We thank Dr. Yonggang Liu for providing precise and valuable feedback on our manuscript. Our complete response with a point-by-point reply to the comments is below in this document. The reviewer's comments are indicated in black text, and our answers follow in blue text. The revised sentences in the manuscript are indicated in red text.

This is my second review of this manuscript. The authors have addressed my previous concerns diligently and responsibly, and I believe the paper has been significantly improved. It will be ready for publication once the following two inaccurate expressions are corrected (see below).
We thank for the effort and the very helpful comments on our manuscript. We have addressed the comments by revising sentences.

Comment:
L1: "In the past, Earth experienced snowball events …"
The hypothesis of a fully ice-covered "snowball Earth" is not yet conclusively proven. To reflect this scientific uncertainty, it is recommended to use a more cautious formulation such as "its surface might have been completely covered with ice". Moreover, "where" may be replaced with "during which"
We agree on improving the description of the snowball Earth hypothesis. We have revised the sentence:
L1-2 (tracked changes file): It has been hypothesized that the Earth may have experienced snowball events in the past, during which its surface became completely covered with ice.

L119-120: the influence of sea ice thickness on albedo is actually quite strong, negligence of which might have strengthened the positive sea-ice albedo feedback and rendered the Earth more susceptible to entering a snowball state. This needs to be discussed either here or near the end of the manuscript.
We have added one sentence here to state the potential importance of sea ice albedo evolution by sea ice thickness changes, which is not considered in MIROC4m.
L98-100 (tracked changes file): In MIROC4m, the aging and thickness change of the snow and sea ice do not affect the surface albedo, which might lead to stronger sea-ice albedo feedback in a snowball initiation.

L150-151: a speed of 2 cm/sec is still very large for thick sea ice (or sea glaciers), which moves at a speed of 100 m/year. The possible influence of such high mobility of sea ice should be briefly discussed here.
We have added one sentence to state that the simulated sea ice speed is much faster than expected from sea-glacier models (Pollard and Kasting 2005; Tziperman et al. 2012), and it is necessary to use such model for quantitative discussion of the flow of ice that covers the ocean.
L130-133 (tracked changes file): The simulated horizontal flow of sea ice is much faster than that in sea-glacier modeling studies (Pollard and Kasting 2005; Tziperman et al. 2012). We note that it is necessary to incorporate the stress balance of the sea-glacier model to provide a reasonable discussion of the horizontal flow of the ice cover the global ocean.

L210-212: I do not really understand how atmosphere-ocean heat exchange is treated in the model when the ocean is completely covered by ice. In reality, the heat can only be transferred from the ocean into the atmosphere through thermal conduction within ice, which is extremely slow when ice is more than 10 m thick. When sea ice is thin, there can be many cracks or leads through which strong atmosphere-ocean heat exchange can happen, but these leads will presumably disappear once the sea ice is more than 10s of meters thick. The setup of your model could have greatly exaggerated the atmosphere-ocean heat flux and thus the sea-ice formation rate. This high sea-ice formation rate could drive the active MOC after the snowball onset. It is important for the authors to compare the atmosphere-ocean heat exchange rate to the conductive heat flux through ice, and discuss the possible consequence if the former is large.

Actually, the heat can only be transferred from the ocean into the atmosphere through thermal conduction within the sea ice in our post-snowball simulations. In MIROC4m, the air-sea heat flux in the presence of full sea ice cover is determined by the temperature difference between the surface and basal temperature of the sea ice. We have found that the simulated sea ice growth rate is consistent with the thermal conduction of sea ice, based on a 1-dimensional calculation.

Based on the TSI094 experiment, we assume

- Sea ice surface temperature same as annual mean temperature ~233 K (Figure 10a, low-latitudes)
- Sea ice basal temperature as freezing point ~273K
- thermal diffusivity of the ice 2.1 W/m/K, latent heat of ice 334000 J/kg, ice density 910 kg/m3

And if we assume sea ice thickness is 50 meters, the expected sea ice formation rate by thermal conduction within the ice is ~ 0.15 m/a, which is close to the actually simulated sea ice growth rate (Figure 1b). And as the temperature gradient in the sea ice is inversely proportional to the sea ice thickness, the sea ice growth rate slows down as sea ice thickness increases (Figure 1b).

In the revision, we have revised sentences to clarify that the atmosphere-ocean heat exchange in the snowball climate is governed by the heat conduction within the sea ice.

L186-188: Even in the globally sea ice-covered state (Fig. 3c), there is still atmosphere-ocean heat exchange that contributes to an increase in sea ice thickness (Fig. 1b). This gradual sea ice growth is caused by the thermal conduction within the sea ice, driven by the difference between the surface and basal temperature of the sea ice. As the sea ice thickens, the vertical temperature gradient in the sea ice becomes smaller, and the sea ice growth rate also becomes slower.

L295-297: it is not really clear whether the weakening of oceanic heat transport is completely due to the weakening or disappearance of NADW. The change of wind-driven gyre circulation and STC could all have some contribution. Therefore, it is better to soften the statement here.

We agree that NADW may not be the only cause of meridional heat transport changes.

We have revised the sentences:

L249-251: The oceanic meridional heat transport was weakened in TSI096 (Fig. 11b), likely related to the absence of NADW formation (Fig. 5b),…

L331-344: The discussion here is not very helpful because the major problem is there are too few experiments so that we do not know where the threshold is in your model.

We agree that four experiments may not be sufficient to confirm the threshold of snowball conditions in MIROC4m. We have revised the paragraph to focus on the fact that our experiments show that 94% solar flux was a "sufficient condition" for snowball onset and weakened the term "threshold".

L275-278: On the basis of our MIROC4m experiments, we found that 94 % of the present-day value of insolation is one sufficient condition for snowball onset for the modern Earth. Previous AOGCM studies show larger reduction in solar flux in sufficient modern snowball conditions, …

L357: "at" should be "about"

L419: "The" -> "the"

Corrected them (L289, L322)

L448-449: "Donnadieu et al. (2003) and Pollard and Kasting (2004) used ice sheet models asynchronously coupled with an AGCM …"

This description is inaccurate. To my understanding, neither of the two studies implemented a real two-way asynchronous coupling. Instead, they used outputs (temperature and precipitation) from an AGCM to drive their ice sheet models in a one-way, offline manner. Therefore, "coupled with" is misleading and should be replaced with phrasing like "...ice sheet models driven by outputs from an

AGCM...".

We agree on this point. We have corrected the sentence to clarify that both Donnadieu et al. (2003) and Pollard and Kasting (2004) present one-way, offline ice sheet model simulations. We have added one reference here (Benn et al. 2015) to show asynchronous climate-ice sheet coupling simulation.

L339-341: ice sheet modeling studies with one-way, offline ice sheet model simulations using AGCM climate fields (Donnadieu et al. 2003; Pollard and Kasting 2004) or ice sheet models asynchronously coupled with an AGCM (Benn et al., 2015) found that the development of an ice sheet in low-latitude areas is possible because of net precipitation.

L471-473: what geological records?

This sentence presents an outlook. We have revised the sentence to refer to a review study (Hoffman et al., 2017) to present a general discussion.

L359-362: A future study utilizes Sturtian and Marinoan configurations to understand the evolution of the deep-ocean circulation. Moreover, the inclusion of geochemical processes in climate models can be verified by geological and geochemical records (Hoffman et al., 2017).

Figure 2: it may be helpful plotting the MOC strength as negative.

We have revised Figure 2 and defined deep MOC (associated with AABW) strength as negative.

References added:

Pollard, D. and Kasting, J. F.: Snowball Earth: A thin-ice solution with flowing sea glaciers, Journal of Geophysical Research: Oceans, 110, 1–16, https://doi.org/10.1029/2004JC002525, 2005.

Tziperman, E., Abbot, D. S., Ashkenazy, Y., Gildor, H., Pollard, D., Schoof, C. G., and Schrag, D. P.: Continental constriction and oceanic ice-cover thickness in a Snowball-Earth scenario, Journal of Geophysical Research: Oceans, 117, https://doi.org/10.1029/2011JC007730, 2012.