The authors claim to have derived a new physically-based CR via 'rigorous derivation based on physics', unlike previous versions, which they deem only empirical.

In fact, what the authors achieve is making use of several hypothetical and highly speculative assumptions (lines 210-215):

i) The surface temperature of a small, freely evaporating water body is always the same as that of the surrounding drying land (this would require a heat conduction as effective as evaporative cooling, which is highly unlikely under realistic conditions, thus the corresponding potential evaporation rate remains speculative only);

ii) The Bowen ratio (βw) written for such a small water body does not change during drying of the environment (contradicting the constant surface net radiation assumption stated).

None of the above assumptions are valid in general and none been ever confirmed rigorously by any study.

Response:
We respectfully disagree with the reviewer's comment. Assumption i) is essential for formulating a complementary relationship with a physically meaningful parameter, while Assumption ii) has been validated in our previous studies (Zhou and Yu, 2024 Journal of Hydrology; Zhou and Yu, 2025 Global Change Biology).

To derive a complementary relationship (CR) for evapotranspiration (ET), we must:
1) Define and estimate potential ET (PET) and apparent potential ET (PETa).
2) Establish the relationship among ET, PET, and PETa.

<u>Definition of PET and PETa:</u>
PET represents the rate of ET that would occur when water supply is unlimited at the evaporative surface (i.e., a fully saturated surface). Since PET is constrained by available energy (the net radiation), it is estimated using the energy-based approach and termed as PETe in our study. PET cannot be directly observed unless the entire surface is saturated, such as over a lake or the ocean. Our comparison of PET equations, including Priestley-Taylor and Penman equations, demonstrates that PETe, estimated using net radiation and the wet Bowen ratio (β_w), is the most reliable PET estimator (Zhou and Yu, 2024; 2025).

PETa represents the ET rate from a small, saturated surface within a larger, unsaturated area (e.g., an evaporation pan), with energy supplied by both net radiation and the surrounding environment. However, the energy transfer from the surroundings varies with the pan size, leading to variations in the surface temperature of the wet area and the corresponding evaporation rate. Due to the inherent ambiguity in the definition of PETa, it is essentially indeterminate in practice. The lack of a definitive estimator further complicates PETa estimation, making it challenging to develop a physically-based CR formulation. To address this issue, we estimate the upper limit for PETa by assuming that the surface temperature of the small wet area is maximized and equal to that of the surrounding dry area (Ts).

Complementary Relationship Framework:

The complementary relationship in essence describes the interplay between ET, PET, and PETa. While their exact quantitative relationships remain uncertain across different formulations (Table S2), all valid CR formulations must satisfy two boundary conditions:

- ET = PET = PETa under wet conditions.
- ET < PET < PETa as the surface dries up.

Comparison with Previous CR formulations:

We derived the CR formulation using two well-defined estimators of PET and PETa, along with a physically meaningful parameter (β_w):

- Assumption ii) is used to estimate PET, i.e., PETe.
- Assumption i) is used to estimate the upper limit of PETa by maintaining the surface temperature of the small wet area equal to that of the surrounding environment.

Quite dissimilar to our approach, many previous CR studies in fact involve other two assumptions, i.e., Assumptions a) and b) below. These assumptions are so commonly made implicitly that they have been taken for granted. In fact, they have rarely been clearly stated and validated.

- Assumption a): PETa can be estimated using Penman equation (PETpm) or pan evaporation.
- Assumption b): PET can be estimated using the Priestley-Taylor equation (PETpt).

We adopt Assumptions (i) and (ii) instead of Assumptions (a) and (b) for three main reasons:

1) **Reliability of PETe**: Assumption (ii) provides a robust PET estimator, i.e., PETe. Both Assumption ii) and the reliability of PETe have been validated in our previous studies (Zhou and Yu, 2024; 2025). Importantly, these studies have demonstrated that PETe is

much better than PETpt in terms of estimating ET under wet conditions (e.g., over the ocean) and that PETe can be used for estimating the PET over land based on the Budyko framework.

2) **Indeterminacy of PETa**: PETa is indeterminate, as its value varies with the size of the small wet area. Additionally, the Penman equation should not be used to estimate PETa as it neglects energy transfer from the surrounding environment (Zhou and Yu, 2024). We define an upper limit of PETa using Assumption i) to ensure a physically meaningful CR formulation. As discussed in Section 4.1, only when PETa is maximized can we derive the CR with a physical meaningful parameter (β_w). This is because the empirical parameter, k, estimated through calibration with observations would diverge from the Bowen ratio of an evaporation pan when its temperature is lower than the surface temperature of the surrounding environment (Ts), and they converge to and give physical meaning to the parameter k, i.e., $k = \beta_w$, only when the surface temperature of the pan is the same as its surrounding environment. This resolves the empirical nature of many previous CR formulations (Table 2).

3) **Consistency with the boundary conditions**: Our estimates of PETe and PETa allow CR derivation from the fundamental energy balance equation (Rn=ET+H). They also ensure consistency with two key boundary relationships, i.e., ET=PETe=PETa under wet conditions and ET<PETe<PETa when the surface dries up (see Table 2 and Fig. 2). In contrast, previous CR formulations often violate these conditions, particularly when PETpt and PETpm are directly adopted to estimate PET and PETa, respectively. For instance, using PETpt and PETpm directly results in inconsistencies under wet conditions, i.e., ET≠PETpt and PETpt≠PETpm (see Fig. S3 of Zhou and Yu, 2025 and Fig. 5 of Yang et al., 2019). Since PETpt and PETpm fail to meet the required boundary conditions, they should not be used to formulate the CR.

They proceed further and claim that neither the Penman nor the Priestley-Taylor equation is appropriate for estimating the corresponding apparent potential evaporation rate or the evaporation rate of the wet environment, even though that these equations are the backbone of practically all existing CR methods. Yet, when they decide to discuss the practical applications of their version of the CR they turn to a modified version of the Penman equation with an **empirical coefficient** (k') to be determined from measurements (eqs. 25 & 26). Note that the original Penman equation does not have this additional coefficient. Also, as the land surface temperature is typically unknown in practical applications, they introduce **another empirical coefficient** (α) to convert the Bowen ratio of equilibrium evaporation into βw in eq. (24).

One would expect that when a new method is introduced then its practical predictive superiority is showcased over existing similar methods it is supposed to replace. Such a validation is completely missing here.

Response:
We agree that Penman and Priestley-Taylor equations have been extensively used in hydrology, climate, agriculture, and many other relevant fields. However, recent research has questioned their reliability in estimating PETa and PET (Milly et al., 2016; 2017; Greve et al., 2019; Zhou and Yu, 2024; 2025).

The Penman equation (PETpm) combines PETe and PETa while eliminating the term Ts, assuming that 1) the surface is saturated and 2) PET=PETe=PETa, which are only valid under wet conditions (see Section 2 of Zhou and Yu, 2024). Direct application of PETpm over unsaturated land leads to PET overestimation (due to dry atmospheric conditions, such as higher vapor pressure deficit and warmer temperatures) and PETa underestimation (as it neglects energy transfer from the surroundings). To resolve this issue, an adjustment parameter $k' = \frac{1+1/\beta}{1+1/\beta_w}$ (where β is the Bowen ratio and β_w is the wet Bowen ratio) can modify the Penman equation for estimating PET and PETa using routine meteorological data when Ts is unknown (Zhou and Yu, 2024).

The Priestley-Taylor equation (PETpt), a simplified version of PETpm with an empirical coefficient (α), has commonly been used for PET estimation. However, PETpt exhibits large biases in estimating ET over the ocean (Yang et al., 2019; Zhou and Yu, 2025), making it unsuitable for PET estimation. Moreover, PETpt overestimates the sensitivity of PET to temperature, leading to an exaggerated increase in PET under warming climates (Yang et al., 2019; Zhou and Yu, 2025). These issues can be resolved by using PETe instead of PETpt.

This study derives a CR formulation based on PETe, PETa, and the physically meaningful parameter (β_w). Multiple approaches can be used to estimate PETe, PETa, and β_w, depending on data availability:
1) When Ts and sensible heat (H) are known (e.g., flux tower sites and reanalysis products), β_w can be calculated using equation (11), while PETe and PETa can be derived from equations (12) and (15), respectively.
2) When Ts is available (from *in situ* or remote sensing observations) but H is not, PETa can also be estimated using equation (14).
3) When both Ts and H are unknown, β_w can be estimated from routine meteorological observations using equation (23) and PETe and PETa using equations (24) and (25).

Based on these approaches to estimation of PETe, PETa and β_w depending on data availability, the newly derived CR formulation has significant potential for estimating ET. We have discussed its advantages comparing with previous CR formulations (Table 2) in Section 4.3 and discussed its practical implications in Section 4.4. However, applying the CR formulation specifically for ET estimation is not the primary focus of this, for most part, analytical work. A comprehensive and systematic evaluation of its effectiveness for ET estimation, along with comparisons to other established ET estimation methods (such as FLUXCOM and GLEAM), is an important next step. This should be addressed in a dedicated future study to rigorously validate whether the CR formulation can offer improved performance or advantages over existing approaches.

In this study, we aim to advance our understanding of the complementary relationship by clearly defining and estimating PETe and PETa and establishing the quantitative relationships among ET, PETe, and PETa in a CR formulation with a physically meaningful parameter (β_w). **Notably, the CR formulation in equation (17) and the relationships among ET, PETe, and PETa as shown in equations (18) and (19) have been validated using data from 146 Fluxnet sites (see Fig. 2).**

The authors' main equation (eq. 17), when combined with eqs. 12 and 15 yields simply: ET = Rn – H, which is a rather trivial formulation of the energy balance equation. All the authors do is combine this energy balance equation with the definition of the Bowen ratio and express them in a way that looks like a CR equation, i.e., their eq. 17. For βw they use the actual land surface and air temperature plus vapor pressure values (i.e., eq. 11) by capitalizing on assumption ii). An additional problem is that they still need to know H unless they employ the above mentioned modified Penman equation.

So what is the new insight from the authors' 'theoretically sound' CR? I am not sure.

Response:
We respectfully disagree with the characterization of the energy balance equation as a trivial formulation. On the contrary, it plays a fundamental role in the complementary relationship, which is governed by the partitioning of energy between latent and sensible heat under wet and dry conditions (see Section 2). Changes in the partitioning between the latent and sensible heat manifest themselves as a complementary relationship between ET and PETa, as the latent heat is directly related to ET and the sensible heat proportional to PETa with the wet Bowen ratio (β_w).

The key new insight is that by clearly defining and estimating PET and PETa, the complementary relationship naturally emerges from the energy balance equation, which serves as its foundation. This revelation and reification eliminate the need to construct complex, non-linear relationships among ET, PET, and PETa that rely on unknown empirical parameters, as done in many previous studies (see the review by Han and Tian, 2020 HESS). In fact, CR formulations based on PETpt and PETpm fail to satisfy the boundary conditions of the complementary relationship, and many existing CR formulations are either special cases or unrealistic under certain conditions (see our response to the first comment and Section 4.3).

Another new insight is that the physical meaning of the CR parameter k, identified as the wet Bowen ratio (β_w), is explicitly clarified. β_w accounts for the degree of asymmetry in the complementary relationship across diverse environmental conditions. Since β_w can be directly estimated from observed data without calibration, the physically-based CR can be applied for estimating ET across different regions and time scales.

Regarding the estimation of β_w and PETe, Assumption ii) has been validated as β_w remains fairly constant due to coupled changes in temperature and humidity of the air and at the land surface from dry to wet conditions (Zhou and Yu, 2024). Furthermore, β_w provides a robust estimate of the Bowen ratio for wet surfaces. In contrast, the wet Bowen ratio derived from PETpt (β_{pt}) is highly sensitive to temperature variations between wet and dry conditions and exhibits significant biases under wet conditions (Zhou and Yu, 2024; 2025).

As for the estimation of PETa, our formulation offers flexibility by providing three distinct approaches depending on data availability (see our response to the comment above). This adaptability ensures that the CR formulation remains practical and applicable even when direct measurements of sensible heat flux (H) are unavailable. Thus, the reliance on H is not a limitation but rather a feature that enhances the versatility of our approach.

Based on these observations I can only recommend **rejection** of the manuscript. A thoroughly revised version of the manuscript that is not based on highly questionable assumptions [i.e., i) and ii)] could only be publishable if the authors demonstrate its practical predictive superiority (i.e., that it indeed leads to better ET estimates when differences in the number of parameters to calibrate and input requirements are properly accounted for) over existing CR models and drops any claim that it is a 'theoretically sound' and 'rigorously derived' CR version (in opposition to other existing CR versions) as all versions of the CR today are empirical to varying degrees, if not else then for the Penman equation (with its empirically derived wind-function) they employ.

Response:
We appreciate the reviewer's critical feedback and the opportunity to further clarify the focus and contributions of our study. While we understand the concerns raised, we respectfully argue that the theoretical advancements and foundational insights presented in this paper are significant and warrant publication, even in the absence of extensive practical validation at this stage. Below, we outline the key reasons why this study is innovative, scientifically valuable, and deserving of publication:

1) Theoretical focus and novelty
This paper is primarily a theoretical contribution aimed at advancing the fundamental understanding of the nature of the complementary relationship for ET. Unlike previous studies that often rely on empirical formulations and assumptions, our work provides a physically-based derivation of the CR formulation. By clearly defining and estimating PET and PETa, we establish a more robust and theoretically sound foundation for the CR. This represents a significant departure from many existing approaches, which frequently depend on empirical parameters and lack clear physical justification of the relationships among ET, PET, and PETa.

2) Clarification of the CR parameter (β_w)
One of the key innovations of this study is the explicit identification and interpretation of the CR parameter k as the wet Bowen ratio (β_w). This parameter, which accounts for the asymmetry in energy partitioning between latent and sensible heat under varying environmental conditions, is directly estimable from observational data without the need for calibration. This eliminates the need for empirical fitting, which has been a major limitation of many previous CR formulations. By grounding k in physical principles, our approach enhances the generalization and applicability of the CR across diverse regions and time scales.

3) Resolution of boundary condition issues
Our formulation guarantees consistency with the fundamental boundary conditions required by the CR, namely ET=PET=PETa under wet conditions and ET<PET<PETa under dry conditions. This is a critical improvement over many existing CR models, which often violate these conditions, particularly when PETpt and PETpm are used to estimate PET and PETa. By addressing these inconsistencies, our work provides a more robust framework for understanding and modeling the complementary relationship.

4) Flexibility and adaptability

The proposed CR formulation is designed to be flexible and adaptable to different data availability scenarios. Whether surface temperature (Ts) and sensible heat (H) are known (e.g., from flux towers or reanalysis products) or must be estimated from routine meteorological observations, our approach provides multiple pathways for estimating PET, PETa, and β_w. This adaptability ensures that the method can be applied in a wide range of practical settings, even when direct measurements are unavailable.

5) Validation and foundational insights

While this paper is primarily theoretical, we have validated key aspects of our CR formulation using data from 146 Fluxnet sites (see Fig. 2). These results demonstrate the robustness of our approach in capturing the complementary relationship between ET and PETa, and the positive relationship between ET and PETe. Furthermore, our findings have been supported by previous studies (Zhou and Yu, 2024; 2025), which validate the stability of β_w and the reliability of PETe as a PET estimator.

6) Broader implications and future directions

The theoretical advancements presented in this paper have far-reaching implications for the fields of hydrology, climatology, and environmental science. By providing a more rigorous and physically consistent framework for the CR, our work lays the groundwork for future studies to develop improved ET estimation methods. While we acknowledge that practical validation and comparison with existing models are important next steps, these efforts are beyond the scope of this theoretical paper and should be addressed in dedicated follow-up studies.

7) Why this paper should be published

This paper makes a significant contribution to the scientific community by addressing long-standing theoretical challenges in the formulation and interpretation of the complementary relationship. It provides a clear, physically-based framework that resolves many of the empirical shortcomings of the existing CR formulations. While practical applications and comparisons with other methods are important, they do not diminish the value of the theoretical insights presented here. Publishing this work will enable the scientific community to build upon these foundational advancements, ultimately leading to more accurate and reliable ET estimation methods.

In conclusion, we respectfully request that the paper be considered for publication based on its theoretical rigor, innovative insights, the potential to challenge current practice, and ultimately to advance the field. We believe that the original contribution of this study will inspire further research and practical applications, making it a valuable addition to the literature on evaporation.

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