

# Thermobaric circulation in a deep freshwater lake: A conceptual 1D model

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**Abstract.** Numerical lake models are a powerful tool to optimize water management and mitigate changes due to climate change. Hence, detailed implementation of lake specific processes is crucial to ensure optimal results. However, common numerical lake models have so far omitted the effect of thermobaricity despite its significant influence on deep water circulation in deep lakes. The thermobaric effect is based on the temperature dependence of the compressibility of water. As a consequence, deep water can be significantly colder than 4 °C and deep water renewal becomes complex. For a proper investigation, numerical models can be appropriate tools to display and understand such processes better. Inspired by Lake Shikotsu, which is an excellent example for the influence of thermobaricity, we developed a simplified 1D model for thermobaric effects. Here, we used in situ density to replace potential density for stability considerations such as the Brunt-Väisälä frequency. To prevent any competing influences and isolate thermobaric effects, we excluded any external forcing except for the surface temperature input. Accordingly, we excluded salinity, chose a cylindrical bathymetry without shallow areas, and omitted any inflows. Therefore, the model reproduced deep water circulation solely based on thermal forcing at the surface. We were able to identify key features of the deep water circulation: cabbeling occurs at the intersection of the temperature profile with the  $T_{md}$  (temperature of maximum density) line due to diffusion and induces thermobaricity driven deep water circulation; this deep water circulation cell is detached from the surface, but can extend over hundreds of meters to the lake bed; the deep water stays isothermal; and after the winter stratification the temperature profile aligns with the  $T_{md}$  line. Additionally, we investigated the influence of previous deep water renewal events and the current surface temperature on the deep water circulation. Our results emphasize the feasibility and necessity of the implementation of thermobaricity in numerical lake models by basing stability (Brunt-Väisälä frequency) on in situ density.

## 1 Introduction

Across all climate zones, lakes and reservoirs are undergoing changes due to climate change (Adrian et al., 2009; Sun et al., 2025) and heavy human impact (Søndergaard and Jeppesen, 2007; Weyhenmeyer et al., 2024). These water bodies provide central services such as water and food supply, energy production, flood prevention, and space for recreation. In particular, the provision of high quality drinking water is becoming an increasing challenge that humanity has to deal with (Delpla et al., 2009). This accounts globally and awareness is growing. Hence, to ensure optimal management of lakes and reservoirs in

25 the future, proper knowledge of processes and the prediction of changes are crucial. Numerical modeling of lakes serves as a central tool to predict future development and optimize management strategies (e.g. Weber et al., 2019; Mi et al., 2022) to restore and maintain a healthy ecological state of our natural environment. Good management can help adapt our strategies and mitigate the influences of climate change and direct human impact at least in part (e.g. Winton et al., 2019; Regev et al., 2025).

30 Numerical models for lakes are often based on assumptions originating from oceanography. As a consequence, the inclusion of lake specific properties of limnic waters can improve simulations of lakes fundamentally. This spans from using local weather conditions to implementing lake specific solute compositions in density functions (Pawlowicz and Feistel, 2012; Moreira et al., 2016). In this paper, we want to demonstrate the necessity of including the temperature dependence of the compressibility in the water properties, which is called thermobaricity (McDougall, 1987). Thermobaricity has so far not been dealt with in the commonly used lake models. However, some ocean models changed to in situ density when they changed to the new ocean  
35 standard TEOS-10 (IOC et al., 2010), which includes the effect.

The effect of thermobaricity is based on the temperature dependence of the compressibility of water (which is tantamount to the pressure dependence of the thermal expansion of water,  $\partial^2 \rho_{\text{in-situ}} / (\partial \theta \partial p)$ ) (McDougall, 1987). For pure water, this leads to a decrease of the temperature of maximum density  $T_{\text{md}}$  from approximately 3.98 °C at atmospheric pressure by about 0.02 °C bar<sup>-1</sup> (Chen and Millero, 1977). For instance, pure water at the pressure of 500 m depth has its temperature of  
40 maximum density at about 3 °C. Ultimately, water colder than 3.98 °C is less dense than slightly warmer water at the surface but denser from a certain depth where  $T_{\text{md}}$  is low enough. As a consequence, we find deep water significantly colder than 3.98 °C in deep lakes of the temperate climate zone (e.g. Carmack and Weiss, 1991; Crawford and Collier, 1997; Boehrer et al., 2009, 2013).

Also connected to the temperature of maximum density, the process of cabbeling differs from thermobaricity (McDougall,  
45 1987; Carmack and Weiss, 1991; Grace et al., 2023a, b). Cabbeling occurs where mixing of two water parcels across the temperature of maximum density results in even denser water, which itself drives convective circulation, e.g. in the case of thermal bars (Ivey and Hamblin, 1989; Shimaraev et al., 1993). Although deep water renewal in deep lakes experiencing surface water temperatures below 3.98 °C is controlled mainly by thermobaricity, also cabbeling may be involved in the deep mixing and deep water formation.

50 The vertical progression of cold surface water to the abyss is complex and the depiction of density differences becomes difficult as the convenient property of potential density, namely having one reference pressure, is lost when thermobaricity is dealt with. For proper stability considerations, the densities of two neighbouring water parcels need to be compared at intermediate identical pressure conditions, i.e. at the pressure at which the two water parcels are and interact (Ekman, 1934). For this, the adiabatic change of the water parcels from their origin towards the comparison pressure has to be considered  
55 (Osborn and LeBlond, 1974). We show that using in situ density for stability considerations is a proper approach if this adiabatic change is considered.

External forcing by strong winds as driving force for the deep water renewal has been discussed (e.g. Weiss et al., 1991; Boehrer et al., 2013). Here, these winds push cold water beyond the compensation depth and from there it can proceed sinking due to its higher in situ density compared to the surrounding water up to a depth with equally dense water. Killworth et al.

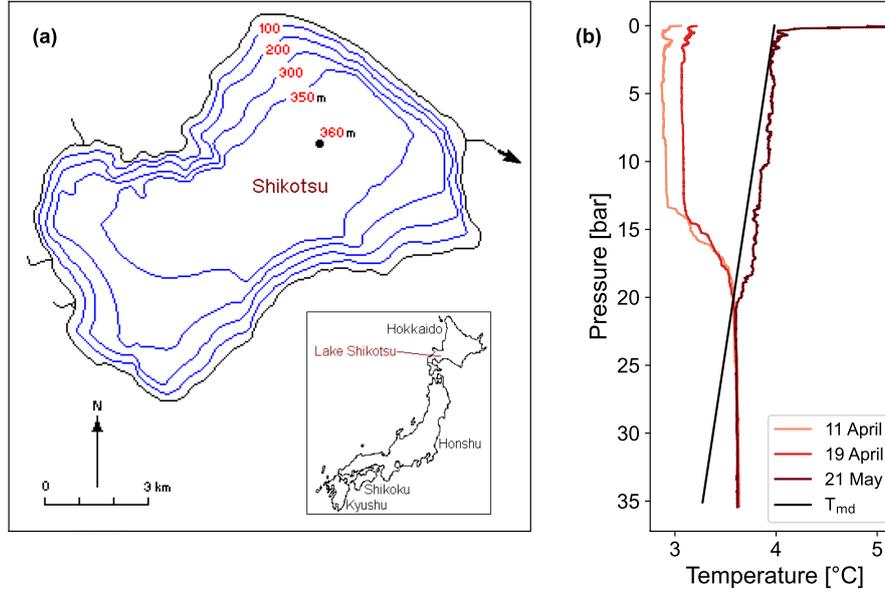
60 (1996) and Piccolroaz and Toffolon (2013) created models to describe such deep water renewal in Lake Baikal. They used wind as external forcing to represent downwelling under conditions of thermobaricity. While Killworth et al. (1996) used a two dimensional model to include the wind in combination with a one dimensional model for vertical tracer distributions, Piccolroaz and Toffolon (2013) parametrized the wind forcing to calculate the forced downwelling before assessing the stability. In contrast to them, we excluded forced plume downwelling while allowing for cabbeling to present the existence of thermobaricity  
65 controlled deep water mixing that is purely one dimensional and not driven by wind. Additionally, we wanted to emphasize the possibility of directly using in situ density for stability considerations to include thermobaricity in lake models by comparing two water parcels at a common depth. Piccolroaz and Toffolon (2013) also used a common depth for comparison but only considered the adiabatic change in in situ temperature, while we could avoid this additional calculation by referring to potential temperatures only.

70 In our model, we wanted to ensure that there was no interference of other effects with the effects of thermobaricity and no misinterpretation of thermobaric effects as secondary effects of other driving forces. Therefore, we decided to keep everything at the minimum complexity to depict the thermobaric effects as well isolated as possible. Consequently, we considered a cylindrical water domain without lateral gradients (on the considered length scale) for our 1D model. We prohibited shallow areas for the formation of waters of different properties, valleys to guide submerged flows, and horizontal gradients at the  
75 surface. Additionally, we implemented pure water properties (no salinity). Lake Shikotsu, Hokkaido, Japan, served as our inspiration, because thermobaric stratification has been documented there (Boehrer et al., 2008, 2009). Hence, we chose the temperature range and size of the basin accordingly, hoping that our numerical 1D model would manage to reproduce water circulation patterns and temperature profiles with some similarities to Lake Shikotsu. As we had presumed that the deep water circulation in this lake could be understood as a one dimensional circulation, we believed that our parsimonious model  
80 should represent the fundamental features there. Nevertheless, we expected not too close similarity, since we abstained from including any detailed information of bathymetry or forcing. Even though Lake Shikotsu is a particularly nice representation of thermobaricity and the necessity of including this effect is obvious, we hope to convince the readers of the importance of including temperature dependent compressibility (thermobaricity) in numerical models, if they want to have a realistic simulation of deep water movements in lakes with deep water temperatures close to the temperature of maximum density.

## 85 **2 Methods**

### **2.1 Inspiration**

Our 1D model was inspired by Lake Shikotsu, Hokkaido, Japan. This lake is located in the temperate zone and consistently experiences surface water temperatures below 3.98 °C during winter. It is a volcanic caldera lake of collapse type, formed by the great eruption of Shikotsu Volcano about 40,000 years ago. Because of that, the lake has a roughly cylindrical shape, very  
90 steep sidewalls, and a maximum depth of 360 m (Fig. 1a). Boehrer et al. (2008, 2009) investigated the deep water circulation and its dependency on thermobaricity. The temperature profiles, measured in 2005, clarify this dependency (Fig. 1b). They cross the  $T_{md}$  line during the winter stratification and resemble it above this intersection shortly after. At the same time, the



**Figure 1.** Location, bathymetry and temperature profiles from 2005 of Lake Shikotsu. (a) presents the bathymetry map of Lake Shikotsu and its location in Japan (adapted from Boehrer et al., 2009), while (b) shows temperature profiles from 2005 (data from Boehrer et al., 2008) and the temperature of maximum density  $T_{md}$ .

deep water temperature below the intersection stays isothermal throughout. The very steep sidewalls and comparatively small horizontal extent of the lake, as well as its symmetric shape, suggest a rather one dimensional governance of the system.

95 Missing shallow areas along the shore and the uniform slope without any trenches make the accumulation and downwelling of cold water plumes induced by external forcing unlikely.

## 2.2 Density and Stability

Potential density  $\rho_{pot} = \rho_{pot}(T)$  is a function of (potential) temperature at a certain reference pressure. In this paper, we follow the usual oceanographic and limnological practice of referring to atmospheric pressure (though sometimes other pressure references have been recommended for use, e.g. Ekman, 1934; Osborn and LeBlond, 1974). On the contrary, densities calculated

100 at other pressures than atmospheric are referred to as in situ density  $\rho_{in-situ} = \rho_{in-situ}(T, p)$  in this paper.

To start with, the sound speed  $c$  is directly connected with the compressibility of water  $\beta_S$  by

$$\beta_S = \frac{1}{\rho_{in-situ}} \left( \frac{\partial \rho_{in-situ}}{\partial p} \right)_S = \frac{1}{\rho_{in-situ} \cdot c^2} \quad (1)$$

(Meschede, 2015), where  $p$  is the pressure and the index S indicates an adiabatic process, and therefore represents the temperature dependency of the compressibility (thermobaricity). Hence, we used this to obtain a formulation of the in situ density.

105

According to Eq. (1), the sound speed can be expressed by

$$\frac{1}{c^2} = \left( \frac{\partial \rho_{\text{in-situ}}}{\partial p} \right)_S. \quad (2)$$

From Eq. (2) the in situ density can be derived as

$$\rho_{\text{in-situ}} = \rho_{\text{pot}} + \int_0^p \frac{1}{c^2} dp. \quad (3)$$

110 Here, in situ density and sound speed depend on temperature, salinity, and pressure while potential density, here with the reference pressure of 0 bar at the water surface, only depends on temperature and salinity. In our approach, we exclude salinity to prevent distraction by competing effects. Hence, salinity is disregarded in the following and we use the formulations of Tanaka et al. (2001) for potential density and Belogol'skii et al. (1999) for sound speed. We are fully aware that for limnic water the solutes must be included in the water properties for the calculation of density (Boehrer et al., 2010; Moreira et al., 115 2016) and sound speed when salinity is considered for a fully realistic representation.

Equation (3) is then further simplified by applying a linear approximation for  $\frac{1}{c^2}$  with respect to pressure

$$\frac{1}{c^2(p)} = mp + n, \text{ with } m = \frac{\partial \frac{1}{c^2(p)}}{\partial p} \text{ and } n = \frac{1}{c^2(p=0)}. \quad (4)$$

We used the surface and bottom values for linearization of the modeled depth range up to 360 m (compare Sect. 2.3). Hence, the maximum deviation occurs at mid depth, regardless of the temperature. For the interesting temperature range close to the 120 temperature of maximum density  $T_{\text{md}}$ , between 3 and 4 °C, a maximum deviation of  $2.13 \cdot 10^{-12} \text{ s}^2 \text{ m}^{-2}$  of the sound speed occurs for 3 °C. This deviation is small relative to the change of  $\frac{1}{c^2}$  with respect to temperature or pressure, which are in the order of  $10^{-9} \text{ s}^2 \text{ m}^{-2}$ .

The final formulation of the in situ density is

$$\rho_{\text{in-situ}}(T, p) = \rho_{\text{pot}}(T) + \frac{p^2}{2} \frac{\partial \frac{1}{c^2(T, p)}}{\partial p} + \frac{p}{c^2(T, p=0)}. \quad (5)$$

125 While we clearly distinguish between  $\rho_{\text{in-situ}}(T, p)$  and  $\rho_{\text{pot}}(T)$ , we only use potential temperature  $T$  (and never in situ temperature  $T_{\text{in-situ}}$ ; differences would be in the range of 1 mK in the presented case).

With the explicit formulation of the in situ density it is now possible to compare neighbouring water parcels at the same pressure to check for density differences. Similarly, the Brunt-Väisälä frequency of a displaced water parcel can be calculated by subtracting the adiabatic density change due to pressure changes from the water column (in situ) density gradient: the 130 Brunt-Väisälä frequency is positive (the water column is stable) when the adiabatic density change of the displaced water parcel exceeds the surrounding water column (in situ) density change (Peeters et al., 1996). This is done straight forward by using the in situ density as

$$N^2 = -\frac{g}{\rho_{\text{in-situ}}} \cdot \left( \frac{\partial \rho_{\text{in-situ}}}{\partial z} - \frac{\partial \rho_{\text{in-situ}}}{\partial p} \frac{\partial p}{\partial z} \right) = -\frac{g}{\rho_{\text{in-situ}}} \frac{\rho_{\text{in-situ}}(T_2, p_2) - \rho_{\text{in-situ}}(T_1, p_2)}{z_2 - z_1}, \quad (6)$$

where  $z$  is the depth and the indices indicating the quantity of the corresponding water parcel. Hence, the adiabatic compensation of the pressure difference formally results in comparing  $\rho_{\text{in-situ}}$  of the two water parcels at a common pressure. As common pressure we chose the depth of one of the water parcels.

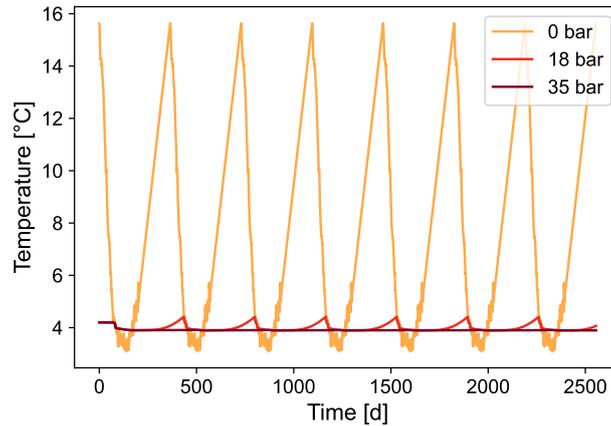
### 2.3 Numerical Model

We created a simple 1D model to conceptually simulate deep water renewal based on thermobaricity. The model domain consisted of 180 layers of 2 m each, which resulted in a maximum depth of 360 m. The values were motivated by Lake Shikotsu because this kind of deep water renewal is suspected to have a significant influence in this lake (compare Fig. 1b). The lateral extent of the layers was not specified, but the volume and hence the horizontal extent of all layers was considered equal. Lateral gradients were excluded by the one dimensional design of the model domain (Boehrer et al., 2008).

Our intention was the demonstration of thermobaricity. Hence, in contrast to common lake models, we removed the atmospheric forcing at the lake surface. Instead, we used surface water temperature as a boundary condition (Boehrer et al., 2000). As a consequence, the numerical model was exclusively forced by temperature, and resulting density differences, in the surface layer of the model. The exclusion of other driving forces prevented their interference with and the misinterpretation of thermobaricity driven effects. The surface temperature input is also based on the measured surface temperature from Lake Shikotsu at a depth of about 1.5 m with one measurement every minute from 18 October 2023 to 8 May 2024 with a temperature sensor RBRsolo<sup>3</sup> T (RBR, Canada), which includes temperatures significantly colder than 4 °C during the winter (compare Fig. 2). We decided to use real surface data to keep heat exchange with the atmosphere in a realistic range. The surface temperature series was then linearly extrapolated to a duration of one year. Although surface temperatures during the summer are unrealistically represented by the linear extrapolation, the thermobaric effect is unaffected by temperatures above 4 °C and the summer temperatures are much warmer. The annual surface temperature is then repeated over seven years which is the total length of the simulation. To include the effect of day-night variation, the time step was set to 1 h.

Starting from an arbitrary isothermal temperature profile, the temperature of the uppermost layer was inserted. We realized the diffusion by exchanging half of the volume of each layer equally with both neighbouring layers and homogenizing each layer correspondingly. By doing so, the time step size and layer size influence the amount of diffusion. For the values used, the diffusion is about  $5.5 \cdot 10^{-4} \text{ m}^2 \text{ s}^{-1}$ , which is roughly in the expected order of magnitude (e.g. Saber et al., 2018). We included diffusion to ensure exchange within the water column itself and include processes that are not directly driven by external forcing like cabbeling.

The special feature of our model is the comparison of in situ densities at given pressure, instead of using potential densities, as basis for the stability, and therefore mixing, of the water column. This means that in situ densities of two neighbouring layers are compared at the pressure of the lower one, while the temperatures (and in principle also the salinity) are kept the same for the two layers, respectively. This comparison is done bottom up. If the stratification is stable, there is no action and the next upper layer is checked. However, if water in the lower layer is less dense than in the upper, the layers are mixed and the temperatures of the two layers are averaged. The mixed layers are then iteratively compared and mixed with all layers below in the water column until they are stable again. Then again, the next upper layer is checked. If the layers are unstable up to the



**Figure 2.** Temperature logs at different depths. The logs show the temperature progression of the controlled surface temperature (0 bar) and at depths of 18 and 35 bar over the entire simulation period.

temperature controlled surface layer, the temperatures of all corresponding layers are set to the prescribed surface temperature. By doing so, all instabilities are removed over the whole water column in each time step. This stabilization takes place after  
 170 the forcing at the surface and successive diffusion.

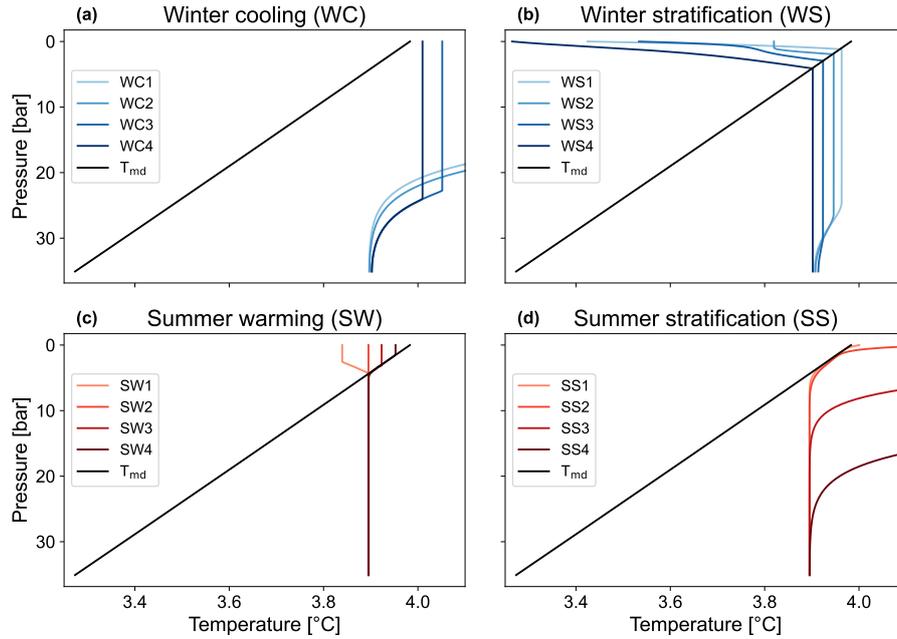
To demonstrate that thermobaric conditions form from an isothermal profile of 4.2 °C and remain thereafter, we ran the model for seven years. Hence, we can use later years when thermobaric conditions have established to discuss the resulting circulation features. In addition, two more runs were done by changing the input surface temperature in the fourth year by adding and subtracting 0.4 °C, respectively. They provide an impression of the stability of thermobaricity and the effect of  
 175 variable winters on deep water renewal.

### 3 Results

The model is driven by the input of the surface temperature. We follow the ocean convention of referring to hydrostatic pressure of 0 bar at the water surface. The simulated temperatures at mid depth (18 bar) and the bottom (35 bar) are also displayed for the entire simulation time (Fig. 2). At mid depth, the temperature rises during the summer stratification and follows the surface  
 180 temperature to a temperature slightly below 4 °C during autumn and winter. This temperature is also equal to the temperature at 35 bar, where only little temperature variation can be observed during the simulation except for the first winter, because the starting temperature profile was set to 4.2 °C.

#### 3.1 Temperature profiles

For the discussion of typical thermobaric circulation features, we refer to the fourth year of simulation, as we considered the  
 185 first three years as possibly affected by the initial conditions. We divide the year into typical stratification phases: winter cooling



**Figure 3.** Temperature profiles of the fourth year of the simulation period. The selected temperature profiles are split into four plots for clarity. (a) contains profiles from the winter cooling (WC) phase, (b) from the winter stratification (WS) phase, (c) from the summer warming (SW) phase, and (d) from the summer stratification (SS) phase. Their points in time are marked in Fig. 4. Additionally, all plots include the temperature of maximum density  $T_{\text{md}}$ .

(WC), winter stratification (WS), summer warming (SW) and summer stratification (SS). The order of these phases is oriented on the starting point of the simulation, which is in October, the beginning of the measured surface temperature input. Figure 3 exhibits four exemplary temperature profiles for each phase.

The four exemplary profiles of the winter cooling (WC1: 12 November, 20:00; WC2: 29 November, 12:00; WC3: 7 January, 01:00; WC4: 7 January, 10:00) are shown in Fig. 3a. During this phase, the surface water cools rapidly and circulates the water below to a depth where colder temperatures still stabilize the water column. Additionally, due to the high surface temperatures compared with the deep water temperatures, the deep water is still subject to warming (compare WC1 and WC2 with WC3 and WC4). This behavior continues until the surface water reaches the temperature of maximum density  $T_{\text{md}}$  at the surface, which is approximately 3.98 °C. From that point on, the inverse winter stratification begins.

During the winter stratification, the surface water is further cooled and it stratifies inversely (compare profiles in Fig. 3b: WS1: 21 January, 14:00; WS2: 29 January, 07:00; WS3: 20 February, 14:00; WS4: 19 March, 16:00). This means that close to the surface colder water floats on warmer water due to its lower density. Inverse stratification is stable to the point where the temperature profile intersects the  $T_{\text{md}}$  line. Here, water mixes across  $T_{\text{md}}$  due to diffusion and produces denser water (Grace et al., 2023a, b). This diffusion induced cabbeling can only occur because of the change of  $T_{\text{md}}$  with depth, i.e. thermobaricity. The denser water then circulates the water column below due to thermobaricity to the depth where lower temperatures sta-

bilize the water column. Since the surface water is further cooled in this phase, cold temperatures diffuse downwards to the intersection with  $T_{\text{md}}$ . This deepens the intersection and ever colder water mixes into the deep water. Eventually, the bottom layer is replaced by colder water from above and the circulation extends to the deepest point. Hence, the intersection with  $T_{\text{md}}$  controls the deep water temperature. Short intermediate warming events at the surface (e.g. WS2) induce small mixing cells at the surface, similar to the winter cooling, but do not influence the deep water mixing cells or their progression. The winter stratification comes to an end when the surface temperature starts to rise significantly.

During the summer warming (profiles in Fig. 3c: SW1: 2 April, 20:00; SW2: 3 April, 09:00; SW3: 3 April, 13:00; SW4: 3 April, 21:00) the inverse stratification gets erased by the rising surface temperatures. Since the surface layer temperature is below 3.98 °C, the warming surface water mixes the colder water below due to the higher density and we see a homogenized surface layer. From the moment when the surface layer temperature exceeds the deep water temperature (SW2), cabbeling comes to an end. The circulation of the deep water stops and a stable density stratification establishes along  $T_{\text{md}}$  above the isothermal deep water (SW3 and SW4).

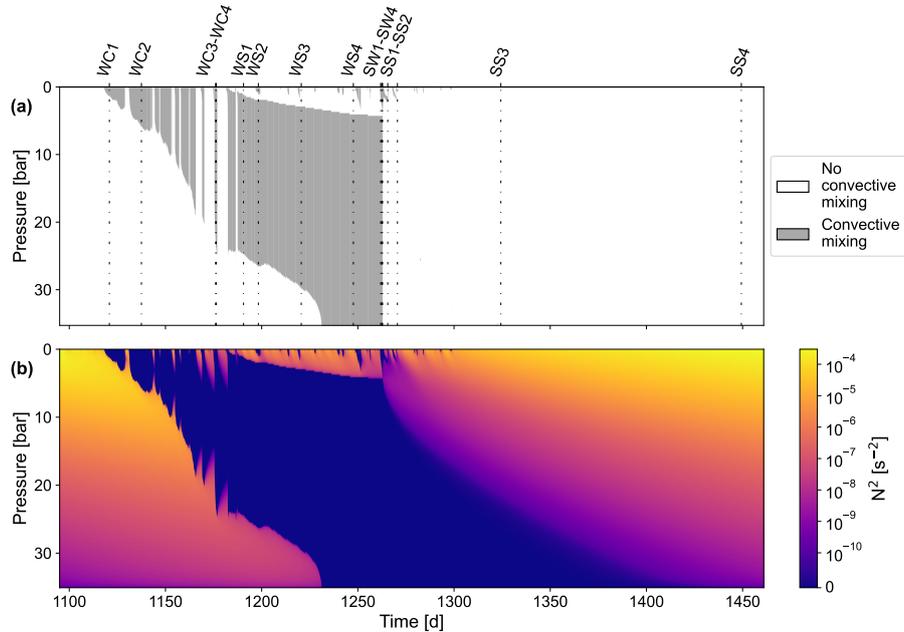
The following summer stratification (profiles in Fig. 3d: SS1: 6 April, 13:00; SS2: 11 April, 14:00; SS3: 4 June, 08:00; SS4: 7 October, 08:00) starts as soon as surface temperatures exceed 3.98 °C. Stable density stratification can establish from the surface. Early in this phase, the temperature profile follows the  $T_{\text{md}}$  line from the surface into the depth where  $T_{\text{md}}$  is equal to the uniform deep water temperature. However, during the summer period, diffusive transport from the surface increases the temperature in ever deeper water top down.

### 3.2 Convective mixing

In autumn, cooling at the surface drives vertical convection due to instabilities in the water column (Fig. 4a) the same way as if thermobaricity was not included. The surface temperature is decreasing and the convective mixing extends vertically to the depth where the surface temperature is equal to the current temperature at this depth. This depth also decreases due to the colder surface temperatures (compare WC1-4 in Fig. 3a). The convective mixing is interrupted from short rewarming events but is always connected to the surface.

Before reaching the bottom, temperatures at the surface fall below 3.98 °C and the stable inverse stratification terminates the circulation at the surface. Hence, the convection cell detaches from the surface (Fig. 4a). As it is permanently overlain by colder temperatures, no interruptions happen due to small rewarming events. These small rewarming events only induce additional small convective mixing cells at the surface, which are well separated from the deep circulation cell. Moreover, the deepening of the intersection of the temperature profile with the  $T_{\text{md}}$  line (compare WS1-3 in Fig. 3b) leads to a deepening of the upper end of the deep circulation cell. Finally the convection cell reaches the bottom (compare WS4 in Fig. 3b).

The convection in the deep water layers stops when the conditions for cabbeling disappear. This happens when surface temperatures rise above deep water temperatures (SW2 in Fig. 3c). The two convective mixing cells, the convective mixing at the surface and the deep water circulation, remain well separated. Afterwards, the deep convection stops and only the surface convective mixing cell remains in place (compare SW3 and SW4 in Fig. 3c), even though its depth decreases due to the change of  $T_{\text{md}}$  with depth. Below the  $T_{\text{md}}$  line, the warmer surface water is less dense than the colder deeper water, which limits the



**Figure 4.** Convective mixing cells and stability of the fourth year of the simulation period. (a) shows the convective mixing cells and the points in time of the profiles shown in Fig. 3 are marked with dash-dotted lines. (b) depicts the corresponding Brunt-Väisälä frequency, calculated with Eq. (6).

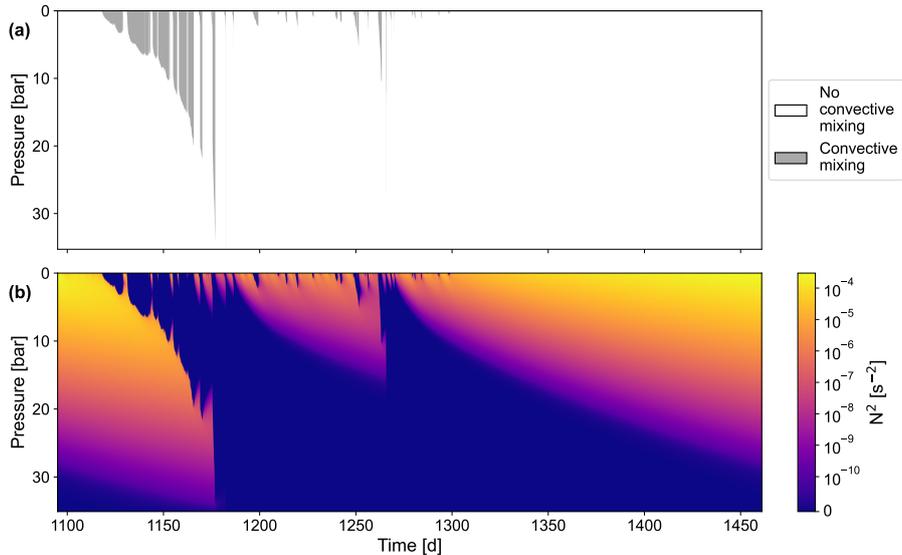
235 extent of the surface convective mixing cell.

After the surface temperature crosses  $3.98\text{ }^{\circ}\text{C}$ , only short intermittent cooling events can induce small surface convective mixing cells. In general, the water column becomes more stable and no convective mixing occurs until the end of the yearly cycle in September (compare SS1-4 in Fig. 3d).

### 3.3 Stability

240 The display of convection cells (Fig. 4a) is instructive for understanding vertical transport, but a quantitative approach to stability is desirable. Potential density is not applicable and in situ density is dominated by compression and hence does not provide much insight into stability either. However, we showed that the Brunt-Väisälä frequency can be calculated (Eq. (6), Fig. 4b). The convection cells can be identified as areas of stability  $0\text{ s}^{-2}$ . They are sharply separated from areas of higher stability. Starting in October, the water column becomes unstable from the surface due to decreasing surface temperatures.

245 Short rewarming events partly restabilize the unstable part at its upper end and diffusion does so at its lower end. After the detachment of the convection cell from the surface, a stable layer develops at the surface. Its lower end corresponds to the depth where the temperature profile intersects with the  $T_{\text{md}}$  line. The surface convective mixing cells originating from rewarming events can be identified by destabilized parts at the surface. After the deep convection ceases and the summer stratification sets in, the whole water column stabilizes due to the increasing surface temperatures and diffusion. Besides that, a marginally



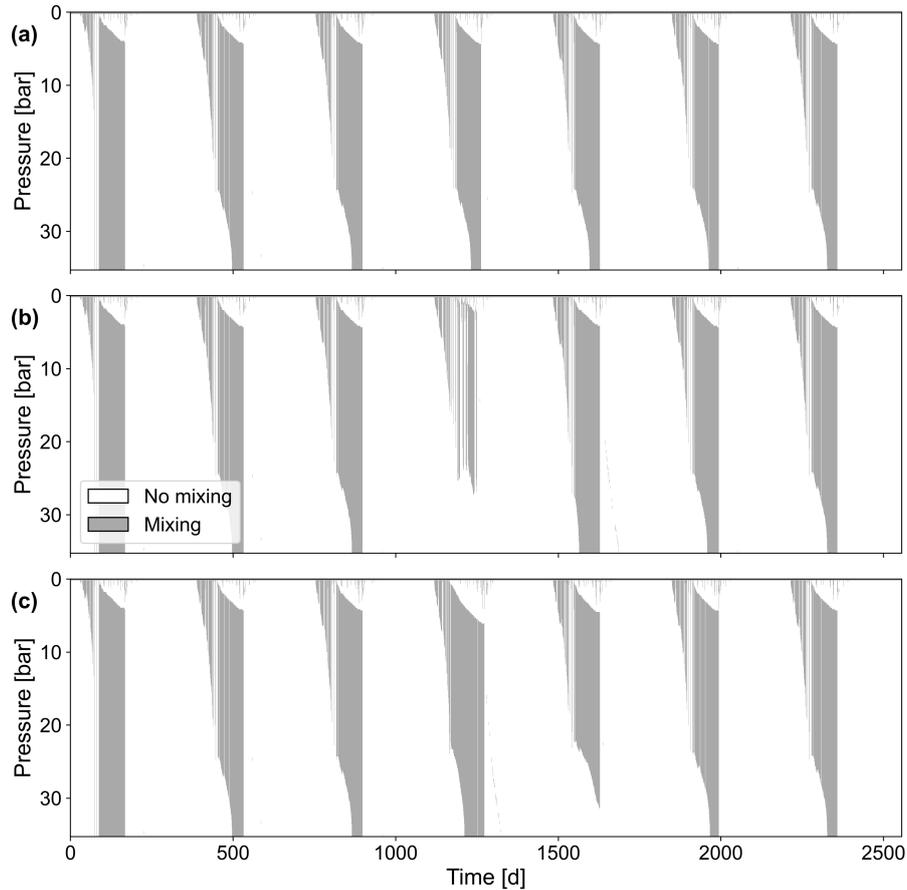
**Figure 5.** Convective mixing cells and stability of the fourth year of the simulation period for the simulation using potential density  $\rho_{\text{pot}}$  instead of in situ density  $\rho_{\text{in-situ}}$  (compare Fig. 4). (a) shows the convective mixing cells and (b) depicts the corresponding Brunt-Väisälä frequency, calculated with Eq. (6).

250 stabilized part remains close to the bottom for a long period. Towards the end of the summer stratification, it also becomes increasingly stable due to diffusion.

### 3.4 Simulation with potential density

To demonstrate the effect of thermobaricity, we conducted the exact same simulation as in Sect. 2.3 only substituting in situ density  $\rho_{\text{in-situ}}$  with potential density  $\rho_{\text{pot}}$  (i.e. neglecting thermobaric effects). We used the formulation of Tanaka et al. (2001) for  $\rho_{\text{pot}}$  as before. Cabbeling was included as an instantaneous destabilizing effect by setting the surface temperature equal to  $T_{\text{md}}$  in one time step for every transition of maximum  $\rho_{\text{pot}}$ , which is equivalent to the surface water temperature crossing 3.98 °C. Neglecting thermobaric effects change the size and the period of convective mixing cells and the water column stability fundamentally (Figure 5). There is only one significant period of overturn after the summer stratification culminating in a brief complete overturn when surface temperatures cross  $T_{\text{md}}$  and go into winter stratification. Due to the missing change of  $T_{\text{md}}$  with depth there is no cabbeling during the winter stratification below the surface and no successive downwards circulation that would be further driven by thermobaricity. Another brief moment of deep overturn occurs when surface temperatures go into summer stratification, again crossing  $T_{\text{md}}$ . The small convective mixing cells at the surface still occur due to their origin from small rewarming events which are unaffected by the thermobaric effect.

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**Figure 6.** Mixing cells over the entire simulation period for three variants of surface temperature input. The surface temperature of the fourth year of the simulation period was (a) not modified, modified by (b)  $+0.4\text{ }^{\circ}\text{C}$ , and (c)  $-0.4\text{ }^{\circ}\text{C}$ .

### 3.5 Interannual Variations

265 To check the effect of interannual variability for the simulation using in situ density, we added two more runs by elevating  
 or lowering the surface temperature of the fourth year of the simulation period by  $0.4\text{ }^{\circ}\text{C}$ , respectively. Increasing the surface  
 temperature leads to a weaker convective mixing during winter (Fig. 6b). A shorter and warmer cold period leads to less cold  
 water at the surface. Hence, the intersection of the temperature profile with the  $T_{\text{md}}$  line deepens less and the deep water  
 circulation is weakened. Depending on the strength of this change, convective mixing may not reach the bottom (compare Fig.  
 270 6b). Consequently, the temperature of the deep water is increased in the following winter and the convective mixing reaches  
 the bottom earlier in that year.

In contrast, decreased winter temperatures enhance the convective mixing during winter (Fig. 6c). The colder temperatures  
 and prolonged cold period leads to further deepening of the intersection of the temperature profile with  $T_{\text{md}}$ . Therefore, colder

water is convectively mixed into the deep water and at the end of the deep circulation period colder isothermal deep water has  
275 been formed. As a result, the following year with the original surface temperature input has less deep convective mixing cells  
since the colder deep water impedes deep convective mixing to the bottom (compare Fig. 6c).

In general, the variations in the surface temperature of successive years are in interplay with each other. They can either  
reinforce (e.g. a prior strong summer stratification and current cold winter period) or weaken (e.g. a prior weak summer  
stratification and current cold winter period) each other. Therefore, the model not only exhibits the different behavior of the  
280 convective mixing phases based on the variations in the forcing, the surface temperature, for the manipulated year, but also the  
dependency on the prior history. In general the stratification returns to normal after only a few years.

#### 4 Discussion

With our numerical approach we intended to demonstrate (a) the effect of thermobaricity on a simple one dimensional system  
and (b) the proper representation of thermobaricity by stability considerations based on in situ density. Key features of this  
285 idealized thermobaric system were (1) diffusion induced cabbeling at the intersection of the temperature profile with the  $T_{md}$   
line and its control of the deep water temperature by thermobaricity driven downwelling, (2) the period when surface water  
and deep water convection cells are separated, (3) the isothermal deep water, as well as (4) the alignment of the temperature  
profile with the  $T_{md}$  line in the upper water shortly after the winter stratification.

Despite the simplicity of our numerical approach, the key features (1)-(4) are remarkably clearly visible in the temperature  
290 profiles of Lake Shikotsu in 2005 (Fig. 1). The basic agreement of these features show that these thermobaric features in  
Lake Shikotsu (and other deep lakes with deep water temperatures below 3.98 °C) can be understood and basing stability  
considerations on in situ density can represent thermobaric effects.

Clearly, the vertical dimension of the surface water and the deep water sections differs greatly between the numerical model  
and Lake Shikotsu. The inclusion of wind forcing at the surface as well as turbulent mixing in the upper water column in the  
295 model would result in a thicker surface water layer. Additionally, it would induce deeper turbulent mixing of cold water. Hence,  
the intersection with  $T_{md}$  would deepen and the deep water temperatures would decrease correspondingly. Additionally, a more  
sophisticated approach for diffusion based on the stratification strength of the water column could further refine the temperature  
profiles, especially in summer. Due to strong summer stratification the heat transport into the deep water is presumably weaker  
compared to the model. Similarly, a limitation of the convective mixing range in each time step could further refine the evolution  
300 of the convective mixing cells. These extensions would produce more realistic timescales for the mixing and its different phases.  
Also, other factors such as salinity would need to be included in a realistic simulation.

We tested the resilience of the thermobaric stratification against interannual variability. The simulations demonstrated the  
dependence of the deep water circulation on the surface temperature as driving force. They also proved that this type of  
circulation was quite resilient against disturbances and returned to the usual circulation pattern within two or three years. Of  
305 course, changes in the governing influences of the circulation as mentioned above would modify the systems responses but the  
general behavior would be maintained.

## 5 Conclusions

In this paper, we presented an approach for implementing the effect of thermobaricity into a numerical model using in situ density instead of potential density. We created a simplified 1D model excluding any external forcing except the surface temperature. Below the surface layer, dynamics were exclusively driven by diffusion induced cabbeling around  $T_{md}$  and successive vertical convective mixing controlled by thermobaricity below, until stable density stratification was achieved. Moreover, we reduced the complexity to a minimum by omitting salinity, using horizontal homogeneity (one dimensional), a cylindrical bathymetry without shallow areas, and excluding any inflows. Hence, the thermobaric effects could be displayed without competing influences. To remain in a realistic range and have a good chance to reproduce thermobaric effects, we used Lake Shikotsu as inspiration and used its maximum depth and surface temperature as input.

The model was able to conceptually characterize key features of the deep water circulation. It elucidated different phases of vertical mixing and defined the vertical and temporal extent of convection cells. Generally, the deep water circulation differed fundamentally from the conventional understanding (i.e. the mixing behavior when thermobaric effects are excluded). During winter stratification, cabbeling occurred at the intersection of the temperature profile with the  $T_{md}$  line due to diffusion which induced convective mixing below due to thermobaricity. This convection cell was disconnected from the surface and extended into a depth where the temperature was equal to  $T_{md}$  at the intersection or to the bottom. Hence, the depth of the intersection of the temperature profile with the  $T_{md}$  line determined the resulting deep water temperature. This circulation behavior was not only influenced by the current surface temperature but also by the previous history of deep water renewal events. For instance, colder winter enhanced deep water circulation but led to weaker convective mixing in subsequent years.

In summary, this simplified model exhibits the necessity and the feasibility of including thermobaricity in the simulation of deep water circulation. This can be achieved by the implementation of in situ density (instead of potential density) for stability considerations. Despite the fact that this parsimonious approach could nicely reproduce the typical circulation features induced by thermobaricity, we are fully aware that a realistic representation of processes in Lake Shikotsu requires a proper lake model with complete forcing and accurate bathymetry. Hence, the proper solution for future lake modeling is the appropriate inclusion of stability considerations based on in situ density into established lake models. This feasible approach will provide new insights into deep water formation in thermobarically stratified lakes such as Lake Shikotsu and will improve the modeling of complex deep water renewal processes that are linked to thermobaricity as they occur in many large lakes.

*Code and data availability.* The model code and input data used in this study is publicly available at GitHub: [https://github.com/JMarks840/Thermobaric\\_Circulation](https://github.com/JMarks840/Thermobaric_Circulation).

*Author contributions.* JM and BB conceptualized the study and designed the model approach. JM created the model, performed the simulations, analyzed and visualized the data, and wrote the original draft of the manuscript. BB acquired the funding, provided supervision and reviewed and edited the manuscript. KC helped with the logistics, acquiring the data and reviewed the manuscript.

*Competing interests.* The authors declare that they have no conflict of interest.

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