

Response to reviewer comments

We are sincerely grateful to the editor and reviewers for their valuable time spent on reviewing our manuscript. The comments are very helpful and valuable, and we have addressed the issues raised by the reviewers in the revised manuscript. Please find our point-by-point response (in blue font) to the comments (in black font) raised by the editor and reviewers.

COMMENTS TO THE AUTHOR(S)

- Justification for using a Mie-scattering lidar: Please clarify why this system was selected. Is it superior to other lidar types for aerosol detection? In which specific aspects?

Response: We are delighted to see that the reviewer is interested in this aspect. The questions raised by the expert are fundamental and core issues in the field of atmospheric optics.

Firstly, Mie-scattering LiDAR is based on the Mie- scattering principle, which refers to the scattering phenomenon that occurs when light waves encounter objects with particle diameters much larger than the wavelength of the light waves in the atmosphere. The wave nature of light and the size effect of particles have been taken into account. Secondly, below the stratosphere, LiDARs based on Mie scattering, Raman scattering, and Rayleigh scattering are commonly used to detect atmospheric components. Mie-scattering LiDAR is often used to detect the concentration and distribution of particulate matter in the atmosphere (such as aerosols, dust, clouds, etc.). Raman-scattering LiDAR is suitable for detecting water vapor and trace gases (such as CO₂, O₃, SO₂, NO₂, etc.) in the atmosphere. Rayleigh-scattering LiDAR is suitable for detecting the density and temperature of gas molecules.

Mie-scattering LiDAR can be used to monitor aerosols by emitting laser wavelengths of 355 nm, 532 nm, or 1064 nm. Its core advantage lies in its ability to precisely detect the vertical fine structure of aerosols and clouds. It is primarily used in scenarios requiring the acquisition of vertical distributions of atmospheric pollutants, dust, clouds, and fog, as well as the fine structure of the boundary layer, such as environmental monitoring, climate research, and aviation meteorology support. It enables minute-level continuous detection with high resolution, operates day and night, and accurately reveals how pollutants are vertically transported and accumulated. Numerous studies have employed Mie-scattering LiDAR for monitoring dust events and atmospheric pollution. Refer to articles such as:

Yang, H.; Fang, Z.; Cao, Y.; Xie, C.; Zhou, T.; Wang, B.; Xing, K.; Lolli, S. Impacts of Transboundary Dust Transport on Aerosol Pollution in the Western Yangtze River Delta Region, China: Insights Gained From Ground - Based Lidar and Satellite Observations.

Earth and Space Science, 8 (3), e2020EA001533. <https://doi.org/10.1029/2020EA001533>, 2021.

Zhou, C., Liu, Y., He, Q., Zhong, X., Zhu, Q., Yang, F., Huo, W., Mamtimin, A., Yang, X., Wang, Y., Meng, L., Dust Characteristics Observed by Unmanned Aerial Vehicle over the Taklimakan Desert. Remote Sensing 14, 990. <https://doi.org/10.3390/rs14040990>, 2022.

- Autonomous driving versus alternative platforms: Provide a concise discussion on the advantages of autonomous driving combined with lidar, particularly in comparison with other state-of-the-art mobile platforms, such as UAV-based systems.

Response: Thank you for the suggestions you provided.

The intelligent driving platform with our Mie-scattering lidar platform offers three main advantages. Firstly, it can replace traditional engineer-driven mobile surveys, enabling intelligent cruising detection over larger areas (avoiding the need for engineers to maintain prolonged on-site duty). Secondly, compared to unmanned aerial vehicle (UAV) detection platforms, it offers multiple advantages in terms of technology (longer endurance, easier system implementation), policy (no airspace restrictions), and safety (higher safety). Finally, compared to other alternative platforms (such as UAV platforms), it offers an advantage in operational costs.

- Lidar specifications: Include a brief summary of the lidar model characteristics or cite the corresponding reference (e.g., <https://doi.org/10.3390/s23020892>). Important parameters such as the temporal resolution of both scanning modes are currently missing.

In Sect. 3.2., please indicate the time periods corresponding to each measurement and specify the temporal resolution of the scans shown. Additional discussion is needed to justify the conclusion that the system is capable of long-term 0 – 360° scanning.

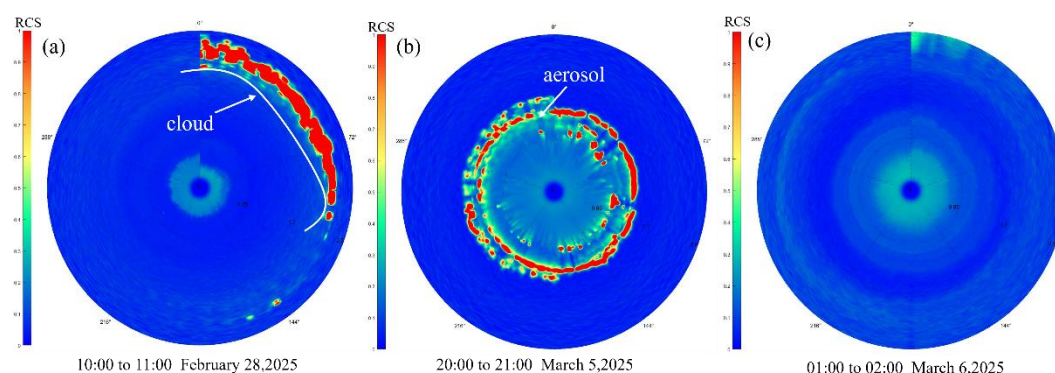
Response: Thank you for your improvement and assistance with our manuscript. After discussion and careful revision, our team has made the following changes:

The LiDAR we utilize is a compact polarization Mie-scattering LiDAR. If you are interested in the LiDAR hardware specifications, you may refer to this paper, as it elaborates on the parameters of the LiDAR used.

Kuang, Z.; Liu, D.; Wu, D.; Wang, Z.; Li, C.; Deng, Q. Parameter Optimization and Development of Mini Infrared Lidar for Atmospheric Three-Dimensional Detection. *Sensors*, **23** (2), 892. <https://doi.org/10.3390/s23020892>, 2023.

The scanning detection experiment is conducted with an elevation angle of 75° and a stepping angle of 1° , and a full 360° scan takes 1 hour.

Of course, the duration for a full scan can be adjusted based on actual detection requirements. Temporal resolution: 20 s. We have also updated the revised manuscript to specify the time period for each measurement segment; details can be found in Section 3.2. We conducted continuous long-term scanning detection experiments (continuous scanning for over 24 hours), which have the capability of $0\text{--}360^\circ$ detection over extended periods.



- Hardware information: Specify the hardware components used in the system. For example, the rebuttal letter mentions the PFAutoECU-APF8000R computing platform. Please list all major hardware parts, provide their model names, relevant specifications, and references.

Response: Thank you for your very professional evaluation and providing us with excellent suggestions.

The technical parameters of the main components related to the thesis topic have been included in the thesis. For details, please refer to Section 2.1. Previously, when responding to the reviewer's request, we had considered including the model numbers of the equipment hardware, but this would have made our manuscript resemble a product manual, thereby reducing the readability of the article. If the expert is interested in the detailed parameters of our hardware platform, we can explore further collaboration in the future.

- Collision avoidance explanation: The clarification provided in the rebuttal letter must be integrated into the main text. Readers should be able to understand the collision avoidance strategy without consulting the review response document.

Response: Thank you for your valuable and professional feedback.

We have described the obstacle avoidance constraints in the paper, which can be found in Section 2.3. The content regarding sensors from the previous draft has also been incorporated into the main text, as referenced in Section 2.1.

- Method limitations: A dedicated discussion of the systems limitations is still required. Describe potential challenges in real-world operation and under what conditions the method may fail or degrade in performance.

Response: We think it's a good proposal.

In light of the system limitations and potential challenges, a “Discussion” section has been added to the revised manuscript, with details provided in Section 4.

- MERRA-2 correlation: The discussion relating MERRA-2 aerosol data to lidar-measured aerosol concentrations, currently only mentioned in the rebuttal letter, must be transferred into the manuscript. Please expand and explain the relationship between the two datasets.

Response: Thank you for your professional and meticulous suggestion.

The revised manuscript has been updated accordingly, as seen in Section 3.1.

MERRA-2 reanalysis data can serve as a large-scale, long-term indirect validation reference. By comparing the overall consistency in spatiotemporal trends and vertical structures with lidar observations, it helps verify the reliability of the lidar data. However, it is important to note the inherent differences between the two in terms of data nature (model assimilation vs. direct observation) and resolution. Emphasis should be placed on trend agreement rather than absolute numerical matching. Refer to articles such as:

Wang, T., Tang, J., Sun, M., Liu, X., Huang, Y., Huang, J., Han, Y., Cheng, Y., Huang, Z., Li, J., Identifying a transport mechanism of dust aerosols over South Asia to the Tibetan Plateau: A case study. Science of The Total Environment 758,143714. <https://doi.org/10.1016/j.scitotenv.2020.143714>,.2021

Sect. 2.4. Unmanned Mobile Detection Path Planning Algorithm requires extensive editing. This section requires extensive editing to improve readability. In its

current form, it feels disconnected from the rest of the manuscript. Many readers of this journal may not have a strong background in control theory; therefore, additional plain-language explanations are needed to bridge the mathematical formulation and its application to your path-planning problem.

Response: Thank you for your professional and meticulous suggestion.

We have streamlined this section to enhance its logical coherence with the overall paper. We obtain the vehicle trajectory by solving optimal control problem.

The detailed formulation for solving the optimal control problem using the Gaussian pseudospectral method is provided below.

(1) time-domain transformations: The initial-final time domain $t \in [t_0, t_f]$ is partitioned into U subintervals. In the GPM algorithm, the intervals of time-variable discrete points are $\tau \in [-1, 1]$, so the subinterval moments need to be transformed in the time domain. For any arbitrary time instant $t \in [t_{v-1}, t_v]$ within the v -th subinterval, conversion by expression to $\tau \in [-1, 1]$.

$$\tau = \frac{2 \bullet t - (t_{v-1} + t_v)}{t_v - t_{v-1}}$$

After time-domain transformation, the optimal control problem within subinterval x can be formally stated as follows:

$$\begin{aligned} \min \psi^{(v)} &= \phi^{(v)}(\xi^{(v)}(\tau_0), t_{v-1}, \xi^{(v)}(\tau_f), t_v) + \frac{t_v - t_{v-1}}{2} \int_{\tau_0}^{\tau_f} g(\xi^{(v)}(\tau), \mu^{(v)}(\tau), \tau : t_{v-1}, t_v) d\tau \\ s.t. \begin{cases} \dot{\xi}^{(v)}(\tau) = \frac{t_v - t_{v-1}}{2} f(\xi^{(v)}(\tau), \mu^{(v)}(\tau), \tau : t_{v-1}, t_v) \\ M(\xi^{(v)}(\xi), \mu^{(v)}(\tau), \tau : t_{v-1}, t_v) \leq 0 \\ N(\xi^{(v)}(\tau_0), \tau_0, \xi^{(v)}(\tau_f), \tau_f) = 0 \end{cases} & \tau \in [-1, 1] \end{aligned}$$

In the formula, $\xi^{(v)}(\tau)$ and $\mu^{(v)}(\tau)$ represent the state variable and control variable, respectively, after time-domain transformation at time t in the v -th subinterval $[t_{v-1}, t_v]$.

(2) variable discretization: Discretize the state and control variables at Legendre-Gauss (LG) collocation points and construct Lagrange polynomial approximations to represent the states ξ and controls μ during the maneuvering process. Legendre polynomials of order N on the interval $[\tau_0, \tau_f]$ are:

$$P_N(\tau) = \frac{1}{2^N \bullet N!} \times \frac{d^N}{d\tau^N} [(\tau^2 - 1)^N]$$

With the zeros $(\tau_1, \tau_2, \tau_3, \dots, \tau_N)$ and the initial point τ_0 forming the constellation

of N+1 collocation points, the state and control variables at each collocation point are defined as:

$$\begin{cases} \xi(\tau_0), \xi(\tau_1), \dots, \xi(\tau_N) \\ \mu(\tau_0), \mu(\tau_1), \dots, \mu(\tau_N) \end{cases}$$

At N +1 collocation points, construct a Lagrange interpolating polynomial using the following basis functions.

$$\begin{cases} L_i(\tau) = \prod_{j=0, j \neq i}^N \frac{\tau - \tau_j}{\tau_i - \tau_j} \\ \tilde{L}_i(\tau) = \prod_{j=0, j \neq i}^N \frac{\tau - \tau_j}{\tau_i - \tau_j} \end{cases}$$

The approximate polynomials for the state variables, control variables, are expressed as follows:

$$\begin{cases} \xi^{(v)}(\tau) \approx \sum_{i=0}^N L_i^{(\mu)}(\tau) \bullet \zeta^{(v)}(\tau_i) \\ \mu^{(v)}(\tau) \approx \sum_{i=0}^N \tilde{L}_i^{(\mu)}(\tau) \bullet U^{(v)}(\tau_i) \end{cases}$$

By differentiating the aforementioned expression, the derivative of the state variable can be approximated using the derivative of the interpolation polynomial. For example, the derivative of the state variable at a time point can be expressed as:

$$\dot{\xi}^{(u)}(\tau_k) \approx \sum_{i=0}^N L_i^{(u)}(\tau_k) \bullet \zeta^{(v)}(\tau_i)$$

Transformation of kinematic differential equation constraints into algebraic equality constraints:

$$\dot{\xi}^{(u)}(\tau_k) - \frac{t_v - t_{v-1}}{2} f(\zeta(\tau_k), U(\tau_k), \tau_k) = 0$$

(3) Under discretization, the equality and inequality constraints for system variables are formulated respectively as:

$$\begin{cases} M(\zeta^{(v)}(\tau_k), U^{(v)}(\tau_k), \tau_k : t_{v-1}, t_v) \leq 0 \\ N(\zeta^{(v)}(\tau_0), \tau_0, \zeta^{(v)}(\tau_f), \tau_f) = 0 \end{cases}$$

Through the discretization, the optimal control problem of vehicle trajectory planning can be transformed into a nonlinear programming problem. Performance metrics can be obtained for discrete conditions:

$$\min \psi^{(v)} = \phi^{(v)}(\xi^{(v)}(\tau_0), t_{v-1}, \xi^{(v)}(\tau_f), t_v) + \frac{t_v - t_{v-1}}{2} \sum_{k=0}^N \rho_k \bullet g(\zeta(\tau_k), U(\tau_k), \tau_k)$$

In summary, through discretization at the collocation points, the Bolza optimal control problem is transformed into a standard nonlinear programming (NLP) problem. This establishes the foundation for subsequent approximate solving of optimal control problems.

You may consider presenting only the most relevant equations in the main text and moving the detailed derivations to an appendix. Suggested improvements include (but are not limited to):

(i) Add a paragraph explaining, in conceptual terms, how the equations in Sect. 2.4 determine the vehicle's motion (e.g., how obstacle detection inputs are translated into wheel angular velocities through the control laws);

Response: Thank you for your professional and meticulous suggestion.

We have added explanations in the reversed manuscript; please refer to Section 2.4.

(ii) Clarify that the performance index function quantifies the deviation between the system output and the desired reference signal.;

Response: We think it's a good proposal.

This study falls within the domain of path planning, with the objective of generating a time-optimal path from the start point to the destination, rather than focusing on path tracking. In this study, the optimization objective is formulated as a time-optimal control problem, aiming to minimize the terminal time t_f . The objective is to enable the vehicle to travel from the starting point to the destination in the shortest possible time while satisfying all physical and obstacle avoidance constraints. Unlike tracking problems that quantify deviations from a dynamic reference signal by integrating a cost function, our planning approach enforces a "zero-deviation" requirement through strict boundary constraints at the terminal time t_f . That is, the system is required to achieve exact convergence to the target point at the terminal time t_f . Any deviation from the target would violate the constraints,

thereby compelling the optimizer to adjust the control inputs and consequently eliminating terminal deviation.

(iii) expand the explanation of $N(\bullet)$ and $M(\bullet)$ of Eq. (9) in the context of path planning;

Response: Thank you for your valuable and professional feedback.

$M(\xi(t), \mu(t), t) \leq 0$ represents the inequality constraints in the vehicle's path planning, including collision avoidance constraints and the physical limitations of the vehicle system (such as maximum speed, maximum steering angle, acceleration limits, etc.). In the third line, $N(\xi(t_0), t_0, \xi(t_f), t_f) = 0$ represents the equality constraints in the vehicle's path planning, including the condition that the vehicle is stationary at both the initial and final times, as well as the continuity constraints in multi-stage planning (i.e., the terminal state of the previous stage must equal the initial state of the subsequent stage).

(iv) Explain the implications of converting the Bolza optimal control problem into a standard nonlinear programming (NLP) problem, and why this transformation is useful for real-time implementation.

Response: Thank you for your very professional evaluation and providing us with excellent suggestions.

1. **Dimensionality Reduction and Simplification:** The Bolza problem involves continuous differential equation constraints, which constitute an infinite-dimensional problem that cannot be directly solved by computers. By discretizing via the Legendre-Gauss (LG) collocation method, we approximate the continuous state and control variables using Lagrange polynomials, thereby transforming the differential equation constraints into algebraic equality constraints.

2. **By employing efficient solvers:** The transformed NLP problem exhibits a highly sparse structure, which can be rapidly solved using established nonlinear programming solvers (such as IPOPT) that leverage gradient information.

3. **Significance for real-time applications:** The GPM method offers the advantages of a fast convergence rate and strong robustness. After transforming the problem into a Nonlinear Programming (NLP) formulation, algebraic operations replace the time-consuming numerical integration process, significantly reducing computation time, which is crucial for real-time obstacle avoidance in unmanned vehicles.

In Sect. 3.3., include a 2D pcolor (time – range) plot showing the range-corrected signal as a function of time. The information in Fig. 8(a) is difficult to interpret without such a visualization.

Response: Thank you for your professional and meticulous suggestion.

A two-dimensional figure, labeled as Fig. 8(c), has been added in the revised manuscript. For details, please refer to Section 3.3.