# Dear Referee.

Thanks a lot for taking the time to review our paper and for your very useful thoughts and comments. We have revised our manuscript following your suggestions and detail here these modifications. We provide our answers in **blue**.

## - General Comments:

The author should consider also using BGEP ULS moorings for an additional SIT validation product, as they only use OIB for this, and OIB missions were only run during April 2019, while BGEP ULS data are available throughout their experimental period.

We fully agree with the reviewer that we should extend our validations. We have added these validations against BGEP ULS data in the revised manuscript. The validations are presented below:

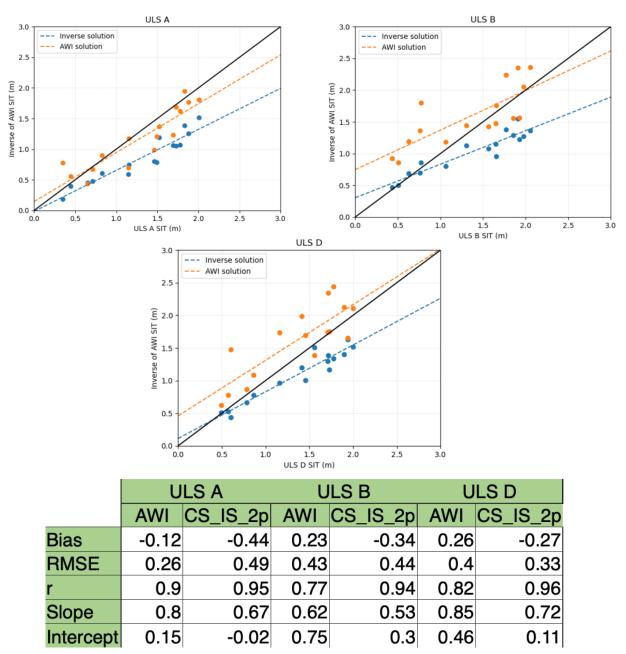


Table 1: Statistics of the validation

The upward-looking sonar (ULS) moorings were deployed by the Beaufort Gyre Exploration Project (BGEP). The BGEP data were collected at three locations in the Beaufort Sea (BGEP-A, BGEP-B, and BGEP-D) between 2010 and 2024. Sea ice thickness was derived from draft measurements by applying a ratio of 0.89, following the method of Rothrock et al (https://doi.org/10.1029/2001JC001208).

The inversion was then performed at mid-month intervals for the winter periods from 2018 to 2021, corresponding to the years for which comparison data are available from AWI.

The authors find unphysical values of penetration factors (though it can be explained by laser scattering above the snow layer), do they also find similar for ice thickness or snow depth post-inversion? Or do the authors correct for these in some post-processing?

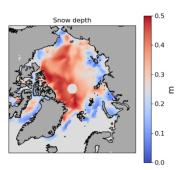
No unphysical values (for example negative values) were found for ice thickness or snow depth, so we don't correct for this after the inversion. This can be explained by the choice of the priors for ice and snow, that are both only positive whereas we allow negative values for the penetration factors (L177+ in the manuscript).

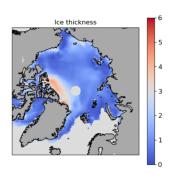
The only post-processing performed is the computation of the mean map we show in the manuscript.

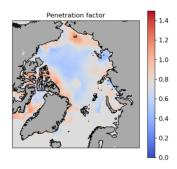
Did the authors test how the inversion method results vary if the prior distributions are changed. It could be particularly useful to analyse how the results change if the penetration factor bounds are chosen to be above 0 in the prior.

We performed some tests with other bounds for the priors, especially for the penetration factors.

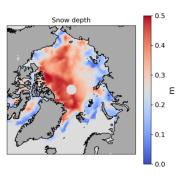
- Inversion for the model CS-IS-3p for 2019/04 with the following prior for  $\alpha_{cs}$ : [0, 1.5] (instead of [-0.5, 1.5] in the manuscript):

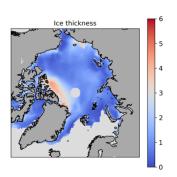


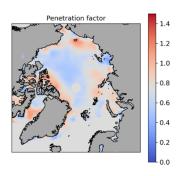




Inversion for the model CS-IS-3p for 2019/04 with the following prior for  $\alpha_{cs}$ : [-0.5, 1.5] (same than the one in the manuscript):







As we can see with these 2 examples, the results look very similar for the 3 parameters. Thus, the inversion seems to be very robust even with a slight change in the penetration factor prior. Note that we didn't change the priors for ice and snow depth.

The authors should use panel labels (a), (b) etc. and then refer to them in the caption instead of using (left), (right), (first column) etc. This would also help for references to the figures in the text. In some cases (e.g. figure 8) the captions of the figures are missing some key details about the figure.

# The manuscript has been modified according to this comment.

The authors should use a different colour range when plotting figures of maps of the penetration factor, as using the Red-Blue symmetric scheme makes these plots harder to read. The colour range they use for the snow depth/thickness maps would be better for this. I added specifics about the figures at the bottom of this report.

## The manuscript has been modified according to this comment.

### - Minor Comments:

Line 7: Statistically encouraging is a bit strange wording.

We propose to reformulate as: "The inversion results demonstrate statistical coherence with snow and ice evaluation products."

Line 21: Could just say "especially in the Arctic". Can also add a sentence or two looking at impact of climate change on the Arctic region in comparison to other regions.

We propose this formulation: "Over the past decades, climate change has had a significant impact on Earth and especially in the Arctic."

Line 58: Good to mention CPOM-CS2 modifies the warren climatology by halving snow depth over first year ice.

## We have added this precision in the revised manuscript.

Line 83: SIT retrieval can also depend on the choice of retracker. It would be good to mention the impact of different retracker choices here.

### The revised manuscript has been modified to include this comment.

Table 2: May be good to also mention that r is linear correlation coefficient in the table caption.

## The caption has been modified

Line 229-230: confirmed -> be confirmed

#### The sentence has been modified.

Line 280-284: What does it mean when the authors say SIT and SD are "important"? Just thick?

Yes exactly, we have rephrased these sentences as followed:

"There are some similarities when comparing the maps for  $\alpha_{CS}$  and the results for SIT and SD. Especially for April, the bias factor is close to 1 around Wrangel Island. In this same area, SIT is higher (close to 3 m). Similarly, north of Greenland where  $\alpha_{CS} \simeq 1$ , the SIT is substantial (around 4.5 m in some places), but here the SD is also significant (even though thinner ice is found near the Canadian Archipelago)."

Line 337: Sentence that starts with "Thanks to figure 12" needs rewording.

#### The sentences have been modified.

Line 375: Should be "Given a user-defined"

#### The sentence has been modified.

Line 425-440: The language in the conclusion is a bit informal in some cases, and there are also some spelling errors: e.g. "could even be unknown", "of course not exhaustive", "in order of efficiently construct".

We propose this new writing of the conclusion (note that we have changed "penetration factor" by "bias factor" following the comments of the other reviewer):

We introduced a novel application of a Bayesian method for retrieving ice and snow parameters over the Arctic. This approach proved suitable for estimating SIT and SD without requiring assumptions on SD, which typically represent the main source of uncertainty. The probabilistic nature of the solution enables further investigation of covariances and joint probability distributions of ice and snow depth. Comparisons of the inverted SIT and SD maps with existing products (AWI, AMSR2, OIB, UiT) and independent datasets (MOSAiC, IceBird, BGEP-ULS) indicate promising consistency, although additional refinement of the algorithm is required to achieve an operational product for the joint inversion of SIT and SD. In particular, this will necessitate an improved understanding of the influence of the bias factor on both ice and snow depth.

The algorithm is capable of producing maps for the bias factors of the satellites used. This could help the understanding of the behavior of  $\alpha$  and its link with SIT and SD (using the covariances for example or with more sensitivity studies). In the meantime, this will allow further investigation on temporal and spatial variability of the penetration factor.

One of the main advantages of this code is that it can be extensively customized and further improved. The forward model could be refined to invert for ice mass instead of the SIT or to invert for a deviation to a background instead of looking at absolute of the physical parameters. The error on the input could be changed and could even be unknown for the inversion. This list of further investigations of this method is of course not exhaustive and paves the way for future works in order to efficiently construct maps of ice and snow depth over the poles.