Reply to Dr. Karl Kortum (Referee #1)'s comments (RC1)

The authors would like to thank Dr. Karl Kortum for the invaluable comments and suggestions. The following are the replies for each point of the comment, together with specific revisions that are made. The original comments are in *blue italic* font and listed in paragraphs, with our reply following each paragraph separately. The revisions are also highlighted in the revised manuscript in blue and marked by ***RC1***.

The authors present a study on the local extrapolation of freeboard statistics using Sentinel-1 backscatter. I found the paper to be well written and researched. The results add value and new insight into a recently emerging topic in the scientific discourse concerning the combination of altimetry and SAR instruments. The methodologies used are well suited to the investigation.

Overall, I think the paper certainly merits publication and should be ready for it after a few additions/clarifications.

Major:

1. Concerning smaller scales:
I think it would be beneficial for the evaluation of the extrapolation scheme to also assess the accuracy of the predicted freeboard distribution on a smaller scale or per-pixel basis. Especially concerning the ICESat-2 results, the 27km segment length is quite large and it would be very interesting to know how well the relationships from HH brightness to freeboard distribution hold on a smaller scale. It would also make it easier to compare the accuracy of this extrapolation to existing methods (Macdonald 2024, Kortum 2024).

Reply: Thanks for the helpful comments. We would like to first make clarifications on the predictions we carry out. Specifically, we have developed two prediction algorithms. The first prediction method is the traditional, "pixel-wise" prediction of the freeboard, similar to Macdonald et al. (2024) and Kortum et al. (2024). This method yields freeboard maps, and it is introduced in Sections 3.1 and 3.2.

The second method targets at the prediction of the freeboard distribution at the altimeter's native resolution (e.g., 1-meter for OIB ATM, covered in Sec. 3.3). For instance, with OIB ATM the freeboard distribution's resolution is 1m, while for IS2 (ATL07) the resolution is the beam segments' nominal size (i.e., about 30 m for the strong beams). The second method is based on: (1) binning freeboard samples according to backscatter, and (2) the fitting of freeboard distribution in each bin. Therefore, this method is free from regression models and their potential problems.

For either method, we accumulate collocated samples between the altimeter and the SAR image to train the prediction model. And the samples are from a local region of 9km (along-track). Hence 9km is not the resolution of the freeboard map (for the first method).

For the traditional method which predicts the freeboard map, in the manuscript we did not show the prediction results, but we did show the regression model's parameters (Fig. 6).

Practical resolution of freeboard maps with Sentinel-1 images is about 200m, at which the regression model's R2 is sufficiently high. As pointed out by the referee, this method is comparable to existing ones in Macdonald et al. (2024) and Kortum et al. (2024).

Therefore, according to the referee's suggestion, we have conducted a comparative analysis of the statistical relationships across these studies. It should be noted that the sampling scale in existing literature is based on basin-scale observations, which is much larger than ours (at 9km or 27km). To improve comparability, in this comparison, we extended the sampling scale for our training data to the entire OIB track (~200 km in length). We further aligned the resolution of the statistical relationships with the other two works. The comparison results are presented in the following table:

R	40m Fs VS 40m backscatter			100m Fs VS 100m backscatter		
	Macdonald et al. 2024	This study		*Kortum et al. 2024*	This study	
		April 8	April 12		April 8	April 12
HH	\	0.48	0.40	0.49	0.58	0.47
HH(FYI)	0.37	0.53	0.44	0.18	0.63	0.50
HH(MYI)	0.48	0.40	0.24	0.34	0.49	0.30
HV	\	0.49	0.47	0.62	0.58	0.59
HV(FYI)	0.40	0.53	0.50	0.32	0.62	0.60
HV(MYI)	0.49	0.41	0.37	0.49	0.50	0.48

There are two spatial scales for comparing our results with existing studies: 40m (for comparing with Macdonald et al. (2024)) and 100m (for comparing with Kortum et al. (2024)). In both comparisons, we found that the correlation of the freeboard with the HV-channel backscatter is slightly higher than with the HH-channel. The correlations (Pearson's *r*) in our study are broadly consistent with those reported in Macdonald et al. (2024), ranging from 0.37 to 0.53. This consistency provides confidence in the reliability of our results. And we also observe that the correlation for MYI using HH backscatter on April 12, 2019 is lower than correlation in Macdonald et al. (2024). Our correlations are higher than those presented in Kortum et al. (2024). The discrepancy may be due to the differences in study area, sampling scale, and SAR data preprocessing workflows, among other factors.

Minor:

2. Concerning the matching of OIB/ICESat-2 and Sentinel-1:
a. I think the time differences between the different acquisitions should be mentioned, especially when discussing the coregistration/collocation. Furthermore, I think it would be beneficial to plot the corrections (meters offset) along the ICESat-2 track. Also, the size of segments used for the matching adjustment should be mentioned (is it the same 3 km as used for the inter OIB matching found in the Appendix?).

Reply: First, we agree with your suggestion that it is necessary to mention the time differences between the different acquisitions in the manuscript. As Table B1 presents the

observation times of OIB, ICESat-2, and Sentinel-1, the acquisition time differences are shown below:

- OIB passes collocation:
    - 1.25 hours between left and middle pass;
    - 2.5 hours between right and middle pass.
- OIB-Sentinel-1 collocation:
    - 2019-04-08 track: ~40 minute separation;
    - 2019-04-12 track: ~4 hour separation.

We had added these details in the revised manuscript.

Second, the drift corrections (in meters) along/across the ICESat-2 track are displayed in the correlation map (Fig. S9, panels a, b, e, and f).

Third, regarding the size of segments used for collocation, the 3km segment length is used for OIB pass collocation in Appendix B2, while the 9 km segment length is used for the collocation between OIB and Sentinel-1. We have further cleared this information to the revised manuscript and corresponding figures.

b. I am quite surprised how good the linear correlations are (after correction) between OIB freeboard and sigma_0(HH). These are in large part significantly higher than reported in previous studies (Cafarella 2019, Segal 2020, Macdonald 2024, Kortum 2024). Maybe you could also give the regression without binning for easier comparison with these existing studies.

Reply: In response to your comment, we would like to highlight that Figure 6 provides all the the linear regression parameters and the corresponding Pearson's *r* (**before binning**) between the 200m-scale freeboard and 40m-scale σ0 for all segments on April 8th (panels a, b, and c) and April 12th (panels d, e, and f).

3. HV vs HH:
In the supplement I see that the observed HV correlations seem similar/stronger than HH, yet in the manuscript you focus entirely on the HH band. I suppose you deem the HH band to be more robust, due to the better signal to noise ratios – but this in my opinion should be reasoned before continuing only with the HH band.

Reply: Thanks for the helpful comment. Indeed, we have also analyzed the statistical relationship between freeboard and backscatter in the HV-channel for the whole OIB track. Our results show general consistency with previous studies (Macdonald et al., 2024; Kortum et al., 2024), that freeboard generally correlates slightly better with the HV-channel than with the HH-channel backscatter. Besides, HV-channel in C-band has been shown to be crucial for the FYI-MYI classification (Komanov & Buehner, 2019), which can be combined with HH-channel for future studies of their relationship to ice topography.

The main reasons we focus on the HH-channel in this study are:

1. The HV-channel backscatter is generally much weaker than the HH-channel. This is particularly evident for FYI, where HV backscatter often falls below the nominal

noise floor (Segal et al., 2020). Additionally, the sub-swath artifacts are more evident (i.e., on sub-swath boundaries) for the HV-channel for Sentinel-1 EW mode images (Lohse et al., 2021).

2. Despite the stronger correlation observed in the HV band, the qualitative statistical relationship between freeboard and backscatter is similar when using either the HH or HV channel.

Given these considerations, we have conservatively focused on the S-1 HH channel in our manuscript.

Segal RA, Scharien RK, Cafarella S, Tedstone A. Characterizing winter landfast sea-ice surface roughness in the Canadian Arctic Archipelago using Sentinel-1 synthetic aperture radar and the Multi-angle Imaging SpectroRadiometer. Annals of Glaciology, 61(83):284-298. doi:10.1017/aog.2020.48, 2020.

Lohse, J., Doulgeris, A. P., and Dierking, W.: Incident Angle Dependence of Sentinel-1 Texture Features for Sea Ice Classification, Remote Sensing, 13, https://doi.org/10.3390/rs13040552, 2021.

Komarov A. S. and M. Buehner: Detection of First-Year and Multi-Year Sea Ice from Dual-Polarization SAR Images Under Cold Conditions. IEEE Transactions on Geoscience and Remote Sensing, 57(11), 9109-9123, doi: 10.1109/TGRS.2019.2924868, 2019.

4. Paper summary:
In the Kortum et al 2024 paper, extrapolation is done not for temporally coincident scenes, but with an allowance of up to 24 hours in time difference between Sentinel-1 and ICESat-2 measurements.

Reply: It has been revised in the manuscript.

5. Segment sizes:
As scale is quite important with these observations, I think the dimensions of segments (and size of bins) should be mentioned in every figure where they appear, as it is sometimes not obvious at the moment.

Reply: Thanks, we have revised the figures accordingly.

6. Linear regression:
A large part of the paper deals with linear relationships between backscatter and freeboard. It should be briefly reasoned somewhere when or why this is a reasonable assumption: It seems the relationship becomes more linear at larger scales for example, but at smaller scales linearity might be stronger simplification.

Reply: We agree that there are limitations of using the linear regression model. However, there are several reasons, listed below:

1. The linear regression model is simple and reduces the potential of overfitting. We recognize that complex microwave backscattering mechanisms drive the differences in the SAR-measured backscatter among the various sea ice types (FYI, MYI, level and ridged ice, etc). Hence the relationship between freeboard and backscatter should not be linear. However, we don't have a comprehensive understanding of these mechanisms. Consequently, we adopted a linear fit as a first-order approximation to this relationship.

2. Our analysis reveals that the relationship between freeboard and backscatter becomes much more linear at larger spatial scales (e.g., 200m-scale as in Fig. 2 and 3). Moreover, it is remarkably linear after binning to the backscatter (panel c and e in Fig. 4 and 5). This ensures that the linear regression model is highly effective in predicting the mean freeboard.

3. Lastly, and arguably more importantly, in this study we focus on predicting the freeboard distribution, rather than the pixel-scale freeboard map. It is worthy to differentiate these two types of prediction tasks. The regression model is needed for the prediction of the freeboard map (i.e., point to point prediction). However, it is not needed for the prediction of freeboard distribution in Sec. 3.3. The prediction of the freeboard distribution allows us to fully utilize the altimeter's measurements at their native resolution (e.g., meter-scale of the OIB ATM). Therefore, given our focus of the study, we only adopt the linear regression model in the analysis of Sec. 3.1 and 3.2.

Spelling etc.:
There should be a small non-breaking space between units and numbers (10 m not 10m).

Reply: It has been revised in the manuscript.

L.62 'organised into racetracks'
– I am not familiar with this expression. Maybe 'organised in a racetrack pattern' is easier to understand.

Reply: Agree. This has been revised in the manuscript.

L.183 The segment size (I believe 9 km) should be mentioned here.

Reply: It has been revised in the manuscript.

Fig. 4: I believe it should be 'Binning' not 'Bining'

Reply: Thanks, we have revised the figures accordingly.

L.331: ‘sea ice drift’

Reply: It has been revised in the manuscript.