Reply to Referee Comment #3 (RC3)

The authors would like to thank the referee for the invaluable comments and suggestions. The following are the replies for each point of the comment, together with specific revisions that are made. The original comments are in *red italic* font and listed in paragraphs, with our reply following each paragraph separately. The revisions are also highlighted in the revised manuscript in *red* and marked by ***RC3***.

Review of "On the Statistical Relationship between Sea Ice Freeboard and C-Band Microwave Backscatter – A Study with Sentinel-1 and Operation IceBridge"

Summary

In this paper, the authors use a combination of data from Operation IceBridge, ICESat-2 and Sentinel-1 to investigate the statistical relationship between the altimetric freeboard and C-band backscatter signature for assessing the feasibility to predict the 2D variability of altimeter freeboards that typically offer limited spatial sampling (in comparison to SAR). The analysis carried out in this paper could be a good contribution to the sea ice remote sensing community. However, the paper in its current state is not ready for publication. I recommend major revisions.

Major comments

> *One of my main concerns is the localized aspect of the statistical relationships. You demonstrate that even locally, replicating freeboard with SAR observations is challenging because of many factors: change in local sea ice conditions, scattering mechanisms. Therefore, I am not convinced of the actual usefulness of the work reported here. A more complete analysis would include looking at coincident data between sentinel-1 and ICESat-2 which would provide more confidence on the feasibility of upscaling ICESat-2 measurements.*

Reply: Regarding the referee's concern about the locality of the relationship and the usefulness of the method, we would like to argue that the freeboard prediction can be accurately predicted, including the following aspects:

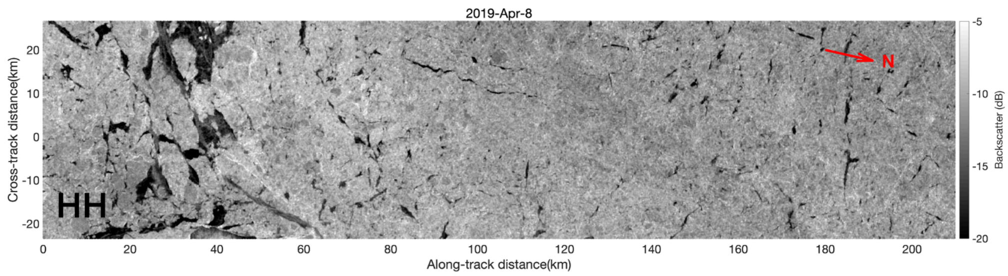
First, the spatial locality of the statistical relationship is mainly caused by the small-scale variability of the sea ice. As shown in our analysis with the 9km segment lengths for deriving the relationships (Fig. 6 of the original manuscript), the regression model parameters (slope and intercept) show gradual changes along the flight track, and they are similar between the inbound and the outbound track. Furthermore, at 27km segment length, the variability of these parameters are effectively reduced, implying that their variability is caused by the limited representation at 9km segment lengths. Therefore, the local spatial window that is large enough for accumulating freeboard-backscatter samples is necessary so as to ensure the stability of the relationship and overcome its locality during prediction.

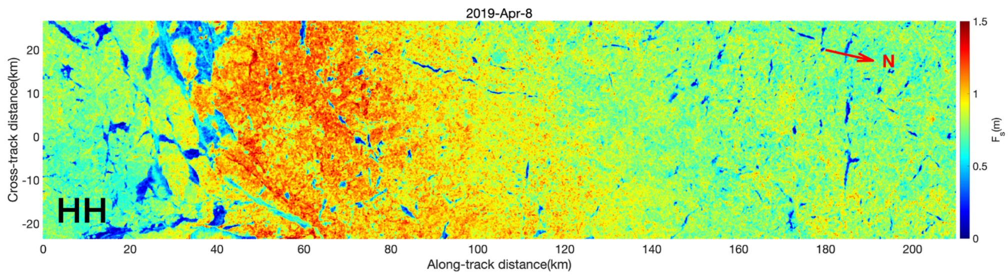
Second, temporal locality is also required for both the collocation between freeboard and backscatter measurements and the derivation of the prediction model. The temporal locality is influenced by various physical processes. Events like ice melt or snowfall can alter the snow-ice interface properties, potentially degrading the freeboard-backscatter relationship at synoptic scales. Ice deformation can also impact this relationship by degrading the collocation between the two measurements (Sec. 4.3 of the manuscript).

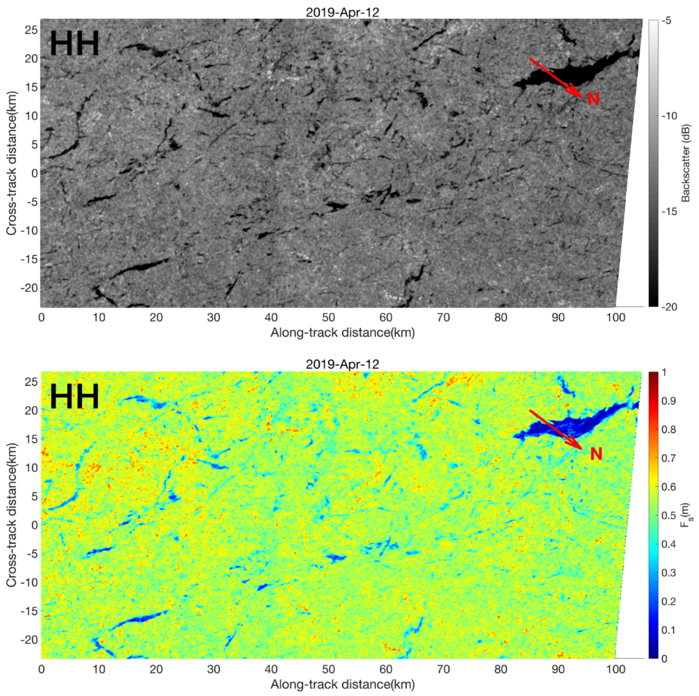
Our supplementary analysis (Fig. S7 and S8) demonstrates significant correlation between freeboard and backscatter over 14-day windows of sample segment on 2019-Apr-8. For the sample segment on April 12th, between April 5th and 12th, significant sea ice drift and deformation is present for the sea ice cover around the sample segment. Correspondingly, the correlation coefficients between Fs and backscatter witness significant drops. This suggests that when the ice conditions permit, the time window for ICESat-2/Sentinel-1 synergy can potentially be extended to 14 days.

In order to accommodate the locality of the statistical relationship, we will carry out large-scale retrieval by both: (1) deriving the relationship with locally collocated observations by the altimeter and SAR satellite, and (2) carrying out prediction within the periphery of these observations.

Finally, although our focus is the prediction of freeboard distributions at the altimeter's native resolution, we prefer to demonstrate the capability of the method with the freeboard map examples using linear regressions and HH-polarized backscatter maps at 200m scale. The results for the OIB campaign on both April 8th and 12th are shown below. We consider that at 200m scale, the linear regression model is good enough for the production of freeboard maps. In the cross track direction, the freeboard maps cover 50km, much wider than the OIB ATM/IS2 swath. In the future study, we will further our study to quantify the optimal temporal and spatial scales for: (1) the collocation between the altimetric scan and SAR maps, and (2) the prediction of freeboard maps and freeboard distributions.







I think the clarity of the paper could be greatly improved if the structure was revised, and some unnecessary text was removed. A lot of relevant information is in the supplement section and should be moved to the main text. In addition, some technical terms are not defined and datasets not introduced which hinder the understanding of the paper.

Reply: We appreciate your suggestions and have made several revisions to improve the paper structure and enhance its readability.

We have added a new section titled "*Methodology*" immediately after Section 2. This section provides a detailed description of the methods and technical detail used in our study. Moverover, we have moved the content of Appendix B into the main text. We have reviewed the manuscript to ensure that all technical terms are clearly defined and all datasets are adequately introduced.

Minor comments/questions

In the introduction, specifically in the problems section, you talk mostly about the limited synergy between ICESat-2 and SMOS, due to the sampling of ICESat-2. You raise that as a problem you want to address, but it is not mentioned again in the paper.

Reply: The topic of ICESat-2 and SMOS synergy is a related topic in terms of altimetry representation. Indeed we are not able to return to this issue in this study, so we have deleted it in the revised manuscript.

Why do you collocate OIB passes from the same day? There is an overlap but do you determine the correction from that overlap and apply it to all the cross-track samples?

Reply: The collocation between OIB passes is necessary: since there are both slight sea ice drift and locating uncertainty, the OIB passes can not be merged directly. The main purpose is to merge the passes and form a freeboard map that spans a wider area in the cross-track direction (~1500m). The merged freeboard map facilitates the analysis of its relationship with the backscatter at a range of spatial scales.

We confirm that the overlap of adjacent OIB passes are carried out by their overlapping part. The ATM swath width is about 500m and the adjacent OIB passes overlap by about 10%. And the collocation is based on maximizing the correlation of the freeboard between the adjacent passes (detailed examples in Appendix A). Specifically, we first divide the OIB track in the along-track direction into small segments, and then carry out the collocation and merging of the freeboard map.

The OIB campaigns on April 8th and 12th were organized as racetracks, so for both the outbound and the inbound passes, the merged freeboard map is about 1500m wide. Furthermore the inbound and outbound freeboard maps are separated by about 3000m (i.e., the same separate between beam #1 and #2 of ICESat-2).

The description of the OIB processing is confusing: you look at the correlation between successive OIB passes on the same day. What does that tell you?

Reply: The correlation is carried out over the overlapping part of adjacent OIB passes. We carry out drift correction to maximize the correlation, in order to merge these passes into a wide freeboard map (examples in Fig. 2 and 3). The correction to maximize the correlation is potentially indicative of the relative drift of the sea ice between the visit OIB times.

For revision and clarity, we add the detail of the correlation between OIB passes before the collocation collocation in Fig. A1 and A2. For the OIB segment on April 12th, the correlation between OIB passes before collocation is indeed low. This is primarily due to the sea ice drift during the different visit times of the OIB passes.

You need to provide more details on the collocation between S-1 and OIB. Your correlation analysis shows values in the along-track direction only (Fig 4 and 5). But OIB provides you maps of freeboard. Do you apply the same drift correction for all the cross-track samples

Reply: We divided the OIB tracks into 9km segments when collocating with S-1 data. We then performed collocation for each 9km outbound segment and each 9km inbound segment independently. Collocation is performed in both the along-track and cross-track directions.

In Figures 4a and 5a, we showed the drift corrections applied to each segment. Each vector represents the drift correction for a specific 9km segment, including along- and cross-track components. The start position of the vector indicates the original position of the OIB segment before drift correction, while the end position of the vector shows the corrected position after applying the drift correction. Specifically, blue vectors represent the drift corrections applied to the outbound segments, and red vectors represent the drift corrections applied to the inbound segments.

As suggested by the referee, we have added a detailed description of the collocation process between OIB Fs and S-1 in the revised manuscript.

You mention that you "validate" your OIB freeboard estimates with the OIB Level 4 product. However, this data product is not described in the text. Does it use the same approach for retrieving freeboard?

Reply: We have added an introduction to the official OIB L4 product in Section 2.1.

How do you determine the reference sea surface elevation for the calculation of the OIB freeboards?

Reply: We adopted the lowest elevation method to retrieve the OIB freeboard. Specifically, we extracted the lowest 1‰ of elevation samples within each 10 km segment and then interpolated these extracted samples to construct the local water level by the Inverse Distance Weighting (IDW) method. The method for determining the sea surface is described in Appendix A. This approach is verified, to first order, through the consistency with the official OIB L4 product.

Please define interquartiles. Also are you referring to interquartile range?

Reply: We confirm that we are indeed referring to the interquartile range (IQR). As suggested, we have added a definition of the interquartile range to the revised manuscript.

Line 193: please explain what you mean by:" After binning to σ0,"

Reply: The phrase "After binning to σ0" refers to the process of grouping the OIB freeboard samples into *σ0* bins. Specifically, the binning is based on 1 dB increments of σ0. Within each bin, we calculated the mean freeboard within the interquartile range. We have added a detailed explanation of this process when the term first appears in the manuscript.

Line 228: you should define the term "heteroskedasticity"

Reply: We have now avoided the use of the term “heteroskedasticity”. It describes the statistical phenomenon where, for larger $\sigma0$ bins, both the mean value of freeboard and its variability increase.

You should define the Kolmogorov-Smirnov (K-S) distance

Reply: We have added the formal definition of the Kolmogorov-Smirnov (K-S) distance to the manuscript.
The K-S distance is a statistical measure used to compare the similarity between two probability distributions, or to assess how well a sample fits a theoretical distribution. When comparing two sample-based distributions, the K-S distance is:

$$D = \sup_x |F_n(x) - G_m(x)|$$

where Fn(x) and Gm(x) are the empirical CDFs of the two samples. The sup (supremum) means the maximum value over all x.

I believe your description of the ICESat-2 data products is not accurate. Please check the along-track resolution for weak and strong beams as well as the footprint. Also you mention considerable uncertainties (line 418). Please clarify what you mean.

Reply: We have made revisions to be more clear on the ICESat-2 data. For clarification: the *considerable uncertainty* is the per-photon height uncertainty, not the typical ATL07 beam segment height. The ATL07 beam segment height is shown to be lower than 3 centimeters, which is highly precise. The sentence has been revised in the revised manuscript: “*The photon-level elevation measurements represent a similarly fine spatial scale to the OIB ATM, but contain higher uncertainty than that of the beam segment elevation product (ATL07)*”.

Line 316: Your comment on ICESat does not seem to belong here.

Reply: It has been deleted in the revised manuscript.

Your summary and outlook section is too short. I believe you should merge it with the discussion section.

Reply: According to the suggestion, we have merged the Summary and Outlook section with the Discussion section in the revised manuscript.