

Supplementary materials: “Centroids in second-order conservative remapping schemes on spherical coordinates”

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In the main paper, mostly the results of experiment A2 are shown and discussed. This supplement includes those of the other two experiments, A1 and A3. Although the results of A2 is presented in the main paper, they are repeated in this supplement for easy comparison.

Tables S1, S2, S3 and S4 are summary of the metrics $|L_g|$, $\|E\|_{L_2}$, $\|E\|_{L_\infty}$ and $\|E\|_{H_1}$, respectively for one-time forward remapping.

Tables S5, S6, S7 and S8 are summary of the metrics $|L_g|$, $\|E\|_{L_2}$, $\|E\|_{L_\infty}$ and $\|E\|_{H_1}$, respectively for the fields after 1000-time iterative remapping.

The tables follow those in the main paper. the second column indicates the offset longitude, where mid, edge, global correspond to the $\phi_{\text{ofs}} = (\phi_0 + \phi_1)/2$, $\phi_{\text{ofs}} = \phi_1$, and $\phi_{\text{ofs}} = 180^\circ$ cases, respectively. Destination grid sizes are $(N_\theta, N_\phi) = (90, 180)$, $(180, 360)$, $(360, 720)$, $(720, 1440)$. The digits which are different from corresponding mid-cell experiments are written in bold. When the first different digit comes after 7 from the decimal point, it is written as $(n\ d)$ where n denotes the largest order to show difference. Also, computation for some cases stops before 1000 iteration, which is denoted as ∞ in the tables.

Figures S1 and S2 are the evolution of the metrics for experiment A1 and A3, respectively, which show $\|E\|_{L_2}$, $\|E\|_{L_\infty}$ and $\|E\|_{H_1}$ through 1000-times iterative remapping.

Table S1: Summary of results of the sensitivity experiments in the paper. The absolute value of relative error in global conservation ($|L_g|$) is shown.

(a) Summary of experiment A1.

Scheme	Offset	(90, 180) [$\times 10^{-15}$]	(180, 360) [$\times 10^{-15}$]	(360, 720) [$\times 10^{-15}$]	(720, 1440) [$\times 10^{-15}$]
N	mid	0.107	0.323	0.012	0.034
N	edge	0.088	0.331	0.005	0.034
N	global	8503.	2114.	5286.	1321.
P	mid	0.155	0.302	0.023	0.026
P	edge	0.096	0.254	0.003	0.033
P	global	0.142	0.406	0.230	0.433
C	mid	0.107	0.292	0.008	0.034
C	edge	0.097	0.309	0.007	0.027
C	global	0.065	0.216	0.112	0.169

(b) Summary of experiment A2.

Scheme	Offset	(90, 180) [$\times 10^{-15}$]	(180, 360) [$\times 10^{-15}$]	(360, 720) [$\times 10^{-15}$]	(720, 1440) [$\times 10^{-15}$]
N	mid	0.989	2.261	0.565	1.272
N	edge	0.706	2.120	0.141	1.272
N	global	3199.	7960.	1990.	498.4
P	mid	0.989	1.837	0.706	1.554
P	edge	0.989	2.120	0.848	1.554
P	global	0.706	1.979	0.989	1.554
C	mid	0.848	1.979	0.565	1.554
C	edge	0.706	2.261	0.565	1.554
C	global	0.565	1.979	0.565	1.554

(c) Summary of experiment A3.

Scheme	Offset	(90, 180) [$\times 10^{-15}$]	(180, 360) [$\times 10^{-15}$]	(360, 720) [$\times 10^{-15}$]	(720, 1440) [$\times 10^{-15}$]
N	mid	10.46	9.046	9.329	7.350
N	edge	10.60	9.188	9.753	3.251
N	global	2527.	6240.	1558.	385.9
P	mid	10.46	9.188	9.188	7.209
P	edge	10.60	9.188	9.329	7.491
P	global	10.60	9.188	9.188	6.785
C	mid	10.46	9.188	10.17	6.926
C	edge	10.46	9.470	10.03	6.926
C	global	10.31	9.188	9.895	7.209

Table S2: The same as Tab. S1 but for metric $\|E\|_{L_2}$.

(a) Summary of experiment A1.

Scheme	Offset	(90, 180) [$\times 10^{-5}$]	(180, 360) [$\times 10^{-5}$]	(360, 720) [$\times 10^{-6}$]	(720, 1440) [$\times 10^{-7}$]
N	mid	5.678173	1.387396	3.140817	7.671703
N	edge	5.678174	1.387417	3.142241	8.433541
N	global	7.418609	9.084217	110.1579	4077.132
P	mid	5.678173	1.387396	3.140817	7.671703
P	edge	(11d)	(11d)	(11d)	(11d)
P	global	(12d)	(11d)	(10d)	(10d)
C	mid	5.678173	1.387396	3.140819	7.686592
C	edge	(11d)	(11d)	(11d)	(9d)
C	global	(11d)	(12d)	(12d)	(9d)

(b) Summary of experiment A2.

Scheme	Offset	(90, 180) [$\times 10^{-5}$]	(180, 360) [$\times 10^{-6}$]	(360, 720) [$\times 10^{-7}$]	(720, 1440) [$\times 10^{-7}$]
N	mid	1.098306	2.683551	6.071390	1.194096
N	edge	1.098307	2.683582	6.073436	1.329164
N	global	1.356547	15.13717	182.9107	677.0403
P	mid	1.098306	2.683551	6.071390	1.194096
P	edge	(11d)	(11d)	(10d)	(11d)
P	global	(11d)	(11d)	(10d)	(11d)
C	mid	1.098306	2.683551	6.071393	1.198306
C	edge	(12d)	(11d)	(9d)	(8d)
C	global	(11d)	(11d)	(9d)	(10d)

(c) Summary of experiment A3.

Scheme	Offset	(90, 180) [$\times 10^{-5}$]	(180, 360) [$\times 10^{-6}$]	(360, 720) [$\times 10^{-6}$]	(720, 1440) [$\times 10^{-6}$]
N	mid	3.747827	9.257410	2.194338	2.685845
N	edge	3.747832	9.258204	2.199037	2.736232
N	global	8.238006	143.6032	169.8547	617.8555
P	mid	3.747827	9.257410	2.194338	2.685845
P	edge	(12d)	(11d)	(12d)	(11d)
P	global	(12d)	(11d)	(11d)	(10d)
C	mid	3.747838	9.257520	2.194442	2.685868
C	edge	(12d)	(11d)	(11d)	(10d)
C	global	(11d)	(11d)	(11d)	(10d)

Table S3: The same as Tab. S1 but for metric $\|E\|_{L_\infty}$.

(a) Summary of experiment A1.

Scheme	Offset	(90, 180) [$\times 10^{-5}$]	(180, 360) [$\times 10^{-5}$]	(360, 720) [$\times 10^{-6}$]	(720, 1440) [$\times 10^{-6}$]
N	mid	6.721206	1.647458	3.777779	1.219158
N	edge	6.721210	1.647879	3.783340	1.222615
N	global	12.73785	24.53750	300.0424	1114.899
P	mid	6.721206	1.647458	3.777779	1.219158
P	edge	(10d)	(15d)	(15d)	(15d)
P	global	(10d)	(9d)	(9d)	(8d)
C	mid	6.721206	1.647444	3.777725	1.219063
C	edge	(15d)	(10d)	(8d)	(15d)
C	global	(9d)	(9d)	(8d)	(8d)

(b) Summary of experiment A2.

Scheme	Offset	(90, 180) [$\times 10^{-5}$]	(180, 360) [$\times 10^{-6}$]	(360, 720) [$\times 10^{-7}$]	(720, 1440) [$\times 10^{-7}$]
N	mid	1.677407	4.105416	9.388161	2.401264
N	edge	1.677408	4.106974	9.410346	2.510583
N	global	2.694874	46.92178	568.6044	2115.962
P	mid	1.677407	4.105416	9.388161	2.401264
P	edge	(10d)	(8d)	(7d)	(15d)
P	global	(9d)	(8d)	(8d)	(8d)
C	mid	1.677407	4.105464	9.388614	2.401393
C	edge	(15d)	(8d)	(8d)	(15d)
C	global	(9d)	(15d)	(8d)	(9d)

(c) Summary of experiment A3.

Scheme	Offset	(90, 180) [$\times 10^{-5}$]	(180, 360) [$\times 10^{-5}$]	(360, 720) [$\times 10^{-6}$]	(720, 1440) [$\times 10^{-5}$]
N	mid	7.854525	2.092929	5.911905	1.047242
N	edge	(10d)	(10d)	5.928570	1.057373
N	global	35.67816	95.16515	1159.350	429.5185
P	mid	7.854525	2.092929	5.911905	1.047242
P	edge	(10d)	(15d)	(15d)	(11d)
P	global	(11d)	(9d)	(7d)	(8d)
C	mid	7.854547	2.092922	5.913253	1.046135
C	edge	(9d)	(15d)	(8d)	(15d)
C	global	(10d)	(10d)	(8d)	(7d)

Table S4: The same as Tab. S1 but for metric $\|E\|_{H_1}$.

(a) Summary of experiment A1.

Scheme	Offset	(90, 180) [$\times 10^{-4}$]	(180, 360) [$\times 10^{-5}$]	(360, 720) [$\times 10^{-5}$]	(720, 1440) [$\times 10^{-5}$]
N	mid	3.192491	7.859755	2.374764	7.364183
N	edge	3.192502	7.870065	2.538142	8.329136
N	global	10.35066	468.8418	1042.859	4529.672
P	mid	3.192491	7.859755	2.374764	7.364183
P	edge	(11d)	(11d)	(11d)	(9d)
P	global	(12d)	(11d)	(10d)	(9d)
C	mid	3.192491	7.859799	2.375813	7.451696
C	edge	(13d)	(11d)	(9d)	(11d)
C	global	(11d)	(10d)	(9d)	(9d)

(b) Summary of experiment A2.

Scheme	Offset	(90, 180) [$\times 10^{-5}$]	(180, 360) [$\times 10^{-5}$]	(360, 720) [$\times 10^{-6}$]	(720, 1440) [$\times 10^{-6}$]
N	mid	4.776152	1.173408	3.381292	9.707323
N	edge	4.776172	1.175321	3.696067	11.67384
N	global	16.95678	77.74871	1731.057	7520.313
P	mid	4.776152	1.173408	3.381292	9.707323
P	edge	(11d)	(10d)	(8d)	(8d)
P	global	(10d)	(10d)	(8d)	(7d)
C	mid	4.776152	1.173418	3.383763	9.928686
C	edge	(10d)	(11d)	(8d)	(7d)
C	global	(12d)	(10d)	(9d)	(8d)

(c) Summary of experiment A3.

Scheme	Offset	(90, 180) [$\times 10^{-4}$]	(180, 360) [$\times 10^{-4}$]	(360, 720) [$\times 10^{-4}$]	(720, 1440) [$\times 10^{-4}$]
N	mid	9.042237	2.350035	1.030696	4.042293
N	edge	9.042256	2.351017	1.040087	4.085654
N	global	23.39566	80.15887	164.7841	701.6693
P	mid	9.042237	2.350035	1.030696	4.042293
P	edge	(11d)	(10d)	(10d)	(10d)
P	global	(11d)	(10d)	(11d)	(10d)
C	mid	9.042265	2.350064	1.030713	4.043904
C	edge	(12d)	(11d)	(12d)	(11d)
C	global	(11d)	(11d)	(10d)	(10d)

Table S5: The same as Tab. S1 but for metric $|L_g|$ after 1000-time iterative remapping.

(a) Summary of experiment A1.

Scheme	Offset	(90, 180) [$\times 10^{-15}$]	(180, 360) [$\times 10^{-15}$]	(360, 720) [$\times 10^{-15}$]	(720, 1440) [$\times 10^{-15}$]
N	mid	67.98	464.1	27.70	43.86
N	edge	93.79	474.2	32.54	43.81
N	global	6308.	3773.	∞	∞
P	mid	23.87	440.4	17.25	51.03
P	edge	82.69	395.1	42.87	43.68
P	global	37.18	549.6	188.8	355.0
C	mid	71.58	432.8	29.69	45.61
C	edge	85.51	448.5	31.60	51.65
C	global	118.7	354.1	151.8	249.4

(b) Summary of experiment A2.

Scheme	Offset	(90, 180) [$\times 10^{-15}$]	(180, 360) [$\times 10^{-15}$]	(360, 720) [$\times 10^{-15}$]	(720, 1440) [$\times 10^{-15}$]
N	mid	227.3	1388.	103.0	150.1
N	edge	76.33	1458.	52.72	136.2
N	global	2392.	4327.	∞	∞
P	mid	269.2	1372.	34.06	149.5
P	edge	325.9	1400.	17.52	134.0
P	global	43.25	1519.	25.86	134.2
C	mid	160.4	1421.	26.85	140.0
C	edge	141.7	1366.	34.49	147.4
C	global	232.8	1498.	69.26	137.9

(c) Summary of experiment A3.

Scheme	Offset	(90, 180) [$\times 10^{-15}$]	(180, 360) [$\times 10^{-15}$]	(360, 720) [$\times 10^{-15}$]	(720, 1440) [$\times 10^{-15}$]
N	mid	138.9	1306.	123.6	132.7
N	edge	22.89	1369.	76.75	115.9
N	global	1683.	1638.	∞	∞
P	mid	224.3	1292.	57.10	132.1
P	edge	255.4	1318.	36.32	113.5
P	global	34.49	1428.	53.57	121.2
C	mid	106.3	1343.	46.50	122.2
C	edge	60.64	1289.	50.46	130.7
C	global	192.2	1420.	71.52	134.7

Table S6: The same as Tab. S2 but for metric $\|E\|_{L_2}$ after 1000-time iterative remapping.

(a) Summary of experiment A1.					
Scheme	Offset	(90, 180) [$\times 10^{-4}$]	(180, 360) [$\times 10^{-4}$]	(360, 720) [$\times 10^{-5}$]	(720, 1440) [$\times 10^{-5}$]
N	mid	5.529522	1.141016	8.849167	1.984378
N	edge	5.529917	1.148703	9.173183	3.845315
N	global	90821.45	∞	∞	∞
P	mid	5.529522	1.141016	8.849167	1.984378
P	edge	(9d)	(9d)	(10d)	(10d)
P	global	(9d)	(10d)	(8d)	(10d)
C	mid	5.516203	1.144484	8.877811	2.000286
C	edge	(9d)	(10d)	(10d)	(10d)
C	global	(9d)	(10d)	(9d)	(9d)
(b) Summary of experiment A2.					
Scheme	Offset	(90, 180) [$\times 10^{-5}$]	(180, 360) [$\times 10^{-5}$]	(360, 720) [$\times 10^{-5}$]	(720, 1440) [$\times 10^{-6}$]
N	mid	3.404520	1.242931	1.153953	2.549738
N	edge	3.406271	1.261852	1.221960	6.049095
N	global	121409.7	∞	∞	∞
P	mid	3.404520	1.242931	1.153953	2.549738
P	edge	(7d)	(10d)	(10d)	(9d)
P	global	(8d)	(8d)	(9d)	(9d)
C	mid	3.379145	1.249970	1.162807	2.600785
C	edge	(9d)	(10d)	(9d)	(9d)
C	global	(9d)	(9d)	(9d)	(8d)
(c) Summary of experiment A3.					
Scheme	Offset	(90, 180) [$\times 10^{-2}$]	(180, 360) [$\times 10^{-3}$]	(360, 720) [$\times 10^{-4}$]	(720, 1440) [$\times 10^{-4}$]
N	mid	2.542678	3.742836	5.796647	1.050506
N	edge	(7d)	3.742923	5.812058	1.164832
N	global	2.786510	∞	∞	∞
P	mid	2.542678	3.742836	5.796647	1.050506
P	edge	(11d)	(10d)	(8d)	(11d)
P	global	(12d)	(10d)	(8d)	(10d)
C	mid	2.546099	3.748990	5.805445	1.051638
C	edge	(11d)	(11d)	(12d)	(12d)
C	global	(12d)	(11d)	(10d)	(10d)

Table S7: The same as Tab. S3 but for metric $\|E\|_{L_\infty}$ after 1000-time iterative remapping.

(a) Summary of experiment A1.					
Scheme	Offset	(90, 180) [$\times 10^{-3}$]	(180, 360) [$\times 10^{-4}$]	(360, 720) [$\times 10^{-4}$]	(720, 1440) [$\times 10^{-5}$]
N	mid	1.143672	1.976176	1.849959	4.872257
N	edge	1.145761	1.977551	1.850817	10.42006
N	global	591567.4	2516259.	∞	∞
P	mid	1.143672	1.976176	1.849959	4.872257
P	edge	(12d)	(7d)	(8d)	(7d)
P	global	(11d)	(8d)	(9d)	(7d)
C	mid	1.134460	1.972254	1.850083	4.870799
C	edge	(11d)	(9d)	(8d)	(8d)
C	global	(11d)	(8d)	(9d)	(7d)
(b) Summary of experiment A2.					
Scheme	Offset	(90, 180) [$\times 10^{-5}$]	(180, 360) [$\times 10^{-5}$]	(360, 720) [$\times 10^{-5}$]	(720, 1440) [$\times 10^{-6}$]
N	mid	7.229935	2.836765	3.495363	8.932623
N	edge	7.337693	2.839733	3.505998	21.10220
N	global	1227662.	5312862.	∞	∞
P	mid	7.229935	2.836765	3.495363	8.932623
P	edge	(7d)	(8d)	(10d)	(7d)
P	global	(7d)	(9d)	(8d)	(7d)
C	mid	7.292156	2.831743	3.494677	8.934358
C	edge	(8d)	(9d)	(8d)	(8d)
C	global	(8d)	(8d)	(8d)	(9d)
(c) Summary of experiment A3.					
Scheme	Offset	(90, 180) [$\times 10^{-2}$]	(180, 360) [$\times 10^{-2}$]	(360, 720) [$\times 10^{-3}$]	(720, 1440) [$\times 10^{-4}$]
N	mid	7.739175	1.106519	2.235436	5.166085
N	edge	7.739280	1.106520	2.290092	6.569416
N	global	8.024978	7186866.	∞	∞
P	mid	7.739175	1.106519	2.235436	5.166085
P	edge	(10d)	(9d)	(10d)	(9d)
P	global	(10d)	(9d)	(8d)	(7d)
C	mid	7.748931	1.107826	2.225731	5.166796
C	edge	(10d)	(9d)	(9d)	(9d)
C	global	(9d)	(9d)	(8d)	(7d)

Table S8: The same as Tab. S4 but for metric $\|E\|_{H_1}$ after 1000-time iterative remapping.

(a) Summary of experiment A1.					
Scheme	Offset	(90, 180) [$\times 10^{-2}$]	(180, 360) [$\times 10^{-3}$]	(360, 720) [$\times 10^{-3}$]	(720, 1440) [$\times 10^{-3}$]
N	mid	1.156071	3.635854	8.923197	3.487481
N	edge	1.156152	3.657658	9.127597	5.592652
N	global	927295.6	∞	∞	∞
P	mid	1.156071	3.635854	8.923197	3.487481
P	edge	(12d)	(9d)	(11d)	(10d)
P	global	(7d)	(8d)	(8d)	(8d)
C	mid	1.146111	3.628485	9.005739	3.576232
C	edge	(11d)	(9d)	(10d)	(9d)
C	global	(10d)	(10d)	(8d)	(9d)
(b) Summary of experiment A2.					
Scheme	Offset	(90, 180) [$\times 10^{-4}$]	(180, 360) [$\times 10^{-4}$]	(360, 720) [$\times 10^{-3}$]	(720, 1440) [$\times 10^{-4}$]
N	mid	7.290697	4.128391	1.173717	4.511081
N	edge	7.294201	4.179597	1.216562	8.569447
N	global	1239478.	∞	∞	∞
P	mid	7.290696	4.128392	1.173717	4.511081
P	edge	(7d)	4.128393	(9d)	(8d)
P	global	(7d)	4.128393	(7d)	(7d)
C	mid	7.059105	4.147361	1.195936	4.737624
C	edge	(7d)	4.147360	(8d)	(7d)
C	global	(7d)	4.147360	(8d)	(7d)
(c) Summary of experiment A3.					
Scheme	Offset	(90, 180) [$\times 10^{-1}$]	(180, 360) [$\times 10^{-1}$]	(360, 720) [$\times 10^{-2}$]	(720, 1440) [$\times 10^{-2}$]
N	mid	5.996567	1.098501	5.243229	2.025716
N	edge	5.996568	1.098548	5.253945	2.132798
N	global	7.093377	∞	∞	∞
P	mid	5.996567	1.098501	5.243229	2.025716
P	edge	(10d)	(12d)	(10d)	(10d)
P	global	(11d)	(11d)	(11d)	(9d)
C	mid	6.004536	1.100441	5.248092	2.027478
C	edge	(11d)	(12d)	(10d)	(12d)
C	global	(10d)	(11d)	(10d)	(10d)

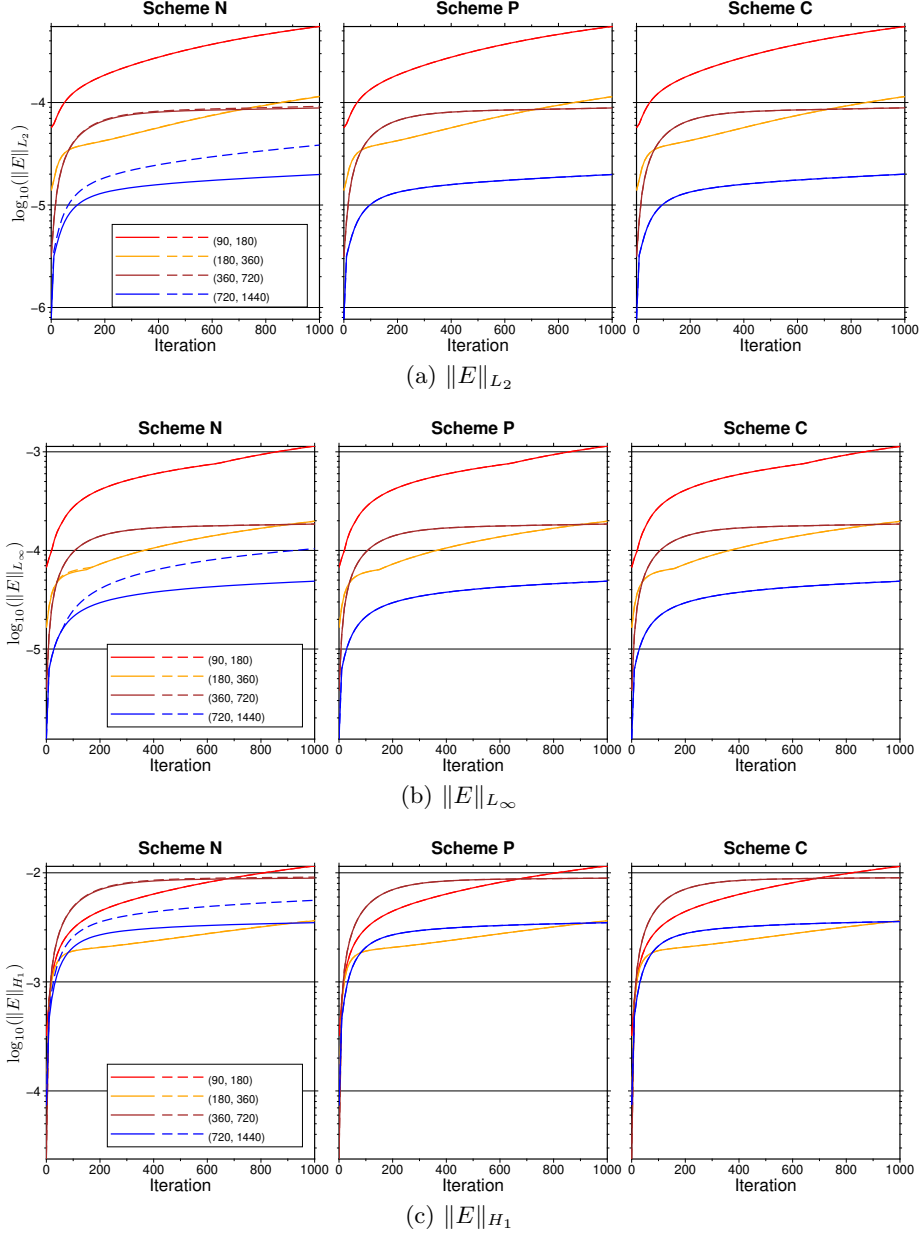


Figure S1: Results of the sensitivity experiments A1 for the offset longitude using Schemes N, P, C. Destination grid sizes are $(N_\theta, N_\phi) = (90, 180), (180, 360), (360, 720), (720, 1440)$. Solid and dashed lines indicate that $\phi_{\text{ofs}} = (\phi_0 + \phi_1)/2$ and $\phi_{\text{ofs}} = \phi_1$ cases, respectively, and mostly they are overlapped except for some particular cases. Global offset cases ($\phi_{\text{ofs}} = 180^\circ$) are excluded from the plot, because the metrics explode at early stage for Scheme N, and because the metrics overlaps with other cases for Scheme P and C. The metric $\|E\|_{L_2}$ as a function of iteration number of two-way remapping is shown.

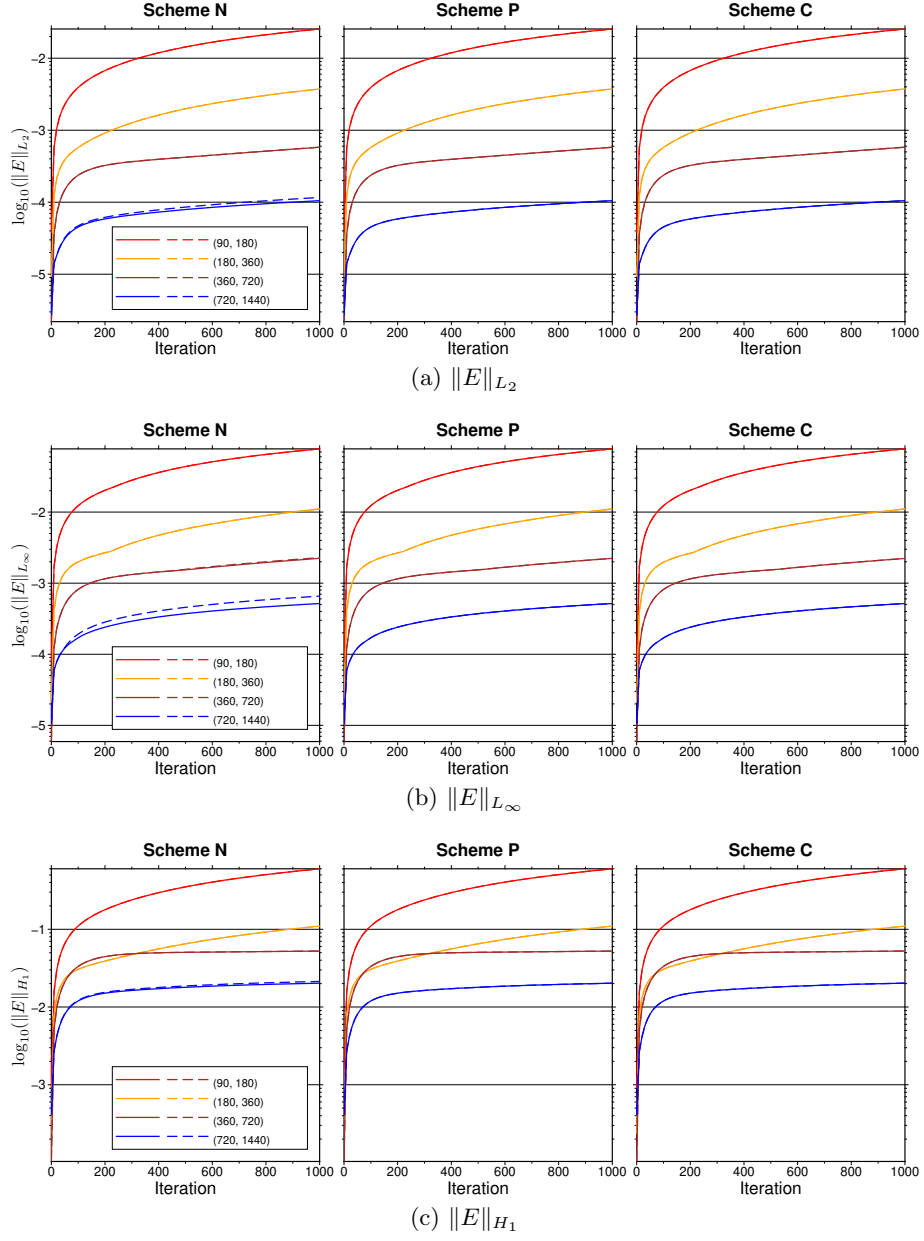


Figure S2: The same as Fig. S1 but for experiment A3.