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I appreciate the authors' thorough response to my previous comments and the substantial revisions made. The manuscript is now clearer and better presented. I have just a few minor suggestions to help further refine the argument.

We would like to thank the reviewer again for his/her review and constructive feedback. We appreciate the effort and time the reviewer has invested in evaluating our work. Please find our point-to-point response below in blue.

Are x_1 and x_2 estimated by fitting or by the location of the vertically averaged center? Line 380 indicates that they are one of the parameters obtained by optimization/fitting, but from the authors' response to my comments, it seems that x_1 and x_2 are estimated as the location of vertically averaged eddy center on cross-sections. Please clarify this.

We thank the reviewer for his/her remark. We apologize if our previous response was not clear enough. The location of the vertically averaged eddy center is used to optimize the vertical structure function only as a starting step. This is consistent with what we wrote in the article between line 385 and 390. x_1 and x_2 are estimated by fitting. It explains the differences in values between x_1 and x_2 .

Equation (9) and others: I found the notation δ^2 to be confusing. It can also mean the square of δ or the second-order derivative operator. To avoid this confusion, I suggest using a different symbol, such as δ^s for spiciness mode.

We thank the reviewer for his/her suggestion. We have adopted the notation δ^s in the revised manuscript.

Line 137: "assume" -> "estimate"

We thank the reviewer for his/her remark. The wording has been corrected in the revised version of the manuscript.

Equation (53) and (54): Is α enforced to be an integer during the optimization? If so, please say that in the manuscript.

We thank the reviewer for bringing this to our attention. Yes, α is enforced to be an integer. We have clarified this point in the revised version.

Equation (27) and (28): α_1 and α_2 have already been used for different variables in equation (22).

We thank the reviewer for bringing this to our attention. Notations have been modified in the revised version.

Line 387: vertically averaged location

We thank the reviewer for his/her suggestion. The wording has been modified in the revised version of the manuscript.

Figure 14: I still think it is uncommon to assign several panels with the same label. It would be clearer to assign them with six different letters such as (a)-(f).

We thank the reviewer for his/her suggestion. We have modified the figure accordingly in the revised version.

Caption of Figure 14: (Nencioli et al., 2008) -> the approach of Nencioli et al., (2008)

We thank the reviewer for bringing this to our attention. The sentence has been modified accordingly.

Line 277: Remove either "at" or "near".

We thank the reviewer for bringing this to our attention. The sentence has been modified accordingly.