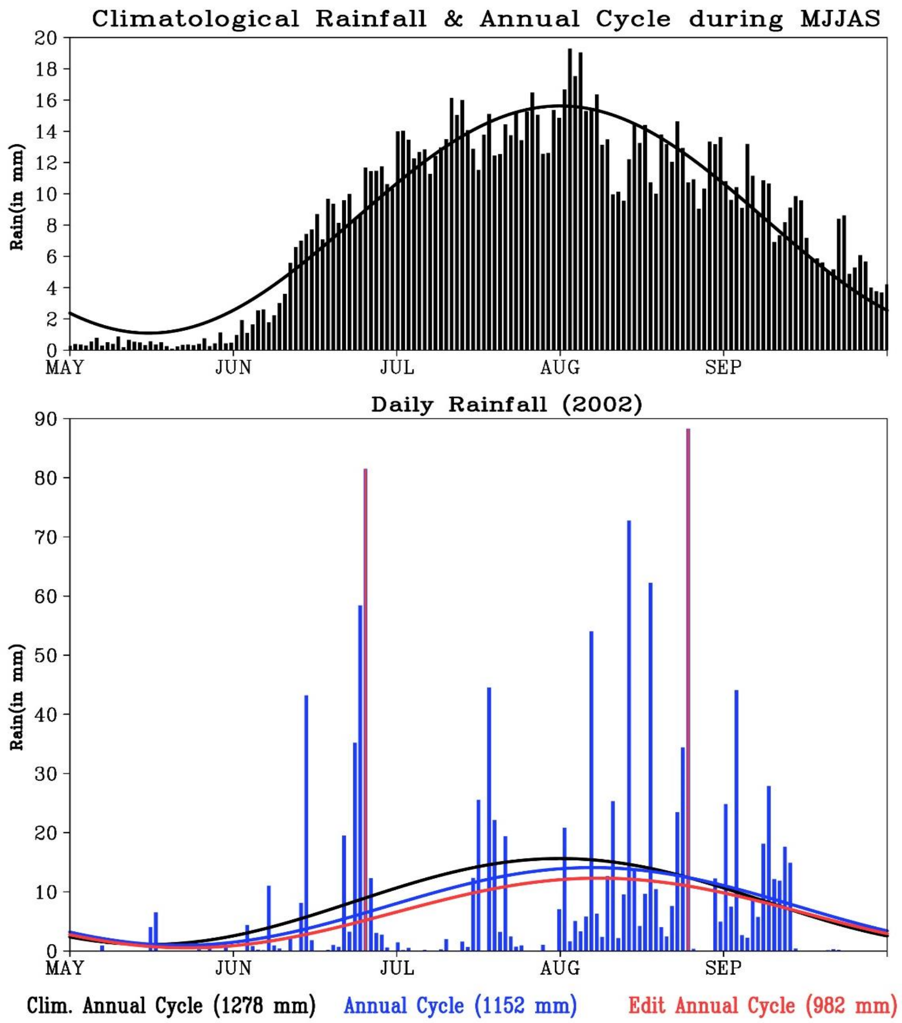

Figure R1: *Climatological mean daily rainfall (black solid bar) and smooth annual cycle (black line) over an area in the homogeneous central India region are shown in the upper panel. Lower panel shows rainfall of a particular year (here 2002) with smooth annual cycle (blue line), smooth annual cycle after removing two days of rainfall events (red line) and climatological mean smooth annual cycle (black line).*

To demonstrate how rainfall event of just one or two days could influence the annual cycle and seasonal anomaly, two-day rainfall event (>80 mm/day) are removed, assuming it arises from sub-seasonal variability. The reconstructed annual cycle, without these two days rainfall becomes visibly weaker (red curve). The resulting change in seasonal mean rainfall amounts to about **59%** of the interannual standard deviation. We also note that only two 1-day rainfall events are removed here; if a complete event is removed, as happens, the impact on the seasonal anomaly would be substantially larger. This highlights why sub-seasonal components are often termed the "building blocks" of the monsoon.

Because rainfall is a discrete phenomenon, it does not possess true, physically persistent modes. Thus, global predictors influence monsoon rainfall primarily by modulating the sub-seasonal components either through their strength, their duration, or both. The "persistent modes" that emerge from various data-analysis techniques are projections or statistical composites of these sub-seasonal rainfall components. We have now modified the manuscript with above discussion and added above figures in the supplementary section.

Regarding the first term on the right-hand side of Eq. (12), we acknowledge that the earlier description may have caused confusion. The seasonal anomaly is treated as an external term, under the assumption that it is entirely generated by the predictors, which act as the drivers of the seasonal anomaly. To address this issue, we have revised the manuscript. Now the manuscript is modified as

The time series of daily rainfall of a year (area average or a single point) can be represented by the following equation

$$x_T = x_c + x_a + \sum_f x_f \tag{7}$$

where, x_T is the total rain, x_c is the climatological mean annual cycle, x_a is the anomalous annual cycle, x_f represents the rest sub-seasonal components consisting of all frequencies f. Using harmonic analysis, the sum of the mean and the first three harmonics represents the 'smooth annual cycle' in the daily time series for a year. Here, x_c is the climatological mean of the 'smooth annual cycle', and x_a is the deviation of the 'smooth annual cycle' of a year from the climatological mean annual cycle. Sum of x_a in a season is the exact seasonal anomaly. Therefore, after re-arrangement, the above equation can be written as

$$(x_T - x_c) = x_a + \sum_f x_f \tag{8}$$

The left-hand term represents the total daily anomaly. In terms of seasonal variance, using daily June-to-September data (122 days) equation 8 for a particular season can be written as

$$\sum_{l=1}^{122}(x_T^l - x_c^l)^2 = \sum_{l=1}^{122}(x_a^l)^2 + \sum_{f=1}^{K}\sum_{l=1}^{122}(2x_a^l.x_f^l) + \sum_{f=1}^{K}\sum_{l=1}^{122}(x_f^l)^2 \tag{9}$$

$$V = V_a + \sum_f V_{cov} + \sum_f V_f \tag{10}$$

where l represents the day, V is the total variance, V_a is the variance of the anomalous annual cycle, V_{cov} is the covariance among sub-seasonal and anomalous annual cycle, V_f represents the sub-seasonal variance, K is the number of sub-seasonal bands (e.g., synoptic, bi-weekly) in a season. However, due to orthogonality, the covariance term becomes negligible. In terms of seasonal anomaly, equation 10 can be written as

$$V' = V'_a + \sum_f V'_f \tag{11}$$

Where, V', V'_a and V'_f are seasonal anomalies of the total variance, variance of anomalous annual cycle, and sub-seasonal variance respectively. Let I' be the anomaly of seasonal-mean rainfall then, the covariance between the seasonal rainfall anomaly and anomaly of total variance can be written as

$$\sum_i V'I' = \sum_i V'_a I' + \sum_i \sum_f V'_f I' \tag{12}$$

The left-hand term of equation 12 represents the interannual covariance between total sub-seasonal variance and seasonal anomaly. The first term on the right-hand side represents the covariance between variance of anomalous annual cycle and the seasonal mean, while the second term represents the covariance between variance of rest sub-seasonal components and the seasonal mean. It is important to note that the first term on the right-hand side explicitly does not contain information on the building blocks of the seasonal mean and is, therefore, not used in our analysis. On the other hand, the last term is of particular interest, as it represents the interannual covariance between seasonal mean and sub-seasonal bands.

Although the anomalous annual cycle (x_a) and sub-seasonal components (x_f) are orthogonal, their seasonal variances (equation 11) are interlinked on year-to-year time scale (Figure R2), indicating role of sub-seasonal components on seasonal anomaly (Figure R2). Correlations > 0.35 (< -0.35) are significant at 95% level.

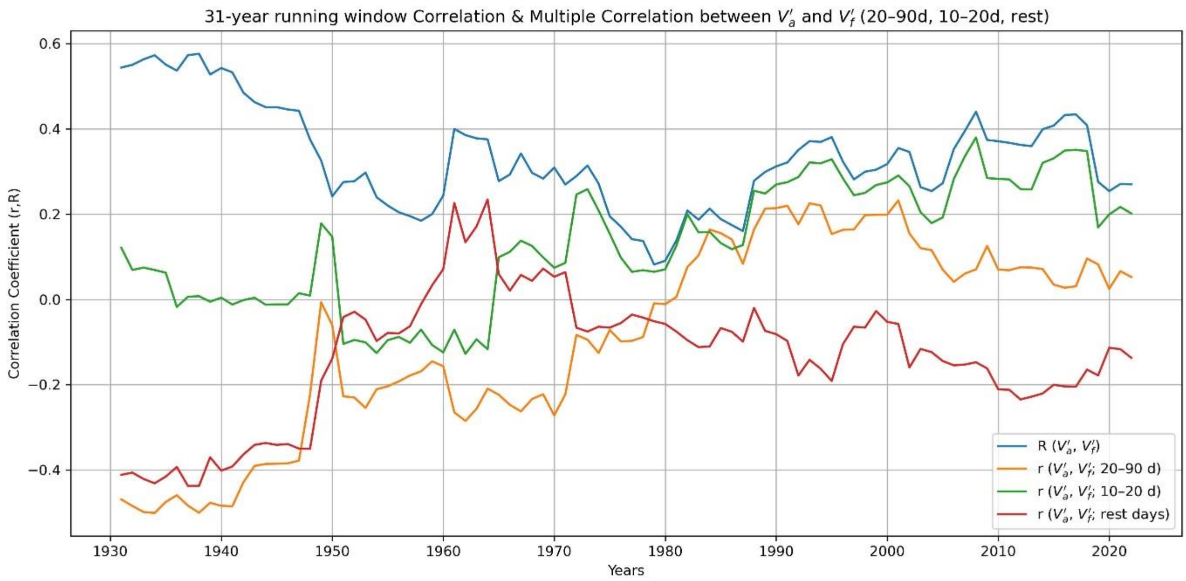

Figure R2: *Linking sub-seasonal components of ISMR with anomalous annual cycle (i.e. seasonal anomaly) in terms of their variances. Moving window correlation (31-years) between V_a and V_f (20-90 days, 10-20 days, <10 days band) and multiple correlation.*

The variance of sub-seasonal components in a season represents its energy or vigour, which also, in principle, should be linked with seasonal rainfall anomaly (i.e. last term in equation 12). A strong correlation of all India seasonal rainfall (i.e. ISMR) anomaly with variance of individual sub-seasonal components (Figure R3) support our arguments that sub-seasonal components are key to generating seasonal anomaly.

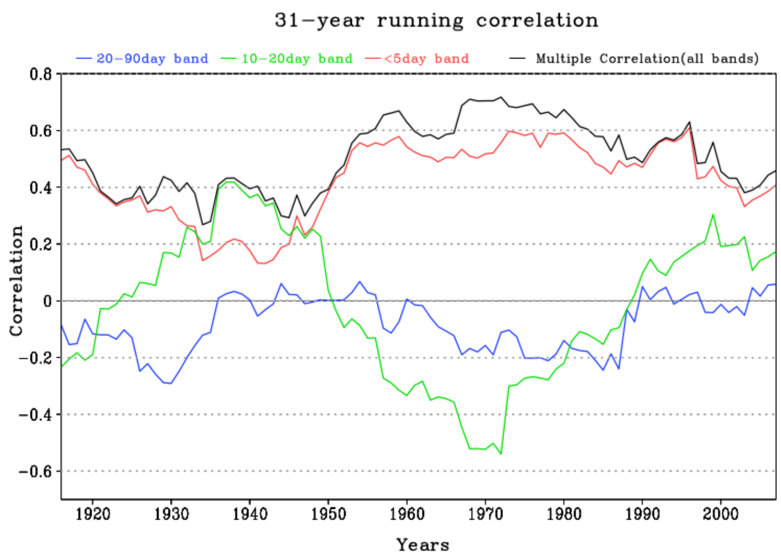

Figure R3:: *Linking sub-seasonal components of ISMR with its seasonal anomaly. Moving window correlation (31-years) between I and V_f (20-90 days, 10-20 days, <5 days band) and multiple correlation.* Correlations > 0.35 (< -0.35) are significant at 95% level.

L234 and abstract: I don't think anyone is saying that partitioning into signal and noise is going to eliminate paradoxical behaviour so I don't really understand this line.

Thank you for pointing this out. We did not intend to claim that perfect partitioning into signal and noise would eliminate paradoxical behaviour entirely. Rather, our point was to highlight that even with accurate separation (as assumed in the perfect model framework), paradoxes persist due to other factors like model imperfections. We have rephrased line 234 and the relevant abstract sentence for clarity.

L319: In this case the total variance is too strong. In our original papers we were careful to say that the paradox only really arises if the total variance is close to the observed variance – otherwise it could simply be a case of overdispersion – is that possibly the case here?

We appreciate this insight and agree that the paradox is most pronounced when total variance matches observations, whereas overdispersion (excessive total variance in the model) could explain apparent paradoxes in some cases. In our analysis, the total variance in the model for rainfall over the study regions (South Asia Monsoon region) is indeed higher than observed (as noted in Section 3.2 and Fig. 5b, 8c, 8d, which may indicate overdispersion rather than a "pure" low signal-to-noise issue. We have expanded the discussion at Line 319 to explicitly report on total variance comparisons with observations, referencing relevant papers (e.g., Scaife et al., 2014; Scaife and Smith, 2018) to distinguish between these scenarios.

Finally, I'm afraid I do not agree that the proposed method of selecting the ensemble members that yield the highest correlation with the observations is a viable algorithm for determining the upper limit of predictability. This can be illustrated if we consider a system with no skill but some random noise. Some combinations of ensemble members (half of them in fact) will then exhibit apparent skill even though none is present. The problem is particularly acute for small ensembles as you find in Fig.10 so I think this section should be removed.

We thank you for your views. Here our objective is to find out the range of actual skill achievable by this particular model. With limited number of ensemble members, we may be able to find out the population statistics using random choice of ensemble members. The distribution of actual skill with possible maximum and minimum defines the model's ability to predict seasonal monsoon rainfall. Improvements of model is likely to shift the whole distribution towards higher correlation side (e.g. Figure 4b in Saha et al., 2019).

We agree that with ensemble members having no skill (half of the members negative correlation and half with positive correlation), could show higher predictability. However, this is not the case here. We have shown minimum, mean, and maximum possible skill and all of them are positive (Figure 10). We do not propose this as a formal, universally applicable algorithm for redefining the PPL, nor as an operational technique. Rather, it is an experimental diagnostic to illustrate the maximum possible skill that can be obtained. We believe, it is an important diagnostic, which helps to understand predicative capability of a model and would like to retain in the manuscript with some modification/caveats.

We have now revised the manuscript to reflect the same. In the abstract, we have modified the sentence “In this context, we propose a novel method to estimate the PPL of seasonal climate, which can be free from paradoxical situation” to “In this context, we present a simple diagnostic approach to estimate the maximum achievable seasonal prediction skill, which may be interpreted as the PPL”. Similarly in result and discussion section we have modified keeping in mind that it is just a diagnostic and not the true PPL.

- **Minor points:**

L7: Regarding the orthogonality of noise and signal, there is a relevant recent paper by Brocker et al in QJRMS 2023 which makes a similar point. It certainly should be referenced and this may make it easier to justify the point about signal and noise not being orthogonal in time. See: https://rmets.onlinelibrary.wiley.com/doi/10.1002/qj.4440

Thank you for suggesting this. We have included this citation.

L30: I think that although it is not perfect, monsoon prediction skill is now well established so is this statement a little negative?

We agree. We have modified the statement.

L33-35: As we both know, the PPL is not an upper limit of predictability (or even the prediction skill of the model) so could rephrase to something like “…it is commonly assumed that the PPL is an upper limit on predictability…”

Thanks for the suggestion. We have made the changes.

L37: “….variance in models is too weak to explain the level of prediction skill.”

We have revised this statement.

L48: “…can arise…” rather than “…arises…”

Will Change to: "...can arise..."

L57: skills

Will be Corrected to "skills".

L114: should it be “…is often an overestimate of external….”

We will rephrase to: "...the variance of the ensemble mean is often an overestimate of external..."

L224: I think we need to be careful about saying internal variability = noise as ENSO, the QBO etc are all internal oscillation but are still predictable on these timescales. Suggest you use “internal unpredictable variability (noise)…” or similar

Thank you very much for pointing out this. We have now used signal and noise terms only and avoided using ‘external’ and ‘internal’ terms.

L253-254: I don think the paradox arises from splitting into signal and noise because the other measure we use is the ratio of Rmo/Rmm in Scaife and Smith 2018 which also exceeds 1, again showing the paradox but with no separation into signal and noise.

We agree, the paradox persists even without explicit signal-noise separation. We have tested the possibility from the point of view how signal and noise are estimated. As the current method considers only role of initial error and no other sources of error, the estimate of signal and noise is not accurate. As a result, signal and noise are not orthogonal. From this perspective also, it suggests perfect model framework is not adequate for estimating PPL. We have now revised the manuscript and included discussion related to finding using Rmo/Rmm.

L273: agreed could this be due to mean bias for example?

As mean is related with variance, yes, it is a possibility. mean bias could contribute. we will add a check for mean bias in the revised analysis and discuss it.

L302: bad not band

The typo will be corrected to "bad".

We believe these changes will address your concerns and improve the manuscript's clarity, rigor, and scope. We will submit a revised version incorporating these revisions shortly. Thank you very much for the constructive comments.

Thanking you,

Yashas Shivamurthy