egusphere-2025-109-v3 Correction

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Dear Marko,

This document addresses two typographical errors identified during the proofreading of our manuscript. These corrections do not alter the article's content but are important to avoid potential confusion for readers.

The first error is in the TS4 point of version 3 of the manuscript, specifically in the sentence on lines 27–32 of page 7. The original sentence reads:

> In both cases, we aim to balance the cost function between the background term and the observation term, but we no longer seek $\boldsymbol{x}$ such that $\boldsymbol{\mathcal{H}}(\boldsymbol{x}) \approx \boldsymbol{y}$. Instead, we now seek $\boldsymbol{w}$ that determines the linear combination $\mathbf{H}\mathbf{X}_{\mathrm{b}}'\boldsymbol{w}$, which is equal to the distance $\delta\boldsymbol{y}$ such that $\delta\boldsymbol{y} \approx \boldsymbol{\mathcal{H}}(\boldsymbol{x}) - \boldsymbol{y}$.

The error lies in the last equation within this sentence, which should be corrected to:

$$\delta\boldsymbol{y} \approx \boldsymbol{\mathcal{H}}(\boldsymbol{x_b}) - \boldsymbol{y}. \tag{1}$$

This correction aligns with equation (18) of the article, which defines the cost function:

$$J(\boldsymbol{w}) = \frac{1}{2}\left(\mathbf{H}\mathbf{X}_{\mathrm{b}}'\boldsymbol{w} + \boldsymbol{\mathcal{H}}(\boldsymbol{x_b}) - \boldsymbol{y}\right)^T \mathbf{R}^{-1}\left(\mathbf{H}\mathbf{X}_{\mathrm{b}}'\boldsymbol{w} + \boldsymbol{\mathcal{H}}(\boldsymbol{x_b}) - \boldsymbol{y}\right) + \frac{1}{2}\boldsymbol{w}^T\boldsymbol{w}. \tag{2}$$

From this, it is clear that $\mathbf{H}\mathbf{X}_{\mathrm{b}}'\boldsymbol{w} = \boldsymbol{\mathcal{H}}(\boldsymbol{x_b}) - \boldsymbol{y}$ ensures the observational term becomes zero.

The second error is in equation (5) on page 6, which presents the following cost function:

$$J(\boldsymbol{x}) = \frac{1}{2}\left(\sum_{t}^{N_t} \boldsymbol{\mathcal{H}}_t(\boldsymbol{x}) - \boldsymbol{y}_t\right)^T \mathbf{R}_t^{-1}\left(\boldsymbol{\mathcal{H}}_t(\boldsymbol{x}) - \boldsymbol{y}_t\right) + \frac{1}{2}\left(\boldsymbol{x} - \boldsymbol{x_b}\right)^T \mathbf{B}^{-1}\left(\boldsymbol{x} - \boldsymbol{x_b}\right). \tag{3}$$

The brackets in this equation are incorrectly placed, reducing readability. We propose the following revision:

$$J(\boldsymbol{x}) = \frac{1}{2}\sum_{t}^{N_t}\left(\boldsymbol{\mathcal{H}}_t(\boldsymbol{x}) - \boldsymbol{y}_t\right)^T \mathbf{R}_t^{-1}\left(\boldsymbol{\mathcal{H}}_t(\boldsymbol{x}) - \boldsymbol{y}_t\right) + \frac{1}{2}\left(\boldsymbol{x} - \boldsymbol{x_b}\right)^T \mathbf{B}^{-1}\left(\boldsymbol{x} - \boldsymbol{x_b}\right). \tag{4}$$

We apologies for these two typos and kindly request your approval of these corrections. We thank you for your understanding.

Best regards,

Simon Beylat