

REVISION NOTES

We sincerely thank the reviewers for recognizing the progress made in the revised manuscript and for their further evaluation of our work. We did our best to address the unresolved issues given in the comments. As suggested by Reviewer 2, we have also revised the text to assure consistent flow between the revised and original sections. All changes are highlighted in red for clarity.

We are grateful to the editor and reviewers for your valuable input, and we hope that the revised version of the manuscript meets your expectations.

Sincerely,

The Authors

POINT-TO-POINT REPLIES TO COMMENTS OF REVIEWER #1

The authors have made significant revisions to the paper, and the new version is significantly better; however, I still feel that several comments were not fully addressed.

General Comment 1: Although there is a “good agreement” between MODFLOW and the authors’ solver, details are needed regarding the grid size sensitivity, convergence, and stability of the numerical code, as these are standard practices. That is why the added S1 does not address my comment. Stating that “We also explored the model's behaviour under different flow regimes by simulating the coexistence of turbulent and laminar flows” is not sufficient; one should show the outcome of this exploration in a quantitative way, how many iterations were required to solve the flow equation? How well did they reproduce the benchmark of MODFLOW?, etc.,

Reply: We have taken additional steps to evaluate the performance of our model. Specifically, we varied grid sizes and recharge rates and calculated the root mean square difference (RMS) in water heads at water table nodes between our solution and that obtained with MODFLOW (Harbaugh et al., 2000). The results are summarized in Table S1. Three grid sizes and three recharge rates were tested while keeping the aperture fixed. To ensure comparability between models, we used an equivalent hydraulic conductivity in MODFLOW and the corresponding discretization. The RMS difference of water heads is approximately one quarter of the grid size and increases with recharge rate as shown in Table S1. We have updated the Supplement with text and Table.

$$RMS = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2}, \quad (S2)$$

where y_i is the MODFLOW water head, and $\hat{y}_i$ is our method's water head at a water table node, and n is the number of water heads.

Table S1 The model fit between our method and the MODFLOW solution with different grid sizes and recharge rates

aperture(cm)	grid size (m)	equivalent hydraulic conductivity K (m/d)	recharge rate (mm/a)	layer-by- layer iterations	RMS difference of water heads at water table nodes (m)
0.01	10	0.00705	200	19	1.27
0.01	10	0.00705	400	23	2.45
0.01	10	0.00705	800	34	2.77
0.01	15	0.00473	400	20	2.11
0.01	20	0.00353	400	19	5.07

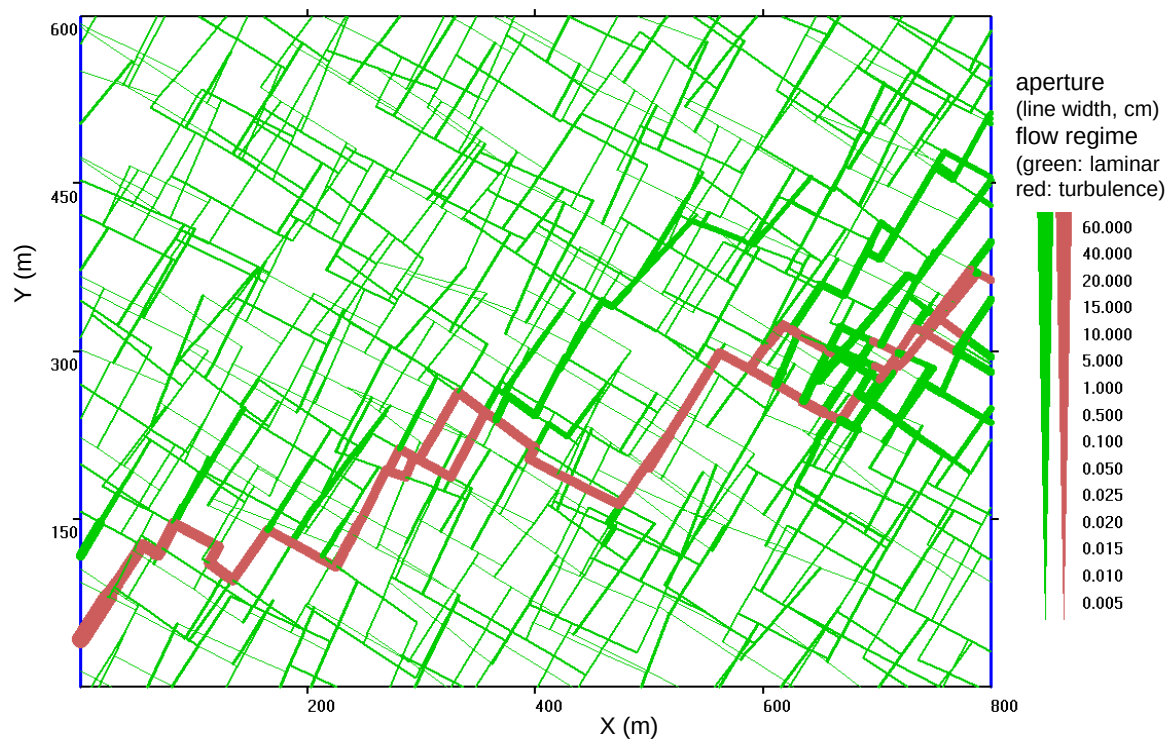


Figure S2.2: The iterative solution of turbulent and laminar flow (Jiao et. al, 2022).

Although turbulence is not present in the cases discussed in this work, the model accounts for the possible transition to turbulent flow. More details on this can be found in Jiao et al. (2022). Figure S2.2 illustrates a network exhibiting the coexistence of laminar and turbulent flow. For a typical case involving approximately 3,000 fractures, the number of iterations in the Newton–Raphson scheme is about two when the entire domain remains in the laminar regime. At the onset of turbulence, the number of iterations increases sharply to around 15. During the turbulent regime, the number of iterations is kept relatively low (below 10) by maintaining a sufficiently small evolution time step—that is, by ensuring that hydraulic properties change only slightly within each time step.

General Comment 2: In the same sense, stating that this is just one realization out of many other possibilities in response to my comment on the relatively high fracture length does not really address my comment. Every realization is one out of many possibilities, which is what makes it a realization from a statistical standpoint. That is why we aim to simulate a “representative case” and justify why it is representative, which was my request. Otherwise, we deal with a toy model intended to outline a process rather than reproduce it. This is even more important when the fracture lengths clearly dominate the whole regime, in a way that may suggest that the results presented are not representative. This should be further clarified.

Reply: Thank you for pointing this out. The fracture lengths presented in Table 1 are indeed misleading, as they show the values *before segmentation by the random procedure*. In reality, the network consists of 7,401 fracture segments, with lengths ranging from 0.01 m to 143 m. The average segment length is 15.74 m. We agree that the configuration of the initial fracture network strongly influences the evolution of the aquifer. To assess this sensitivity, we tested several realizations (Figure S5.1) and obtained conceptually identical results.

Characterizing a fracture network for a specific field site remains a significant challenge. In this study, we employed field-relevant fracture orientations, while the initial aperture and length distributions were assigned heuristically, consistent with approaches used by other researchers.

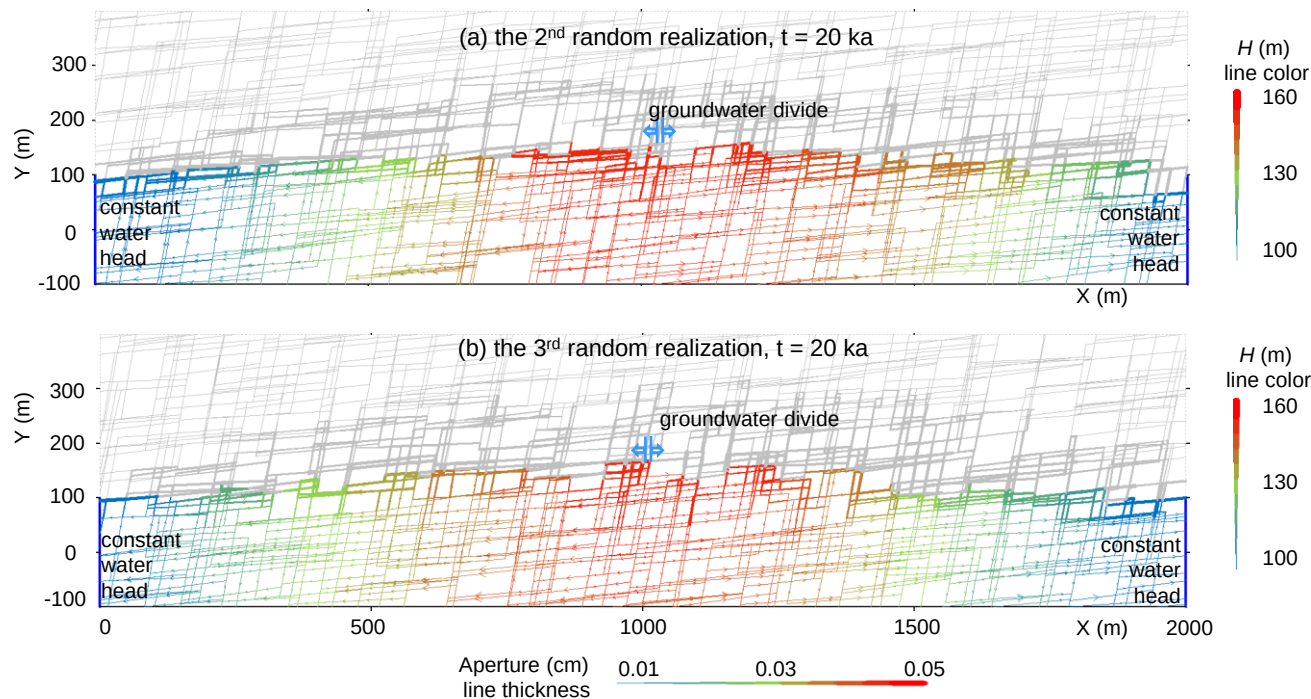


Figure S5.1: The other two random realizations of karstification in the interfluvial aquifer.

Replies to specific comments

- 1. I believe that the Yes /No aspect is missing in the “No nodes wet?” part of the chart.

Reply: We have corrected the Figure.

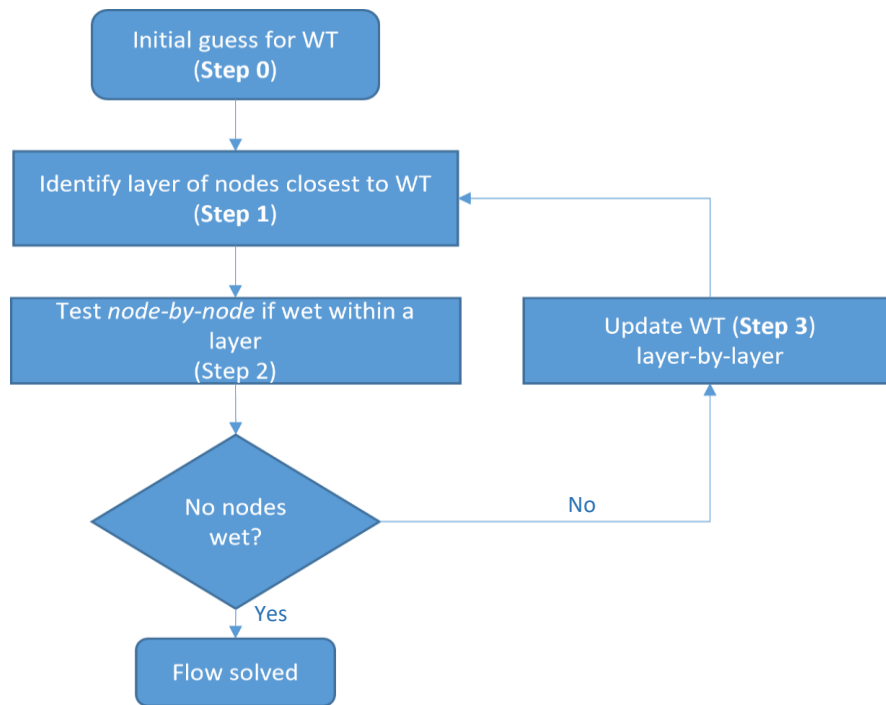


Figure S1.1: Flowchart of the flow solution.

2. “This approximation is valid for situations where the solutions are close to equilibrium, which is mostly the case in the presented scenarios”. Can you provide a reference to this approximation?

Reply: Chemical analyses of the borehole samples indicate that the water is nearly saturated with respect to calcite (Yuan et al., 2002; Wan et al., 1999), as shown in Figure S5.2. Numerous studies have reported high saturation levels of infiltrating water within the vadose zone (e.g., Fairchild & Baker, 2012). Furthermore, during the initial stages of karst aquifer development, equilibrium with calcite is typically achieved over very short distances (Dreybrodt et al., 2005). Thus, even when the initial calcium concentration is low, saturation conditions—under which such an approximation remains valid—are usually reached within a few meters.

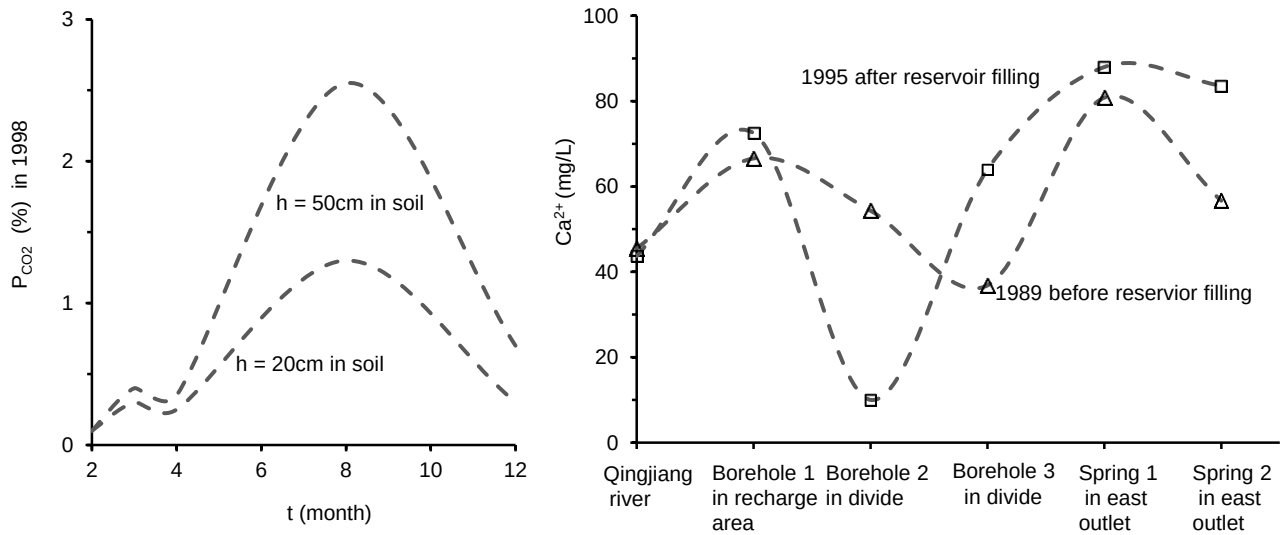


Figure S5.2: The CO_2 partial pressure in the soil at the Luojiaao observation site and the Ca^{2+} content in 3 boreholes and 2 springs in the Luojiaao interfluve aquifer (Yuan et al., 2002; Wan et al., 1999)

3. Figure 7. It seems that the heat map range does not cover the entire range (the red color is missing from the heat map).

Reply: We appreciate the observation. We have redrawn the map.

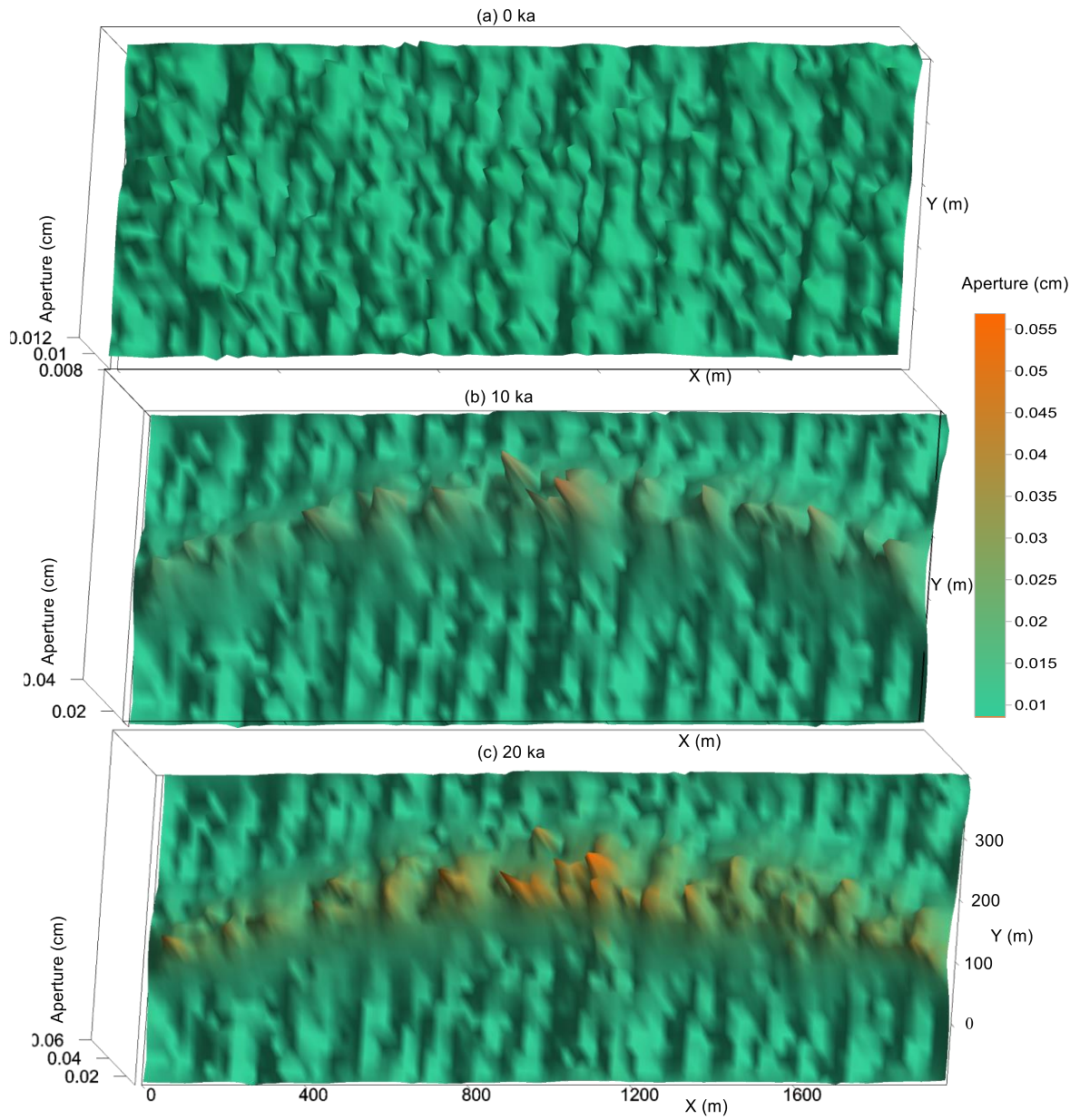


Figure 6. Evolution of fracture apertures under natural karstification at 0 ka (thousand years) (a), at 10 ka (b) and at 20 ka (c).

References:

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- Harbaugh, A. W., Banta, E. R., Hill, M. C., and McDonald, M. G.: MODFLOW-2000, the US Geological Survey modular ground-water model: User guide to modularization concepts and the ground-water flow process, U.S. GEOLOGICAL SURVEY, Open-File Report 00-92, <https://doi.org/10.3133/ofr200092>, 2000.
- Jiao, Y., Huang, Q., and Yu, Q.: Influence of initial fractures on the occurrence of karst turbulent flow, *Carsologica Sinica*, 41, 501-510, (in Chinese), <https://doi.org/10.11932/karst20220401>, 2022.
- Wan, J., Chao, N., Shen, J., and Cai, J.: Study on the carbon cycle in a karst system at the Luojiaao interfluvium along the Qingjiang River of western Hubei, China, *Carsologica Sinica*, 18, 123-128, (in Chinese), <https://doi.org/10.3969/j.issn.1001-4810.1999.02.004>, 1999.
- Yuan, D., Liu, Z., Lin, Y., Shen, J., He, S., Xu, S., Yang, L., Li, B., Qin, J., Cai, W., Cao, J., Zhang, M., Jiang, Z., and Zhao, J.: Karst dynamic system of China, Geological Publishing House, Beijing, (in Chinese), 2002.

POINT-TO-POINT REPLIES TO COMMENTS OF REVIEWER #2

The authors have sufficiently addressed my comments and I think the manuscript improved significantly. I only have a few minor comments regarding some of the modifications.

1. Please check carefully the language and style of the revised new sections, as they are often read very differently compared to the original parts, and break the flow of the text.

Reply: We appreciate this observation. We have carefully reviewed the language throughout the manuscript and made additional edits to improve readability and ensure a smooth, consistent flow between the revised and original sections. In addition to numerous minor corrections (all highlighted in red in the revised manuscript), we have made some moderate changes as listed below:

- We moved the paragraph at Line 170 (First revision) before the description of algorithm (now Line 150),
- We reformulated Section 3.1 to make it more concise and readable, and to avoid repetitions,
- We moved the paragraph between lines 275 and 285 (First revision, beginning with "Karstification represents a form...") to the Discussion and conclusion section (now Lines 350 to 360, as it discusses the results in a broader context.
- We deleted the paragraph (lines 350-355 in first revision), and included its message into Discussion and conclusion section.
- To avoid unnecessary segmentation, we have merged the Discussion and Conclusion sections into a single unified section. These previously separate sections included overlapping content that logically belonged to both.
- We have also removed a few redundant sentences that repeated similar content (see also reply to Comment 8).

Importantly, we ensured that all changes were limited to improving style and readability. No reviewed content has been altered.

2. Figure 1. - caption is a bit unclear: what do you mean here by flow solution? In b) and c) you talk about processes but here about modeling? Please clarify this.

Reply: We agree that the original wording may have been somewhat misleading. Figure 1a illustrates the position of the water table and the flow lines, which indicate the presence of a stagnant flow zone at the groundwater divide. This is a fundamental outcome of the flow field solution in such settings. To avoid confusion, we have revised the figure caption and removed the reference to “flow solution.”

3. Figure 2 - The flowchart really helps with the understanding with the methodology. It was a great idea to link the flowchart elements to the different sections, but then do it for section 2.1 as well.

Reply: Thank you for pointing this out. We have corrected the figure accordingly.

4. The text in the figure could be a bit reduced in size. I would also recommend a more figure friendly font (e.g. Arial - see the new flowchart in the supplements (Fig. R2) which reads a bit nicer due to this).

Reply: We agree. Done as suggested.

5. Section 2.2.1 - Don't forget the reference to the new supplementary figure (R2) and the inclusion of the figure to the supplements.

Reply: Thank you for a reminder. We have included the new revised figures (Figure S1.1, Table S1, Figure S2.2, Figure S5.1 and Figure S5.2) in the supplements.

6. Section 2.4 I would reduce this part significantly, because it currently reads a bit strange. Also, because this only refers to the flow solution it could be just added to the end of section 2.2.2 - because here it is confusing as a verification to the karstification too. A shorter version could be:

"The numerical model was verified successfully against a MODFLOW model and an analytical model using the Dupuit assumption. Please see the Supplements for details."

Reply: We agree and have implemented the reviewer's suggestion. The proposed text has been incorporated *verbatim* at the end of Section 2.2.2, (Line 165).

7. L394: Do you mean that different possible initial aquifer structures can be tested out using the numerical model, to see their impacts later? If so you may use the approach for scenario analysis or even assessing the uncertainties in karst evolution.

Reply: Yes, different realizations can indeed be used to explore the parameter space. We have tested several cases and added the results of two additional realizations to the Supplementary Material (Figure S5.1; see also our reply to Reviewer 1). We agree that further work in this direction would be valuable to broaden the concept, and we plan to pursue this in future studies.

8. L402: "These low karstified..." - this sentence really stands out from here, I think you can delete it (too introductory to the end of the paper and also breaks the linkage between the sentences before and after).

Reply: We agree with the comment and have deleted the sentence.