Title: Numerical analysis of dynamics between debris flows and wave propagation using multi-layer shallow water equations

First of all, we would like to thank all referees for very constructive and knowledgeable reviews. The manuscript has greatly benefited from this and we are happy with comments about the scientific and applied relevance of the work. Their comments and our responses (in blue) are listed below.

Referee #1:

Landslides or debris flows rushing into reservoirs can cause surges and pose damage to dams. The manuscript uses the so-called multi-layer shallow water equations to model the interaction between debris flows and water, and numerically solves them using 1-D FVM. Although the model and method is verified in the experiments and the simulation of the 2020 Sanyang Reservoir collapse event, there are some issues that need to be considered. First, the authors mention landslides and debris flows in the introduction and aim to study the reservoir collapse event caused by landslide–debris flows. Landslides and debris flows are two different kinds of mass movements that may describe by different equations. In the left part, the authors only establish the governing equations of debris flows. Second, there is a distinct interface between debris flows and their intruding water in an ideal scenario. In most cases, debris flows would mix strongly with water. The interface between layers is not clear, and the multi-layer model has limited capability to analyze the dynamic process. Third, the novelty is not enough and the conclusions are trivial.. 1-D dimensional simulation and conventional algorithm FVM are common. Assumption of the constant erosion and deposition rates is simple. Overall, the manuscript presents a conventional study and can be improved greatly by addressing these issues.

Specific comments:

Referee's comment 1:

Introduction: What is the meaning of "landslide-debris flow"? landslide and debris flow, or debris flow transformed from landslide?

Author's Response:

We appreciate the referee's comment regarding the terminology of "landslide–debris flow." In our manuscript, we intended this expression to mean "debris flow transformed from a landslide." However, we agree that this terminology may reduce readability and cause confusion for readers. Therefore, we have carefully revised the manuscript and replaced all occurrences of "landslide–debris flow" with "debris flow" to improve clarity.

Referee's comment 2:

Introduction: "Despite the potential for large-scale complex hazards caused by landslide–debris flow events in dam/reservoir basins, they have received less attention and research than urban areas." I disagree with it. There are many studies and references related to this subject.

Author's Response:

We thank the referee for pointing out this issue. We agree with the referee that numerous studies have already addressed landslide- and debris flow-induced hazards in dam and reservoir basins. Therefore, to avoid overstatement and redundancy, we decided to delete the following sentence in the Introduction:

"Despite the potential for large-scale complex hazards caused by debris flow events in dam/reservoir basins, they have received less attention and research than urban areas."

This deletion does not affect the overall context or flow of the Introduction, and we believe the revised version provides a clearer and more accurate description of the research background.

Referee's comment 3:

Introduction: "without considering critical processes such as erosion, entrainment, and deposition"?? Many studies consider erosion, entrainment, and deposition, such as Cao, Z., Pender, G., Wallis, S., & Carling, P. (2004). Computational dam-break hydraulics over erodible sediment bed. Journal of hydraulic engineering, 130(7), 689-703. https://doi.org/10.1061/(ASCE)0733-9429(2004)13; Iverson, R. M., & Ouyang, C. (2015). Entrainment of bed material by Earth‐surface mass flows: Review and reformulation of depth‐integrated theory. Reviews of geophysics, 53(1), 27-58. https:// doi.org/10.1002/2013 RG000447; Pudasaini, S. P., & Fischer, J. T. (2020). A mechanical erosion model for two-phase mass flows. International Journal of Multiphase Flow, 132, 103416. https://doi.org/10.1016/j.ijmultiphaseflow.2020.103416.; Baggio, T., Mergili, M., & D'Agostino, V. (2021). Advances in the simulation of debris flow erosion: The case study of the Rio Gere (Italy) event of the 4th August 2017. Geomorphology, 381, 107664. https://doi.org/10.1016/j.geomorph.2021.107664

Author's Response:

We fully agree that numerous studies have already considered erosion, entrainment, and deposition processes in debris flow modeling, as the referee pointed out. Indeed, we ourselves have also investigated these mechanisms in detail in our previous works [1-4].
However, what we intended to emphasize in the Introduction is that in the context of debris-flow-induced impulsive wave analysis, most numerical approaches—such as three-dimensional Navier–Stokes solvers, or particle-based methods including SPH, DEM, and PFEM—have typically not incorporated erosion, entrainment, and deposition processes. We have revised the sentence to make this point clearer and to avoid misunderstanding.

(*Abstract*) "Debris flows are destructive disasters that often cause casualties and property damage, particularly when occurring near residential areas. In reservoir basins, debris flows can reduce effective storage capacity by raising the dead storage level and generate waves that may induce overtopping or structural collapse. Previous numerical studies have applied three-dimensional hydrodynamics or smoothed particle hydrodynamics to analyze debris-flow-induced waves. However, these studies focused on the direct inflow of soil masses into water, without accounting for erosion and entrainment processes. As a result, they have limitations in simulating debris generated in upland areas that undergo erosion and entrainment before entering reservoirs. To address this gap, we developed a one-dimensional multi-layer simulation model based on shallow water equations that explicitly considers erosion, entrainment, and deposition. The model was validated through theoretical analysis and laboratory experiments, and applied to the 2020 Sanyang Reservoir collapse event in South Korea. Scenario-based simulations were performed under varying debris flow characteristics and reservoir water levels. The results demonstrate that debris flow momentum strongly correlates with the scale of generated waves, with maximum momentum serving as a reliable predictor of wave magnitude. This study offers a practical framework for assessing debris flow impacts on dam and reservoir safety."

[1] Lee, S., An, H., Kim, M., Lee, G., and Shin, H. (2022a) Evaluation of different erosion–entrainment models in debris-flow simulation, Landslides, 19, 2075–2090, doi:10.1007/s10346-022-01901-y.
[2] Lee, S., An, H., Kim, M., Lim, H., and Kim, Y. (2022b) A simple deposition model for debris flow simulation considering the erosion-entrainment-deposition process, Remote Sens., 14, 1904, doi:10.3390/rs14081904.
[3] Lee, S., An, H., Kim, M., Lee, D., Lee, J. (2024) Debris flows analysis through quantitative evaluation of soil depth distribution under limited data. Catena, 246, 108379, doi:10.1016/j.catena.2024.108379.
[4] Jeong, S. W., Lee, S., Oh, H. J., Kim, M. (2024) Determining the debris flow yield strength of weathered soils: a case study of the Miryang debris flow in the Republic of Korea. Scientific Reports, 14(1), 20975, doi:10.1038/s41598-024-71272-y.

Referee's comment 4:

P4: Eq.1, Is the x direction normal with the slope or vertical with the earth surface? What is z in the last term? Bed elevation?

Author's Response:

Yes, that is correct. In our formulation, x represents the coordinate axis parallel to the Earth surface, while z denotes the bed elevation, which is perpendicular to x. The unit system used throughout the equations is the SI unit, i.e., meters.

Referee's comment 5:

P4: Eq.1 Why does the momentum equation of debris flow not include the loss or gain of momentum due to erosion or deposition.

Author's Response:

As the referee correctly pointed out, erosion, entrainment, and deposition processes influence not only the flow volume but also the momentum of debris flows [1]. However, many previous studies have demonstrated that even when these processes are incorporated only into the mass conservation equation related to flow depth, the resulting variation in flow height indirectly affects momentum, thereby allowing sufficient accuracy in numerical simulations [2–7].

Nevertheless, to achieve more precise simulations of debris flow behavior, it is indeed necessary to account for the direct effects of erosion, entrainment, and deposition in the momentum conservation equation, as highlighted in [1]. In the present study, however, our primary objective was to examine the applicability of the newly proposed two-layer shallow water model. For this reason, we adopted the more established and stable approach presented in [2–7], which incorporates erosion and deposition through the mass balance equation.

[1] Pudasaini, S.P., Krautblatter, M. (2021) The mechanics of landslide mobility with erosion, Nature Communication, 12, 6793, doi: 10.1038/s41467-021-26959-5.

[2] McDougall, S., Hungr, O. (2005) Dynamic modelling of entrainment in rapid landslides, Canadian Geotechnical Journal, 42, 1437–1448, doi.org:10.1139/t05-064.

[3] Sovilla, B., Burlando, P., Bartelt, P. (2006) Field experiments and numerical modeling of mass entrainment in snow avalanches, Journal of Geophysical Research, 111, F03007, doi.org:10.1029/2005JF000391.

[4] Medina, V., Hürlimann, M., Bateman, A. (2008) Application of FLATModel, a 2D finite volume code, to debris flows in the northeastern part of the Iberian Peninsula, Landslides, 5, 127–142. doi.org: 10.1007/s10346-007-0102-3.

[5] Frank, F., McArdell, B., Huggel, C., Vieli, A. (2015) The importance of entrainment and bulking on debris flow runout modeling: examples from the Swiss Alps, Natural Hazards Earth System and Science, 15, 2569–2583, doi:10.5194/nhess-15-2569-2015.

[6] Lee, S., An, H., Kim, M., Lee, G., and Shin, H. (2022a) Evaluation of different erosion–entrainment models in debris-flow simulation, Landslides, 19, 2075–2090, doi:10.1007/s10346-022-01901-y.

[7] Lee, S., An, H., Kim, M., Lim, H., and Kim, Y. (2022b) A simple deposition model for debris flow simulation considering the erosion-entrainment-deposition process, Remote Sens., 14, 1904, doi:10.3390/rs14081904.

Referee's comment 6:

P6: The authors describe their numerical scheme in three pages. If the UDCHR scheme are the same with the original reference, maybe the authors could make this section more concise.

Author's Response:

In the current version of the manuscript, we have already removed the theoretical background and stability analysis of the UDCHR scheme, leaving only the description of how it was applied to the discretization of the one-dimensional two-layer shallow water equations. We believe that further reduction would prevent readers from obtaining the minimum necessary information to understand the numerical implementation.

Referee's comments 7 & 9:

P6: "the constant erosion-entrainment rate; dz/dtd is the constant deposition rate" The erosion-entrainment rate is not constant. It depends flow depth, velocity, and other factors such as channel slope, bed composition.

Table 1: how to determine the values of tau_e and tau_d? how to get the density of debris flows? By field survey?

Author's Response:

We appreciate the referee's constructive comments. As noted, the assumption of constant erosion and entrainment rates may not fully represent actual physical processes. Nonetheless, this simplification has been widely adopted in previous studies of debris flow erosion [e.g., 1–3] and has proven effective in ensuring numerical stability and reasonable accuracy, particularly when compared with variable-rate formulations that often increase instability without clear physical justification. In this study, since our primary objective was to assess the applicability of the proposed two-layer shallow water model, we followed this established approach for consistency and stability.

For parameterization, τ_e and τ_d were determined with reference to experimental and theoretical works (Lambe and Whitman, 1991; Boukpeti et al., 2012; Jeong et al., 2018). In this context, τ_e represents the liquid limit state where soil particles are entrained into the debris flow, and τ_d denotes the transition from fluid-like to solid-like behavior. Accordingly, their values were set to 1.0 kPa and 0.5 kPa, respectively. The bulk density of debris flows was estimated from field survey data at the Sanyang Reservoir site, supported by laboratory tests on collected soil samples.

To address the referee's concern and improve clarity, we have revised the manuscript to provide further explanation of equation (6) and its applicability, as follows:

(*Lines 149-156*) "Eq. (6), derived from the static equilibrium framework of Medina et al. (2008) and Frank et al. (2015), describe erosion and entrainment when the debris flow shear stress (τ) exceeds τ_e, and deposition when τ falls below τ_d. These processes depend on soil depth and can be explained by frictional interactions between the flow and the bed, consistent with Newton's third law. When $\tau > \tau_e$, soil cohesion fails and particles are entrained; when $\tau < \tau_d$, debris flow cohesion fails and material is deposited. Here, τ_e represents the liquid limit state and τ_d the transition from solid to liquid. Following Lambe and Whitman (1991), Boukpeti et al. (2012), and Jeong et al. (2018), τ_e and τ_d were set to 1.0 kPa and 0.5 kPa, respectively. While the use of constant erosion, entrainment, and deposition rates may seem idealized, previous work (Lee et al., 2022a) showed that this assumption enhances numerical stability and accuracy compared to variable-rate models without explicit physical constraints."

[1] Medina, V., Hürlimann, M., Bateman, A. (2008) Application of FLATModel, a 2D finite volume code, to debris flows in the northeastern part of the Iberian Peninsula, Landslides, 5, 127–142. doi.org: 10.1007/s10346-007-0102-3.
[2] Frank, F., McArdell, B., Huggel, C., Vieli, A. (2015) The importance of entrainment and bulking on debris flow runout modeling: examples from the Swiss Alps, Natural Hazards Earth System and Science, 15, 2569–2583, doi:10.5194/nhess-15-2569-2015.
[3] Lee, S., An, H., Kim, M., Lim, H., and Kim, Y. (2022b) A simple deposition model for debris flow simulation considering the erosion-entrainment-deposition process, Remote Sens., 14, 1904, doi:10.3390/rs14081904.

Referee's comment 8:

P11: Voellmy miu = 0.15, zeta = 50, fint = 10. "In this context, the value of zeta is significantly lower than those used in previous studies while fint is notably higher". Why is the value significantly lower than previous studies? How to explain it?

Author's Response:

We thank the referee for this insightful comment. The reason why μ was obtained at a relatively higher value and ξ at a lower value than those reported in previous studies may be attributed to the fact that the debris flow in this study originated and propagated within a reservoir environment, rather than on natural slopes. Consequently, the debris flow experienced higher overall pressure and additional resistance from the surrounding water. In contrast, the higher calculated value of f^{int} is likely because, unlike previous studies where water-dominated flows transported shallow surface deposits such as sand or silt, the present case involved debris-dominated flows, where comparable volumes of sediment and water strongly interacted. We acknowledge that this explanation is conceptual, and further analysis—particularly through parameter sensitivity tests—will be necessary to provide more quantitative insights. To clarify this point for readers, we have added the following statement to the revised manuscript:

(*Lines 265-270*) "In this study, μ was estimated to be higher and ξ lower than in previous works, which may be due to the debris flow being generated and propagating under submerged conditions, thereby experiencing higher overall pressure and additional resistance from water. Conversely, the larger value of f^{int} is likely related to the debris-dominated nature of the flow, where comparable volumes of sediment and water interact, unlike in water-dominated flows observed in earlier studies. This explanation is conceptual, and further sensitivity analysis will be required to verify these findings."

Referee's comment 10:

Figure 4: The labels (a) (b) should be fig. 5a, fig. 5b. What kinds of the dam? It looks like an earthen dam.

Author's Response:

As suggested, we have corrected the labels in Figure 4. In addition, as the referee anticipated, the structure is indeed an earth-fill dam. To improve clarity for readers, we also revised the terminology in the manuscript by replacing fill dam with earthen dam. The revised Figure 4 is provided below for reference.

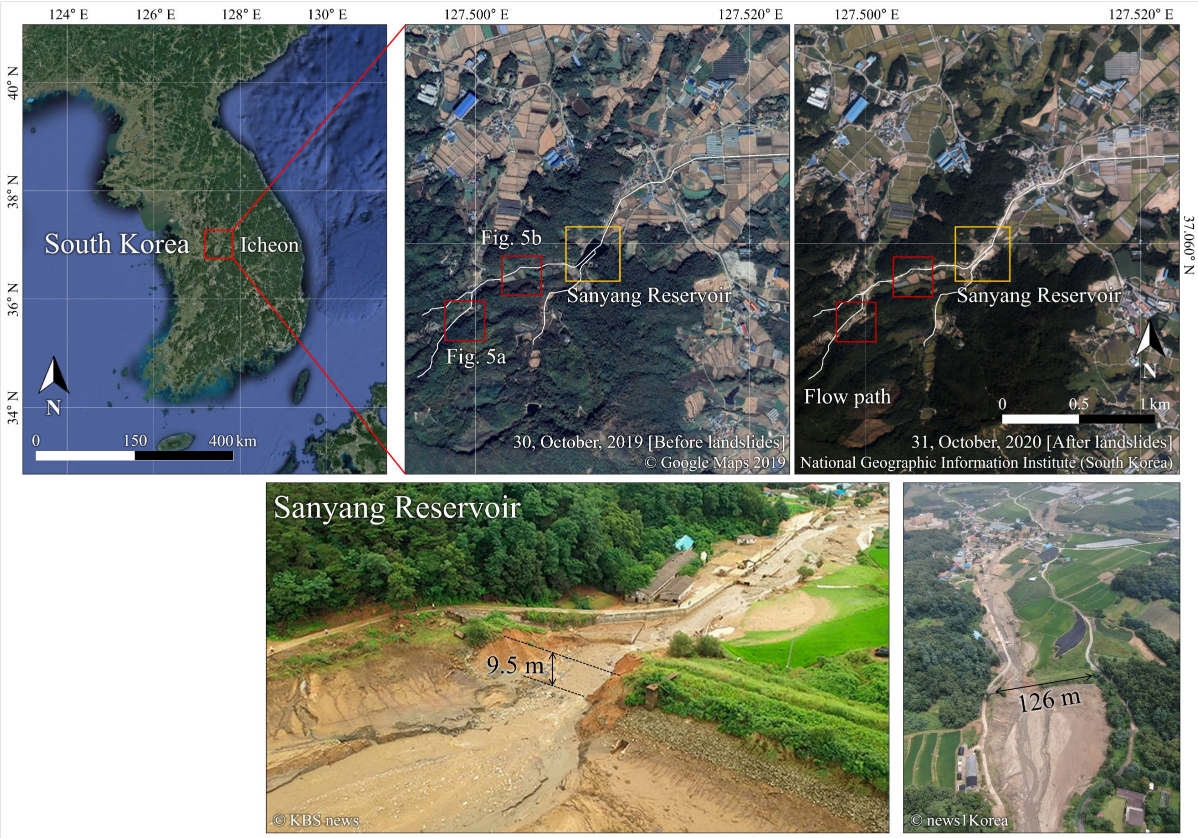

Figure 4. Study area and events: 2020 Sanyang Reservoir collapse event at Icheon in South Korea; Fig. 5a and Fig. 5b evidence of landslides and debris flows (from © Google Maps 2019, National Geographic Information Institute of South Korea, © KBS news, and © news1Korea).

Referee's comment 11:

From Figure 4 and 5, there are actually traces of bank landslides. But we are not sure whether the debris flows happened. Can you provide more pictures or evidence?

Author's Response:

Traces near the stream channel were identified, and they are considered more consistent with debris flow deposits than with landslide features. Accordingly, we have revised the figure and its caption to explicitly indicate them as debris flow traces. However, because the affected area is under the jurisdiction of Icheon City and the observed debris flow traces were located on private land, an on-site field survey could not be conducted. Instead, supplementary evidence was collected using aerial photographs and orthophotos.

Referee's comment 12:

L308: Figures 8a and 8b? maybe Figures 5a and 5b.

Author's Response:

We thank the referee for the kind observation. As correctly pointed out, this was a typographical error, and it has been corrected in the revised manuscript.

Referee's comment 13:

Figure 6c: it is longitudinal section, not cross-section.

Author's Response:

We thank the referee for pointing out this mistake. As suggested, we have corrected the typographical errors in the manuscript and ensured that the section is now properly labeled.

Referee's comment 14:

L330: Are all of slopes > 40 degree the landslide area?

Author's Response:

We agree that the adopted approach is not common and may appear less rigorous. However, in the study area, while traces of debris flow occurrence were clearly identifiable (as shown in Figure 5), it was difficult to determine the precise initiation points. Therefore, following the approach of Ma et al. (2022), we assumed that debris flows were initiated in areas with slope angles greater than 40° (154–160 m and 223–229 m), as illustrated in Figure 7. To improve clarity for readers, we have added the following statement to the revised manuscript:

(*Lines 328–330*) "In this study area, although traces of debris flow were clearly observed (Fig. 5), the precise initiation points could not be identified. Therefore, based on Ma et al. (2022), this study assumed that debris flows originated from zones with slopes greater than 40°, as illustrated in Fig. 7."

Referee's comment 15:

L301: how to determine the initial depth or volume of debris flows?

Author's Response:

We agree that the original manuscript lacked sufficient description of the geological and soil characteristics of the study area. To address this, we have revised the manuscript by adding a description to support the assumptions on soil depth and debris flow volume (Table 1). The following statement has been included:

(*Lines 301–306*) "According to the National Geographic Information Institute of Korea (NGI), the study area is widely covered with sandy loam and silty clay loam soils, with soil depths ranging from 0.5 m to 2.0 m and an average depth of approximately 1.0 m. In addition, geological survey data from the Korea Institute of Geoscience and Mineral Resources indicate that the area is predominantly underlain by Precambrian banded biotite gneiss, with minor distributions of Jurassic granitic rocks and Quaternary alluvial deposits nearby. Based on these conditions, a soil depth of 1.0 m was adopted for debris flow analysis in this study, and the failure depth was also set to 1.0 m (Table 1)."