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New derivation and interpretation of the complementary relationship for evapotranspiration

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Abstract. The complementary relationship (CR) between actual evapotranspiration (ET) and apparent potential evapotranspiration (PETa) is widely used as a simple yet effective method for ET estimation. However, most existing CR formulations are empirical, lacking rigorous derivation based on physics. In this study, the complementary relationship was derived analytically with a physically meaningful parameter: the wet Bowen ratio, defined as the Bowen ratio when the surface becomes saturated. This parameter can be computed from observations without calibration. Fundamentally, the CR is shown to originate from partitioning of the net radiation, with ET directly linked to the latent heat and PETa proportional to the sensible heat. Additionally, ET is linearly related to and constrained by the energy-based potential evapotranspiration (PETe). The physically-based relationship among ET, PETa, and PETe has important implications for our understanding of the spatial and temporal variations in ET and would promote practical application of the complementary relationship for ET estimation across different environments.

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1. Introduction

Terrestrial evapotranspiration (ET) plays a vital role in land-atmosphere exchanges of water, energy, and carbon fluxes and thereby influences weather and climate as well as the water and carbon cycling (Ault, 2020; Gentine et al., 2019; Miralles et al., 2019; Zhou et al., 2022, 2023). While many approaches have been attempted to estimate ET, the complementary relationship (CR) for ET, which was first proposed by Bouchet (1963) and operationalized by Morton (1983), provides a simple conceptual framework for estimating ET using routine meteorological observations (Han and Tian, 2020; Ma et al., 2021; Zhang and Brutsaert, 2021).

The CR essentially describes the relationship between three types of ET over land surface (Brutsaert, 2015). The first type is the actual ET, which is the water vapor flux from open water, soil, and vegetation over an area. The second type is the potential ET (PET), i.e., the ET that would occur from the same area with the same net radiation, but with unlimited water supply, i.e., a saturated evaporative surface. As the PET is limited by available energy, it is termed as PETe in this study. The third type is the apparent potential ET (PETa), which is the ET that would occur from a small, saturated surface within a large and unsaturated area, with abundant energy supply from the net radiation and the surrounding environment (Zhou and Yu, 2024). It is assumed that the saturated evaporative surface is too small to affect aerodynamic conditions, i.e., air temperature, humidity, and wind speed. PETa therefore depends on the prevailing aerodynamic conditions over the large dry area. In practice, PETe is represented by the actual ET over a large lake or reservoir, while PETa by the actual ET measured with a small evaporation pan placed in an otherwise dry environment (Kahler and Brutsaert, 2006). These three types of ET converge when the large area is saturated everywhere. As the large area dries up, ET is limited by available water and falls below

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PETe. Simultaneously, a lower ET reduces the moisture content of the atmosphere and reduced evaporative cooling increases air temperature, resulting in a warmer and drier atmosphere with a higher PETa than PETe. These processes lead to a complementary relationship between ET and PETa when water supply is limited.

Based on the conceptual framework above, a series of CRs have been formulated, with most being empirical, and none of them is fully physically-based (Bouchet, 1963; Brutsaert and Parlange, 1998; Brutsaert, 2015; Crago et al., 2016; Granger, 1989; Szilagyi, 2007; Szilagyi et al., 2017, 2022; Tu et al., 2023). This is because the physical mechanisms underlying the CR and the quantitative relationship between the three types of ET remain unclear, which hinder our understanding of the CR and the derivation of a physically-based CR for accurate estimation of ET. The original CR proposed by Bouchet (1963) assumes that the relationship is symmetric with respect to energy conservation, which is, however, not supported by observations, and several alternative CRs have been proposed with an empirical parameter to account for the departure from the original symmetric CR (Brutsaert and Parlange, 1998; Brutsaert, 2015; Crago et al., 2016; Granger, 1989; Szilagyi, 2007). Due to a lack of physical interpretation of the parameter involved, its values must be estimated through calibration with location-specific observations, which hinders and limits the application of these alternative CRs for ET estimation.

In addition, a lack of definitive estimators of PETe and PETa also hinders development of a physically-based CR (Crago et al., 2016; Tu et al., 2023). As PETe cannot be directly measured unless water supply is unlimited all the time, previous studies used the Priestley-Taylor equation (Priestley and Taylor, 1972) to approximate PETe, which is, however, quite different from

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observed ET over the ocean, indicating that the Priestley-Taylor equation is not entirely appropriate for estimating PETe (Yang and Roderick, 2019; Zhou and Yu, 2024). On the other hand, PETa is generally measured using an evaporation pan or estimated from the Penman equation (Penman, 1948), which is, however, not consistent with the definition of PETa, as the Class A evaporation pan is large enough to have measurable effect on the air temperature and humidity around the pan and the Penman equation does not consider energy transferred from its surrounding environment (Brutsaert, 2015; Kahler and Brutsaert, 2006). An in-depth understanding of the physical processes underlying the complementary relationship and accurate estimation of the three types of ET are therefore crucial for formulating a physically sound complementary relationship.

The objective of this paper is to reappraise the physical foundation of the complementary relationship and derive a physically-based CR for estimating ET over land. Based on theoretical reasoning and analysis, we identify the key physical processes underlying the complementary relationship. By estimating the three types of ET based on their definitions and physical processes, we formulate an alternative CR with a physically meaningful parameter. This study would advance our understanding of the physics behind the complementary relationship to support its practical application for ET estimation over land.

2. Concept of the complementary relationship and empirical formulations

When the land surface is well supplied with water, the magnitude of the three types of ET are identical, depending on the energy supply and aerodynamic conditions.

$$ET = PET_e = PET_a \tag{1}$$

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As the moisture supply at the evaporative surface decreases, the available water is insufficient to meet the evaporative demand, i.e., PET_e, and the energy not expended on ET is shifted to be sensible heat (H) which increases with the difference between surface and air temperatures and causes PET_a, e.g., the evaporation from an arbitrarily small area in a dry environment, to exceed PET_e. In general, we have

$$ET \leq PET_e \leq PET_a \tag{2}$$

The original CR (Bouchet, 1963) assumes that the decrease in ET equals the increase in PET_a, relative to PET_e, from wet to dry conditions.

$$PET_e - ET = PET_a - PET_e \tag{3}$$

If the net radiation remains the same from wet to dry conditions, the decrease in latent heat therefore equals the increase in sensible heat, i.e., $\lambda PET_e - \lambda ET = H - H_w$, where λ is the latent heat of vaporization and H_w the sensible heat under wet conditions. Considering potential variations in the relationship between changes in H and λPET_a, it is reasonable to assume that the left and right hand-sides of equation (3) are proportional (Szilagyi, 2007), resulting in a generalized linear CR in the form of

$$PET_e - ET = k(PET_a - PET_e) \tag{4}$$

or

$$ET = (1 + k)PET_e - kPET_a \tag{5}$$

where k is the coefficient of proportionality, and it can be interpreted as a measure of asymmetry for the CR. Equation (4) is identical to the original CR, i.e., equation (3) with $k = 1$, otherwise the CR becomes asymmetric, the latter has been widely supported with observations (Kahler and Brutsaert, 2006; Szilagyi, 2007, 2021). This asymmetry is likely to have arisen from changes in the net radiation between wet and dry conditions and/or the energy transfer from the surrounding

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environment. For example, as the land surface dries up, the evaporation pan would receive more energy from its side and bottom and local advection of energy, resulting in a larger increase in PET_a than the decrease in ET would suggest (Kahler and Brutsaert, 2006).

It is worth noting that the existing CR formulations can be interpreted in the generalized form with equation (5), and the coefficient of proportionality k for each formulation is shown in Table 1. However, as the physical meaning of the coefficient k remains unknown and unclear, it cannot be directly estimated and need to be calibrated, which limits practical application of the complementary relationship. Here we derive the complementary relationship with a new expression for the coefficient of proportionality, k, that has a clearer physical interpretation.

Table 1. The coefficient of proportionality (k) for different CR formulations.

Coefficient of proportionality (k)	Complementary relationship	References
$k = 1$	$ET = 2PET_e - PET_a$	(Bouchet, 1963)
$k = \frac{1}{b}$	$ET = \left(1 + \frac{1}{b}\right) PET_e - \frac{1}{b} PET_a$	(Brutsaert and Parlange, 1998)
$k = \frac{\gamma}{\Delta}$	$ET = \left(1 + \frac{\gamma}{\Delta}\right) PET_e - \frac{\gamma}{\Delta} PET_a$	(Granger, 1989; Szilagyi, 2007)
$k \approx 0.22$	$ET = \left(\frac{PET_e}{PET_a}\right)^2 (2PET_e - PET_a)$	(Brutsaert, 2015)
$k = \frac{X_{min}}{1 - X_{min}}$	$ET = \left(1 + \frac{X_{min}}{1 - X_{min}}\right) PET_e - \frac{X_{min}}{1 - X_{min}} PET_a$	(Crago et al., 2016)

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$k = \beta_w$	$ET = (1+\beta_w)PET_e - \beta_w PET_a$	This study

3. A physically-based complementary relationship

3.1. Estimation of ET, PETe, and PETa

As ET is controlled by the supply of energy and aerodynamic conditions, two approaches can be used to estimate ET, i.e., the energy-based ET_e and aerodynamics-based ET_a (Chow et al., 1988). The first approach is based on the surface energy balance, i.e., partitioning of the net radiation between sensible (H) and latent (λET_e) heat, and the ratio between the two ($H/\lambda ET_e$) is the Bowen ratio (β). Therefore, the latent and sensible heat for a given area can be expressed as

$$\lambda ET_e = \frac{R_n}{1+\beta} \tag{6}$$

$$H = \frac{\beta R_n}{1+\beta} \tag{7}$$

where R_n ($J \cdot m^{-2} \cdot s^{-1}$) is the net radiation minus ground heat flux (hereafter termed the net radiation for simplicity) and equals the sum of latent and sensible heat.

The second approach is based on the aerodynamics, i.e., transport of water vapor and sensible heat away from the evaporative surface, and the latent (ET_a) and sensible (H) heat are given by

$$\lambda ET_a = \frac{\rho c_p (e_s - e_a)}{\gamma r_a} \tag{8}$$

$$H = \frac{\rho c_p (T_s - T_a)}{r_a} \tag{9}$$

where ρ is the air density ($kg \cdot m^{-3}$), c_p the specific heat of air at constant pressure ($J \cdot kg^{-1} \cdot K^{-1}$), γ the psychrometric constant ($Pa \cdot K^{-1}$), and r_a the aerodynamic resistance ($s \cdot m^{-1}$) that depends on wind speed and land surface characteristics. The term $e_s - e_a$ is the difference in vapor

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pressure (Pa) between the evaporative surface and the air above, and $T_s - T_a$ the difference between surface (T_s) and air (T_a) temperatures (K).

Considering that the latent and sensible heat estimated from the two approaches above should be identical, the Bowen ratio, β, in equations (6) and (7) can be estimated using equations (8) and (9) as

$$\beta = \frac{\gamma(T_s - T_a)}{(e_s - e_a)} \tag{10}$$

To estimate PET_e and PET_a, we introduce a wet Bowen ratio (β_w), i.e., the Bowen ratio when the evaporative surface is saturated (Zhou and Yu, 2024). By replacing e_s in equation (10) with the saturation vapor pressure (e_s^*) at the surface temperature (T_s), we have

$$\beta_w = \frac{\gamma(T_s - T_a)}{(e_s^* - e_a)} \tag{11}$$

For a large area, PET_e can be estimated based on a partition of the net radiation into latent (λPET_e) and sensible (H_w) heat when the whole area becomes saturated:

$$\lambda PET_e = \frac{R_n}{1 + \beta_w} \tag{12}$$

$$H_w = \frac{\beta_w R_n}{1 + \beta_w} \tag{13}$$

The situation becomes complicated when we consider PET_a, i.e., the ET that would occur from a small, saturated area within a large dry area, such as an evaporation pan placed in a desert. The saturated area is considered to be so small that its presence has no practical effect on the

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surrounding environment where the wind speed, air temperature, and humidity are largely dictated by the prevailing meteorological condition over the dry area. The meteorological variables, i.e., e_a, T_a, and r_a, over the small, saturated area are thus identical to those over the surrounding dry environment. The surface temperature of the small, saturated area approaches its maximum value, i.e., T_s of the surrounding dry area, sustained by heat transfer from the surrounding environment. This implies that the sensible heat of the locally saturated area is maximized and equal to that of the large dry area given in equation (9). For the small, saturated area, the wet Bowen ratio, β_w, can be estimated using equation (11) and meteorological variables over the dry area. Consequently, PET_a can be estimated by replacing ET_a with PET_a, and e_s with e_s^* in equation (8):

$$\lambda PET_a = \frac{\rho c_p (e_s^* - e_a)}{\gamma r_a} \tag{14}$$

Equation (14) can be further simplified noting the definition of the sensible heat (H, equation (9)) and that of the wet Bowen ratio (β_w, equation (11)):

$$\lambda PET_a = \frac{H}{\beta_w} \tag{15}$$

As the small wet area has the same surface temperature and atmospheric conditions as the surrounding large dry area, with the only difference being that the small area is saturated with saturation vapor pressure at its surface (e_s^*), evaporation from this small wet area, i.e., PET_a, represents the evaporative demand imposed by atmospheric humidity and aerodynamic conditions (Fig. 1a). Since PET_a is not constrained by available energy and unrelated to land-atmosphere feedbacks, it cannot be realized over a large area where energy supply is limited and surface moisture can significantly impact the atmosphere. However, PET_a could be measured with a small evaporation pan while maintaining its surface temperature equal to that of the surrounding environment (T_s). Alternatively, PET_a can be estimated using H and β_w from eddy covariance and

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meteorological measurements, following equation (15), or using a modified Penman equation based on routine meteorological observations (see Section 4.4) (Zhou and Yu, 2024). In contrast, PET_e represents the maximum ET that would occur over the large area, for the same amount of net radiation but with unlimited water supply (Fig. 1b). PET_e cannot be directly observed unless the entire surface is saturated, such as over a lake or the ocean, but it can be calculated using equation (12) (Zhou and Yu, 2024, 2025).

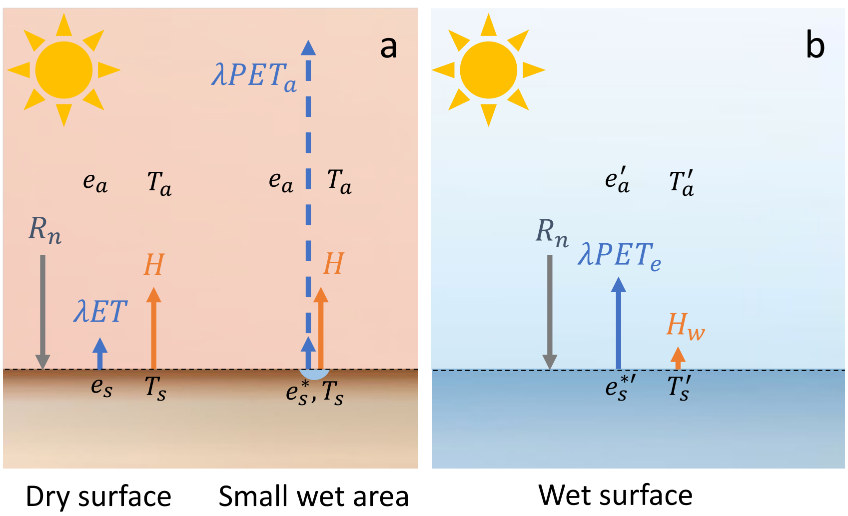

Figure 1. Illustration of the relationships between actual ET and two potential ET (PETe and PETa) under wet and dry conditions. (a) In a dry environment, the net radiation (R_n) is partitioned into latent heat (λET) and sensible heat (H). For an arbitrarily small wet area, the surface temperature (T_s), air temperature (T_a), and vapor pressure (e_a) remain the same as the surrounding dry environment. The only difference is the vapor pressure at the evaporative surface (e_s versus e_s^*). Consequently, H over the small wet area remains unchanged compared to the surrounding environment, but λPET_a is much larger than λET, as both water and energy supply are not limiting

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over the small wet area. (b) When the entire area becomes saturated, both surface and atmospheric conditions cool down (T_s' and T_a') and become more humid ($e_s^{*\prime}$ and e_a') relative to the dry environment. As a result, latent heat (λPET_e) increases while sensible heat (H_w) decreases, constrained by R_n.

Considering coupled changes in temperature and humidity at the evaporative surface and of the air, it has been demonstrated that β_w estimated from meteorological variables corresponding to the large dry area remains relatively constant when the whole surface becomes saturated, provided that the net radiation remains the same under wet and dry conditions (Zhou and Yu, 2024). It is therefore reasonable to assume that β_w in equation (15) for a small, saturated area within a large dry area is the same as that in equation (12) for the large area when the whole area is saturated.

3.2. Derivation of a physically-based complementary relationship

For a certain large area, the energy balance equation is given as

$$R_n = \lambda ET + H \tag{16}$$

The sensible heat, H, is directly related to PET_a as shown in equation (15). Considering that R_n for the given area is assumed to be the same when the whole area becomes saturated, i.e., $\lambda ET + H = \lambda PET_e + H_w$, the only difference is how R_n is partitioned into latent and sensible heat. With R_n from equation (12) and H from equation (15), the energy balance equation can be re-written as:

$$ET = (1 + \beta_w)PET_e - \beta_w PET_a \tag{17}$$

The structural similarity between equations (5) and (17) suggests that the coefficient of proportionality k equals the wet Bowen ratio β_w. Equation (17) clarifies the physical basis for the complementary relationship, and suggests that the actual ET is a linear combination of PET_e and

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PET_a for both wet and dry environments. This is consistent with the boundary condition for equation (2), i.e., $ET = PET_e = PET_a$ for saturated surfaces and $ET < PET_e < PET_a$ for unsaturated surfaces. The complementary relationship emerges as the land surface dries up, ET drops below its potential, PET_e, and the energy not partitioned to λET is shifted to increase the sensible heat, resulting in a higher PET_a, relative to PET_e. The complementary relationship between ET and PET_a essentially reflects the shift in the partitioning of the net radiation between latent and sensible heat under different environmental conditions. In addition, we note ET is linearly related to PET_e, which represents the maximum ET that would occur given the net radiative energy supply. This indicates that PET_e represents the atmospheric evaporative demand or the energy constraint that controls and drives ET over land, and PET_a, on the other hand, is in fact a response of the atmosphere to the reduction in ET where water supply is limited at the land surface.

3.3. Validation of the complementary relationship

The Fluxnet2015 dataset, which provides meteorological measurements and observed land-atmosphere exchanges of water and energy fluxes based on the eddy covariance technique from 212 sites (>1500 site-years) around the globe (Pastorello, 2020), were used to validate the physically-based complementary relationship in equation (17). These sites cover a wide range of climate conditions and vegetation types and were used to examine the relationship between the actual ET and potential ET, i.e., PET_e and PET_a. Data were included in this analysis for site-years where measured or high-quality gap-filled data of air temperature, surface soil temperature, sensible and latent heat fluxes were available. To reduce uncertainties in ET measurements, days with air temperature less than 5 °C or negative sensible/latent fluxes were excluded. Finally, we

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selected 146 Fluxnet sites with effective records of more than 90 days (see Table S1). To validate the complementary relationship for different sites and different seasons, we used data from 7352 site-months, with effective records of more than 15 days for each month. For each site-month, the net radiation minus ground heat flux (R_n) was calculated as the sum of latent and sensible heat. β_w was estimated from equation (11). PET_e and PET_a were estimated from equations (12) and (15), respectively.

To illustrate the complementary relationship between ET and PET_a and the proportional relationship between ET and PET_e, equation (17) can be scaled and re-written as

$$\frac{ET}{PET_e} = (1 + \beta_w) - \beta_w \frac{PET_a}{PET_e} \tag{18}$$

$$\frac{ET}{PET_a} = (1 + \beta_w) \frac{PET_e}{PET_a} - \beta_w \tag{19}$$

Estimation of the three types of ET and their relationships are shown in Table 2 and Fig. 2a-c. Based on observations from the 146 Fluxnet sites (7352 site-months), the complementary relationship between ET and PET_a and the proportional relationship between ET and PET_e across wide-ranging climate conditions are clearly evident (Fig. 2d-f). The scaled ET with PET_e increases from 0 to 1 and the scaled PET_a decreases from 18 to 1 from the driest to the wettest site-months. This provides observational evidence for the complementary principle that ET and PET_a converge towards PET_e and they closely match each other ($ET = PET_e = PET_a$) under wet conditions, while ET falls below PET_e and PET_a rises above PET_e ($ET < PET_e < PET_a$) in a dry environment. As implied by equation (18), a strong negative correlation (r = -0.86) was found between the scaled ET and PET_a, with a low level of nonlinearity induced by variations in β_w (0.05-0.25) across the 7352 site-months (Fig. 2e). The scaled ET and PET_e with PET_a in equation

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(19), on the other hand, are positively correlated (r = 0.99, Fig. 2f), indicating the strong positive control of energy-based PET_e on ET across a wide range of climate environment.

Table 2. Estimation of the three types of ET and their relationships.

Equation	Range
$\lambda ET = \frac{R_n}{1+\beta}$	$\left[0, \frac{R_n}{1+\beta_w}\right]$
$\lambda PET_e = \frac{R_n}{1+\beta_w}$	$\frac{R_n}{1+\beta_w}$
$\lambda PET_a = \frac{H}{\beta_w}$	$\left[\frac{R_n}{1+\beta_w}, \frac{R_n}{\beta_w}\right]$
$\frac{ET}{PET_e} = \frac{1+\beta_w}{1+\beta}$	$[0,1]$
$\frac{ET}{PET_a} = \frac{\beta_w}{\beta}$	$[0,1]$
$\frac{PET_e}{PET_a} = \frac{1+\frac{1}{\beta}}{1+\frac{1}{\beta_w}}$	$\left[\frac{\beta_w}{1+\beta_w}, 1\right]$

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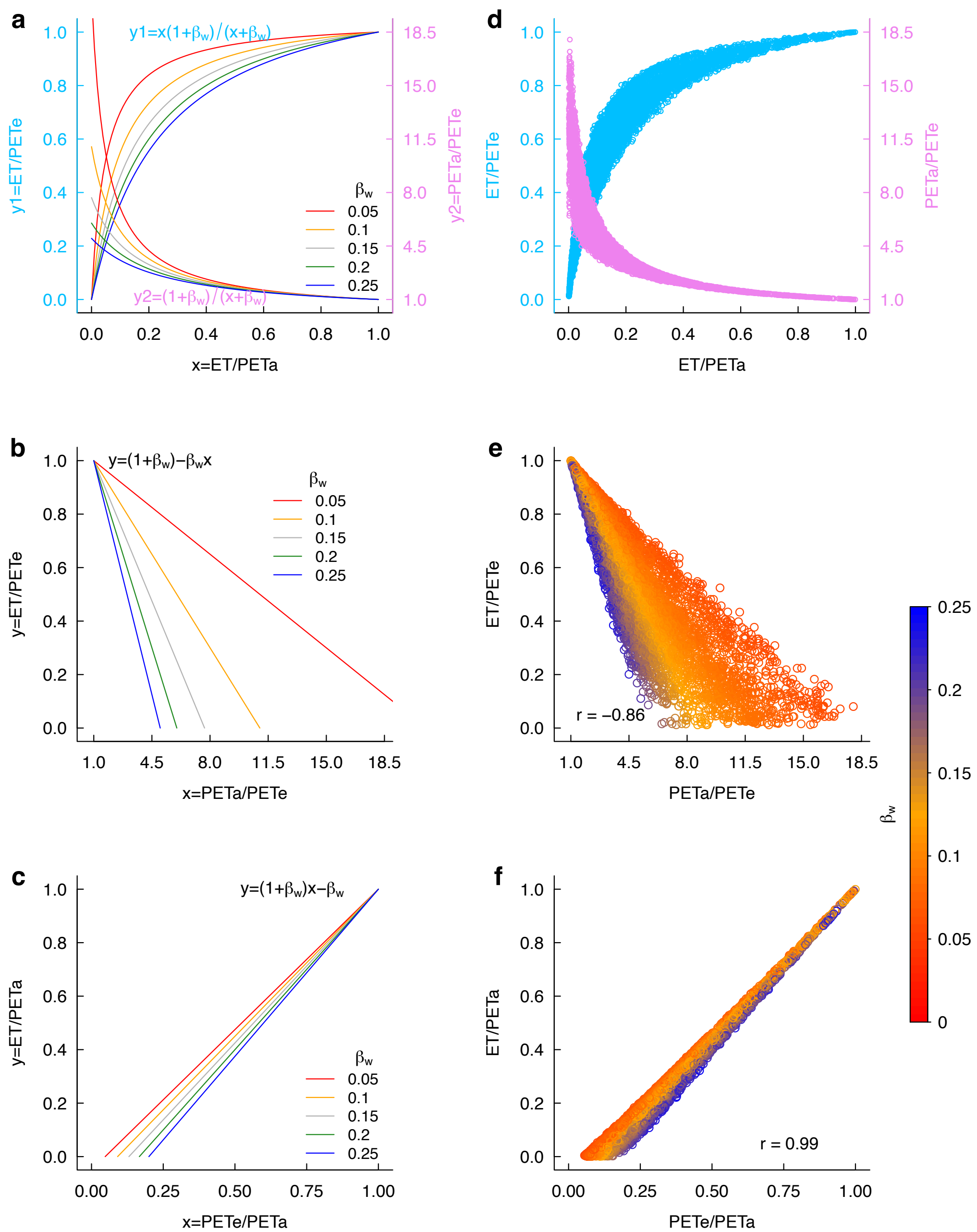

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Figure 2. The complementary relationship between ET and PETa and the positive relationship between ET and PETe. (a-c) Relationships among ET, PETe and PETa with a constant value of β_w ranging from 0.05 to 0.25 at an increment of 0.05. (d-f) Relationships among monthly ET, PETe and PETa estimated using meteorological and flux measurements from the Fluxnet2015 dataset (146 sites and 7352 site-months in total, see Table S1). β_w is shown as variations in color for each site-month (Fig. 2e,f). Since β_w ranged from 0.05 to 0.25 across the 7352 site-months, the slope (β_w) would vary by a factor of 5 in Fig. 2e, while for Fig. 2f the range in slope (1+β_w) varies by 19% only.

4. Discussion

4.1. Assumptions for the physically-based complementary relationship

Assumptions underlying the CR formulation in equation (17) include that 1) the net radiation, R_n, for a given area remains the same under wet and dry conditions; 2) the wet Bowen ratio, β_w, remains the same for the large area when it becomes saturated and for the small, saturated area within the large dry area, i.e.,

$$\beta_w = \frac{H}{\lambda PET_a} = \frac{H_w}{\lambda PET_e} \tag{20}$$

and 3) the surface temperature for the small, wet area is the same as the surface temperature for the large dry area. The first assumption is consistent with that of the original CR and has been adopted for estimating the PET_e, as we do not know in practice what the net radiation would be when the surface becomes saturated (Zhou and Yu, 2024). Even if we forego the assumption of an invariant net radiation and allow the net radiation to change with the surface moisture content, the complementary relationship still holds so long as the latent heat and sensible heat change in opposite directions (Appendix A). In addition, when scaled with PET_e, i.e., the energy constraint,

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a clear complementary relationship between ET and PET_a emerges across different regions/months, as shown in Fig. 2.

The second assumption differs from the assumption of the original CR (equation (3)) in which the increase in λPET_a equals the increase in the sensible heat. The original CR, however, is not supported by observations (Kahler and Brutsaert, 2006; Szilagyi, 2007). In contrast, the assumption underpinning equation (20) has been shown to be reasonable as the difference in β_w under the wet and dry conditions is very small due to coupled changes in temperature and humidity between the land surface and the atmosphere (Zhou and Yu, 2024). Considering the potential variations in β_w under wet and dry conditions, we further demonstrate that the complementary relationship between ET and PET_a and the positive relationship between ET and PET_e remain valid on theoretical grounds (Appendix A).

Equation (20) can be re-written to show that the relative increase, rather than the absolute increase, in the PET_a and sensible heat are identical under wet and dry conditions:

$$\frac{PET_a - PET_e}{PET_e} = \frac{H - H_w}{H_w} \tag{21}$$

This is because the net radiation not expended on ET is shifted to augment H as the land surface dries up. This causes PET_a to increase by a factor of $1/\beta_w$, i.e., the ratio of latent over sensible heat for a saturated surface in equation (15). This provides an explanation of the observed asymmetric relationship between ET and PET_a and clarifies the physical meaning of the coefficient of proportionality in the generalized linear form of the complementary relationship, i.e., $k = \beta_w$. In essence, β_w is the source and a measure of the degree of asymmetry in the complementary relationship.

The third assumption of identical surface temperature of the small wet area and large dry area was invoked in order to calculate the saturation vapor pressure at the wet surface, hence the wet Bowen ratio β_w with equation (11). The surface temperature of the small wet area, such as a Class A evaporation pan, would likely be lower than the surface temperature of the surrounding dry area, so would the associated saturation vapor pressure. Thus, equation (15) is best seen to yield an upper limit for PET_a in practice. A lower PET_a based on pan evaporation measurements would increase the empirically fitted parameter k in equation (4), which becomes closer to unity and makes the observed CR less asymmetric. At the same time, the Bowen ratio of the evaporation pan would be lower than β_w when its surface temperature is lower than T_s of the surrounding dry area. This is because β_w is positively related to T_s, as shown by rewriting equation (11) as follows:

$$\beta_w = \frac{\gamma}{\Delta + \frac{e_a^* - e_a}{T_s - T_a}} \tag{22}$$

where Δ is the slope of the saturation vapor pressure-temperature curve evaluated between T_s and T_a. Thus, the coefficient of proportionality, k, estimated from observations would diverge from the Bowen ratio of an evaporation pan, and they converge to and give physical meaning to the parameter k, i.e., $k = \beta_w$, only when the surface temperature of the pan is the same as its surrounding environment. This explains why previous CRs as well as their parameters remain empirical (Table 2).

4.2. Interpretation of the physically-based complementary relationship

The new derivation of the CR, i.e., equation (17), based on physical reasoning confirms that the complementary relationship is fundamentally governed by how energy is partitioned between latent and sensible heat under wet and dry conditions. Changes in the partitioning between the

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latent and sensible heat manifest themselves as a complementary relationship between ET and PET_a, as the latent heat is directly related to ET and the sensible heat proportional to PET_a with the wet Bowen ratio, β_w, as the coefficient of proportionality. While this physical mechanism is broadly consistent with the concept of the original CR and the existing CR formulations (Bouchet, 1963; Brutsaert and Parlange, 1998; Brutsaert, 2015; Crago et al., 2016; Granger, 1989; Szilagyi, 2007; Szilagyi et al., 2017; Tu et al., 2023), the physically-based CR formulation clarifies the nature of the complementary relationship between ET and PET_a and provides new insight into the physical basis for the complementary relationship.

The theoretically sound complementary relationship also merits further interpretation of the relationship between PET_e and PET_a. Equation (17) can be re-arranged as:

$$PET_a = \left(1 + \frac{1}{\beta_w}\right) PET_e - \frac{1}{\beta_w} ET \tag{22}$$

PET_a that is related to the atmospheric condition can be seen to depend on the energy constrained PET_e and moisture constrained ET. Let us consider two extreme cases: 1) where $ET = PET_e$ for saturated areas, such as the ocean and large lakes, we have the minimum of PET_a as PET_e; 2) where $ET = 0$ for surfaces of maximum dryness without supply of water vapor to the air, we have the maximum of PET_a as $\left(1 + \frac{1}{\beta_w}\right) PET_e$ and the largest difference between PET_a and PET_e, i.e., $\frac{PET_e}{\beta_w}$. This interpretation reinforces the notion that estimation of the PET_a based on the aerodynamics responds to water vapor supply via ET in addition to the energy constraint. The drier the air with reduced ET, the greater the difference between PET_a and PET_e.

4.3. Comparison with previous formulations of the complementary relationship

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Table 1 shows that most of the existing CR formulations share the same structure as equation (17). This is because these CR formulations are developed based on the same physical principles, i.e., partitioning of the net radiation shifting from latent to sensible heat from wet to dry conditions, from which the complementary relationship originates (Bouchet, 1963). The physically-based CR is identical to the two empirical CRs, each involving an empirical parameter, and can be used to interpret and give meaning to these parameters. For the asymmetric CR of Brutsaert and Parlange (1998), we have $b = \frac{1}{\beta_w}$, and $X_{min} = \frac{\beta_w}{1+\beta_w}$ for the rescaled CR of Crago et al. (2016). In particular, the physical meaning of X_{min} in the rescaled CR, i.e., the minimum value of $\frac{PET_e}{PET_a}$ when ET reaches zero, is consistent with our estimation of PET_e and PET_a (Table 2).

For the existing CR formulations without any parameters, or the so-called calibration-free formulations, they are essentially special cases of the physically-based CR. For example, when the air is saturated ($e_a = e_a^*$), we have $\beta_w = \frac{\gamma(T_s - T_a)}{(e_s^* - e_a^*)} = \frac{\gamma}{\Delta}$, the CR of Szilagyi (2007) becomes identical to the physically-based CR. However, this special case rarely occurs, even for saturated surfaces like oceans and lakes.

The non-linear CR formulation is also similar to the physically-based CR when $\beta_w \approx 0.22$ (Brutsaert, 2015). While this non-linear CR formulation has been validated with experimental data, it is not realistic, however, under the driest conditions, i.e., $\frac{PET_e}{PET_a} < \frac{\beta_w}{1+\beta_w}$. This is because the boundary condition of the non-linear CR, i.e., $\frac{PET_e}{PET_a} \to 0$ when $ET \to 0$, is not physically sound, as PET_a cannot be infinitely large and in fact PET_a is limited by $\frac{R_n}{\beta_w}$ as ET reaches zero and H equals

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R_n (Table 2). To overcome this problem, the non-linear CR has been combined with the rescaled CR to develop yet another calibration-free CR (Szilagyi et al., 2017). However, this formulation still represents a special case without physically meaningful parameters such as β_w to account for variations in the complementary relationship under different conditions.

Compared with these earlier formulations, the physically-based CR has many distinct advantages. First, PET_e and PET_a are clearly defined based on physical processes and can be estimated using observed data. Second, the basis for this new CR is physically sound and its derivation is purely based on shifts in the surface energy balance under wet and dry conditions with three assumptions. Third, the physical meaning of the coefficient of proportionality, i.e., the wet Bowen ratio β_w, is clear and it accounts for the degree of asymmetry in the complementary relationship across a wide range of environmental conditions. Moreover, β_w can be directly estimated from observed data without any calibration (equation (11)). This physically-based CR can therefore be widely applied for estimating ET across different regions at various time scales. Finally, the physically-based CR clearly quantifies the complementary relationship between ET and PET_a and the positive relationship between ET and PET_e, i.e., equations (18) and (19). This provides an enhanced understanding of the relationships among the three types of ET over land.

4.4. Implications for practical applications of the complementary relationship

Application of the physically-based CR for ET estimation requires values for β_w, PET_e and PET_a, all of which can be calculated using observations of meteorological variables and surface temperature. However, the fact that surface temperature data are not readily available may restrict

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broader application of this method for ET estimation. To address this limitation, the wet Bowen ratio, β_w, can be approximated as a function of air temperature:

$$\beta_w = \alpha \cdot \frac{\gamma}{\Delta} \tag{23}$$

where Δ is the slope of the curve for saturation vapor pressure as a function of temperature ($Pa \cdot K^{-1}$). The coefficient α typically varies from 0.15 to 0.3, and an approximate value of $\alpha \approx 0.24$ can be adopted (Yang and Roderick, 2019; Zhou and Yu, 2024).

While PET_e and PET_a are defined based on energy balance and aerodynamic principles, they can also be derived using modified versions of the Penman equation with an adjustment parameter k' (Zhou and Yu, 2024). When the evaporative surface is saturated, the Penman equation can be used to estimate both PET_e and PET_a, i.e., $ET = PET_e = PET_a$ (Penman, 1948). However, the direct application of the Penman equation would overestimate PET_e (Milly and Dunne, 2016, 2017; Zhou and Yu, 2025) and simultaneously underestimate PET_a in a dry environment. This occurs because PET_e assumes a large, saturated surface (Fig. 1b), leading to an overestimation of its aerodynamic component when observed meteorological variables corresponding to dry surface conditions are used by the Penmen equation. Concurrently, the energy supply required to sustain PET_a for a small, saturated surface is underestimated (Fig. 1a), as the Penman equation does not account for energy transferred from the surrounding dry environment. These issues can be resolved with the adjustment parameter k' (Zhou and Yu, 2024).

$$\lambda PET_e = \frac{\Delta R_n + k' \frac{\rho c_p (e_a^* - e_a)}{r_a}}{\Delta + \gamma} \tag{24}$$

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$$\lambda PET_a = \frac{\Delta R_n/k' + \frac{\rho c_p (e_a^* - e_a)}{r_a}}{\Delta + \gamma} \tag{25}$$

where e_a^* is the saturation vapor pressure at air temperature (Pa), and $e_a^* - e_a$ is the vapor pressure deficit. The parameter k' can be determined by equating PET_e from equations (12) and (24), and k' so determined can be used to compute PET_a with equation (25). Thus, β_w, PET_e and PET_a can be estimated using routine meteorological variables, enabling broader applications of the physically-based CR for ET estimation.

5. Conclusions

In this study, we proposed definitive estimators of the PET_e and PET_a based on the energy balance and aerodynamic principles and derived an alternative complementary relationship with a physically meaningful parameter, i.e., the wet Bowen ratio, which can be directly calculated from observations. The complementary relationship between ET and PET_a fundamentally originates from partitioning of the net radiation between latent and sensible heat, with ET directly related to the latent heat and PET_a proportional to the sensible heat. The wet Bowen ratio for a small, saturated area quantifies the degree of asymmetry in the complementary relationship. In addition, ET is linearly related to PET_e, with the latter representing the evaporative demand of the atmosphere or the energy constraint on ET over land. By clarifying the quantitative relationship between the three types of ET, this study advances our understanding of the complementary relationship and would promote and facilitate practical application of the complementary relationship for ET estimation across different regions and time scales.

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Appendix A: A variant of the complementary relationship for evapotranspiration

To derive the complementary relationship in equation (17), it has been assumed that 1) the net radiation for a dry area remains the same as the entire area becomes saturated; 2) the wet Bowen ratio for a small, saturated area within a large dry area is the same as that over the entire area if it were saturated. Here we consider and allow potential variations in the net radiation and the wet Bowen ratio under wet and dry conditions and re-examine the effectiveness of the complementary relationship between ET and PET_a and the nature of the relationship between ET and PET_e.

For the first assumption, let R_n be the observed net radiation for the dry area, and the net radiation for the entire area if it were saturated is R_n/k_1 with $k_1 > 0$. Similar to the energy balance for the dry area shown in equation (16), the energy balance for the area when saturated is given by

$$R_n/k_1 = \lambda PET_e + H_w \quad (A1)$$

For the second assumption, let β_w be the wet Bowen ratio over the small, saturated surface, and the Bowen ratio if the entire area were saturated is $k_2\beta_w$ with $k_2 > 0$. The sensible heat for the large saturated area, i.e., H_w, is expressed as

$$H_w = k_2\beta_w\lambda PET_e \quad (A2)$$

Equation (A1) can be re-written as:

$$R_n/k_1 = (1 + k_2\beta_w)\lambda PET_e \quad (A3)$$

Replacing R_n with $\lambda ET + H$, and noting H equals $\beta_w\lambda PET_a$, we have

$$(\lambda ET + \beta_w\lambda PET_a)/k_1 = (1 + k_2\beta_w)\lambda PET_e \quad (A4)$$

or

$$ET = k_1(1 + k_2\beta_w)PET_e - \beta_w PET_a \quad (A5)$$

Equation (A5) suggests that the complementary relationship between ET and PET_a and the proportional relationship between ET and PET_e still hold despite the potential differences in the net radiation and the wet Bowen ratio under wet and dry conditions. It is clear that equation (A5) is reduced to equation (17) when $k_1 = k_2 = 1$. Therefore, equation (A5) represents a more generalized complementary relationship without the restrictive assumptions required by equation (17).

Data availability. The Fluxnet2015 dataset is publicly available from https://fluxnet.org/data/fluxnet2015-dataset/.

Author contribution. S.Z. conceived the study, performed the data analysis, and wrote the initial manuscript. B.Y. provided critical revisions and edits.

Competing interests. The authors declare that they have no conflict of interest.

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