



1 Analysis of Lightning-Induced Currents in Supply Cable Shields and Their Impact on 2 LLS Sensor Site Errors 3

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12 Abstract

13 Lightning location system (LLS) sensors, which detect and locate atmospheric discharges, are typically
14 powered by cables buried up to one meter underground. Within the LLS community, it is well known
15 that these cables can create spurious magnetic fields, which can in turn adversely impact the sensor
16 measurements and the resulting data. This issue arises from currents induced in the cable shield by the
17 lightning electromagnetic fields that penetrate the ground. The magnetic field generated by these
18 currents lead to "site errors," causing inaccuracies in estimating the angle of incidence and the peak
19 current of lightning strokes. Although these sensor-specific errors can be partially corrected, a better
20 understanding of the coupling mechanism between the lightning electromagnetic field and the cable
21 could help in minimizing the site errors. This study presents an analysis of the lightning electromagnetic
22 field interaction with cables and examines the influence of various ground and cable properties on this
23 interaction. This work represents a first step toward understanding the physical mechanism leading to
24 LLS sensor site errors. Considering simplified scenarios involving a single insulated or bare conductor,
25 this work provides practical insights that LLS operators can use to estimate worst-case site errors for a
26 provisioned sensor site. Additionally, we show that some site errors observed in operational sensors can
27 be successfully reproduced with good agreement using the proposed approach.

28 1 Introduction

29 Lightning location systems (LLSs) are networks of sensors whose purpose it is to detect and geolocate
30 lightning discharges. Their main functional principle is based on electric and/or magnetic field sensors
31 detecting an incident electromagnetic (EM) field generated by a lightning discharge. To estimate the
32 strike point location, two techniques can be utilized, either individually or combined (see, for example,
33 Chapter 13 in (Corray et al., 2022)):

- 34 1) Time-of-arrival (ToA): The strike position is estimated using multilateration based on the time
35 difference of arrival at different sensors. The arrival times are determined using precise, GPS-
36 synchronized time stamps.



37 2) Magnetic direction finding (MDF): The strike position is determined by intersecting the
38 estimated directions of the incident field at multiple sensors. This technique is often combined
39 with the ToA technique to achieve optimum positioning results.

40 The MDF technique relies on H-field measurements through crossed coils yielding voltages that
41 represent the amplitude of two (x, y) or three (x, y, z) components of the incident H-field. Recently, it
42 has been shown that estimation errors of the angle of the incident field can be related to the propagation
43 terrain by reflection and diffraction phenomena caused by hills and mountains, see for example
44 (Kohlmann et al., 2021).

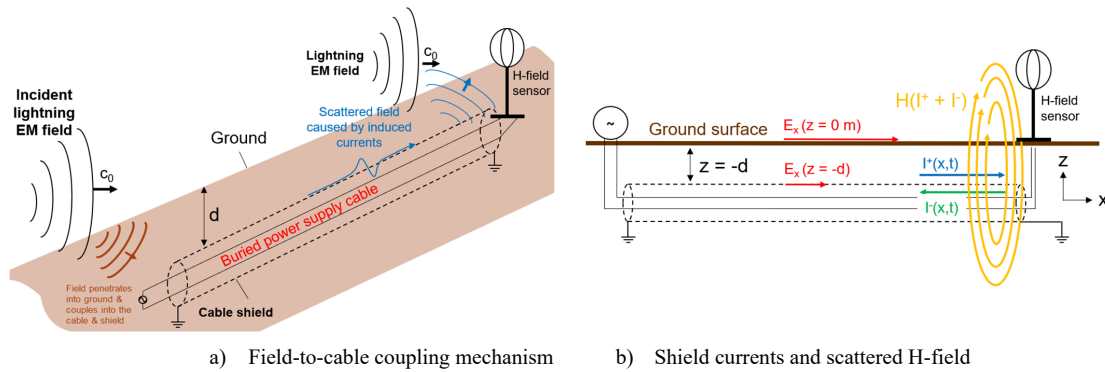


Fig. 1: Mechanism of lightning EM field coupling to a buried LLS sensor power supply cable

45 The present study, in contrast, addresses specific issues related to the MDF technique, which are well-
46 known to the LLS community since the beginning of the application of MDF, namely “angle site errors”
47 and “amplitude site errors” (see e.g., (Schulz, 1997), (Schulz and Diendorfer, 2002)). They are related
48 to spurious magnetic fields, caused by induced currents on the buried power supply cable or nearby
49 conductive objects. These spurious magnetic fields superimpose on the main incident field at the
50 sampling instants, leading to inaccuracies. This type of interference affecting the measurements
51 originates from the very incident field that the LLS sensor is designed to detect. Specifically, the
52 lightning EM field exhibits a horizontal E-field component in the direction of propagation (typically
53 referred to as E_r , but projected onto the x-axis in the present study, thus at the concerned sections also
54 referred to as E_x), resulting from the finite conductivity of the ground. As this field penetrates the ground,
55 it interacts with any metallic structure, such as buried pipes, bare or insulated conductors, cable shields,
56 etc., inducing electric currents. The attenuation of the E-field while penetrating the ground impacts the
57 amplitude of the coupled currents, thus the burial depth of the cable also plays a role, albeit not dominant
58 as the present study will show. The induced currents generate a scattered magnetic field (referred to as
59 $\vec{H}_{err}$ in this study), which, when in close proximity of the H-field sensor, superimpose on the incident
60 lightning magnetic field. An illustration of this electromagnetic environment near an LLS sensor is
61 presented in Fig. 1.



62 The orientation of the scattered magnetic field $\vec{H}_{err}$ depends on the relative position of the
63 underground cable or metallic structure, leading to the distortion in the ratio of the x- and y-components
64 of the detected H-field at the time the sensor samples the signal. Thus, the sensor may estimate an
65 incorrect angle of incidence, which is a critical parameter in lightning location systems that rely on the
66 MDF technique. In the LLS community, this error is referred to as the “angle site error” or “angle error”.
67 Apart from that, the addition of a magnetic field component, affects the measured magnitude of the
68 lightning H-field. This is referred to as “amplitude site error” or “signal error”. Since the measured
69 magnitude serves as an estimator for the peak current of the lightning return stroke, the peak current
70 estimate is also affected, typically leading to an overestimation. This occurs because the amplitude error
71 is positive in most cases, as will be shown in this study. The angle and amplitude errors are tightly related
72 to each other (see also (Schulz and Diendorfer, 2002), respectively the results presented in Section 3.3).
73 As indicated in Fig. 1b, the induced current in the cable shield can have forward and backward
74 propagating waves, the latter being dependent on the termination impedance (i.e., current reflection
75 coefficient). Ideally, a disconnected shield from the ground would yield the smallest currents near the
76 H-field sensor, resulting in minimal angle and amplitude site errors. In real life, however, the cable shield
77 is not always disconnected from the ground. It is the engineer’s task to control and minimize these effects
78 as much as possible, by carefully connecting the sensor to the power supply and thoroughly evaluating
79 the local conditions at the sensor site. This includes the connection of the power supply cable (and
80 potentially a separate communications cable) to the internal circuitry of the sensor’s power cabinet,
81 taking into account the protective earth (PE) wire(s), cable shield(s), and sensor grounding (through
82 earth electrodes) and other structures related to the installation of an LLS sensor. Therefore, a thorough
83 and in-depth understanding of the sensor’s electromagnetic environment and the underlying physical
84 factors causing LLS sensor site errors is of paramount importance.

85 Since site errors at each sensor can be empirically evaluated during a thunderstorm season
86 through reference to the optimum positioning results of the whole LLS network with high location
87 accuracy (which is in the order of 100 m, see (Schulz et al., 2016)), the systematic correction of the site
88 errors is a relatively straightforward task. Consequently, even in the presence of large errors, the
89 correction methods enable angle estimates that contribute meaningfully to location algorithms.
90 However, the qualitative and quantitative characteristics of these errors are still only roughly understood.

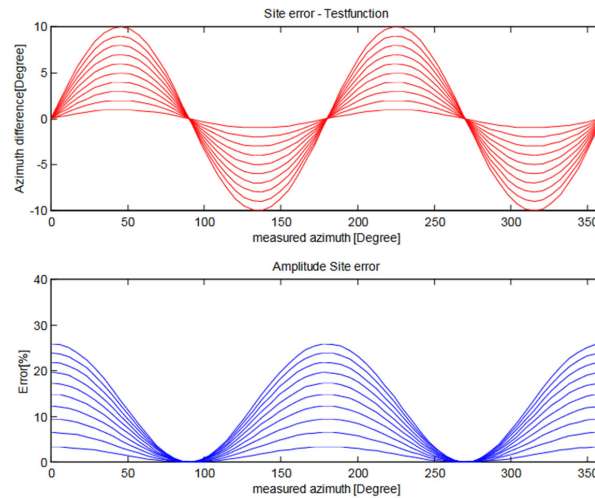


Fig. 2: Theoretical angle site errors (Azimuth difference, top) and amplitude site errors (Error [%], bottom), merely based on simplified geometrical considerations. The shown curves are evaluated assuming different shield current amplitudes, with higher currents corresponding to higher site errors. (Graphic reused with permission from (Schulz and Diendorfer, 2002)).

91

92 It is typically observed in practice that site errors related to induced currents on buried cables
93 exhibit a double-cycle sinusoidal-type of curve, as depicted in Fig. 2. This behavior is expected because
94 during a full azimuthal rotation of the incident field, the induced currents necessarily become zero for
95 particular angles, such as when the field impinges perpendicularly on the power supply cable. For the
96 rest of the angles, sine and cosine functions determine the amplitude of the electromagnetic field
97 impinging on the cable, giving rise to the observed sinusoidal behavior. While in reality, angle site errors
98 of varying levels have been observed, ranging from exceptionally low values below $\pm 1^\circ$, to more
99 typical levels of $\pm 3^\circ$ to $\pm 5^\circ$, and even exceeding $\pm 10^\circ$ in extreme cases. Many of these errors
100 exhibit asymmetric azimuthal behavior due to the complex electromagnetic environment near the sensor
101 and the surrounding topographic terrain (see (Kohlmann et al., 2021)).

102 The primary objective of the present work is to demonstrate, for the first time, the emergence of
103 site errors caused by buried power supply cables, through a semi-analytical approach involving (a)
104 determining the radial lightning EM field at the ground surface and below the ground, (b) computing
105 the shield currents induced by these fields, and, (c) calculating the resulting scattered magnetic fields
106 caused by these induced currents. In other words, the study aims to show that field-to-cable coupling,
107 aside from the terrain-related site errors, is one of the main physical mechanisms contributing to the site
108 errors observed in the LLS sensors employing the MDF technique to locate lightning. We will show that
109 the scattered magnetic fields due to the induced currents on the cable shield can, under specific scenarios
110 of electrical connection of the entering power supply cable, replicate the level of angle and amplitude



site errors observed in actual sites. Furthermore, this work aims to identify the most important parameters impacting the magnitude of these site errors, such as the ground conductivity, the power supply cable length, and the distance of the cable to the sensor.

In this study, a single insulated solid wire, acting as a proxy for a shielded conductor, is considered for the investigation of LLS sensor angle and amplitude site errors. The influence of bare wires is also investigated for comparison. The methods used are comparable to those described in (Aguet et al., 1980), (Bridges, 1995) and (Bridges, 1992), but are adapted to consider incident EM plane waves (with grazing angles of incidence), which are associated with remote lightning strikes.

The rest of the paper is organized as follows. Section 2 presents the methodology followed to obtain the results of this study. It elaborates on the steps to compute remote lightning EM fields (above and below ground), which are used in the following step as the input to the field-to-cable current coupling model. Then, the model for the field-to-cable current coupling and the approach used to evaluate the magnetic fields generated by the induced cable currents is described, as well as their subsequent impact on the resulting site errors. Section 3 presents the results of the individual computation steps. They include key graphs illustrating the expected level of vertical and horizontal E-fields, induced currents, scattered magnetic fields and resulting site errors. Compound graphs are also provided to help readers estimate the maximum expected site errors based on parameters such as supply cable length, ground conductivity and shield termination impedance. Section 4 discusses the practical relevance of the presented results, evaluates the agreement between theoretical predictions and experimental observations, and highlights some relevant real-world insights from the experience of LLS operators. The conclusion summarizes the work and provides an outlook on future work.

2 Methodology

In this Section, we present the procedure for the computation of LLS sensor angle and amplitude site errors, which is carried out in three steps: (a) calculation of the lightning electric fields along the cable, (b) computation of the induced current in the cable shield, and (c) computation of the resulting scattered magnetic field, and the evaluation of the site errors.

2.1 Return stroke modelling, lightning EM field propagation and ground penetration

In order to investigate the induced currents for incident fields typical for lightning discharges, the remote fields associated with the lightning return stroke (RS) have to be obtained in a first step. The geometry of the problem is illustrated in Fig. 3. The objective is to compute the horizontal electric field along the buried cable ($z = -d$), which will serve as source term in the field-to-cable coupling equations (see next subsection). The lightning return stroke is assumed to be a straight vertical antenna located at 100 km from the cable (typical distance covered by LLS sensors). The average ground conductivity along the propagation path is assumed to be σ_p , while the local ground conductivity at the sensor site is σ_{loc} . This



145 latter will determine the amplitude and waveshape of the horizontal E_x -field driving the cable shield
146 current.

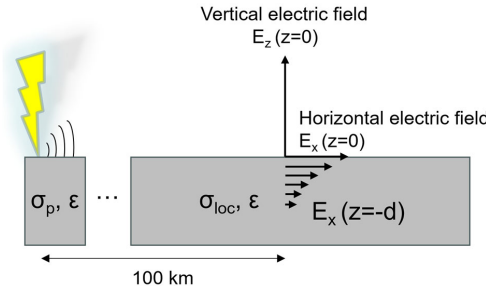


Fig. 3: Lightning EM field propagation towards the sensor site at a distance of 100 km.

147 To represent the lightning return stroke channel, the Modified Transmission Line model with
148 Exponential Decay (MTLE, (Nucci et al., 1988) and (Rachidi and Nucci, 1990)) was used. The
149 parameters of the model were set to $\lambda = 2$ km (exponential decay of the RS current with height),
150 $v_{RS} = 1.5 \cdot 10^8$ m/s (RS wavefront speed). The channel-base current is represented by the sum of two

151 Heidler's functions, described by the following formula: $I(0, t) = \frac{I_1}{\xi_1} \frac{\left(\frac{t}{\tau_{11}}\right)^{n_1}}{\left(\frac{t}{\tau_{11}}\right)^{n_1} + 1} e^{-\frac{t}{\tau_{12}}} + \frac{I_2}{\xi_2} \frac{\left(\frac{t}{\tau_{21}}\right)^{n_2}}{\left(\frac{t}{\tau_{21}}\right)^{n_2} + 1} e^{-\frac{t}{\tau_{22}}}$,

152 $\xi_i = \exp\left(-\frac{\tau_{i1}}{\tau_{i2}} \left[n_i \frac{\tau_{i2}}{\tau_{i1}}\right]^{1/n_i}\right)$. The parameters were chosen to form a channel-base current with
153 characteristics of a typical subsequent RS: $I_1 = 10.7$ kA, $\tau_{11} = 0.25$ μ s, $\tau_{12} = 2.5$ μ s, $I_2 = 6.5$ kA, $\tau_{21} = 2$ μ s,
154 $\tau_{22} = 230$ μ s and $n_1 = n_2 = 2$ (see (Rachidi et al., 2001)). The corresponding subsequent RS-type current
155 waveform, with its short rise time of less than 1 μ s, is depicted in Fig. 4.

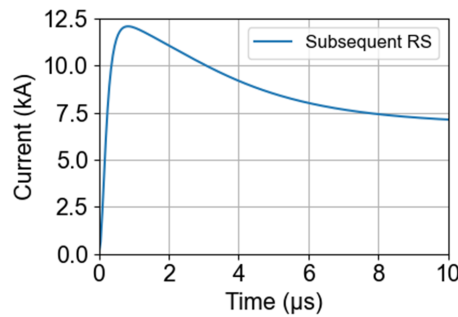


Fig. 4: Return stroke current waveform representing a typical subsequent RS.

156 The vertical electric field and the horizontal magnetic field generated by the return stroke are first
157 computed assuming a propagation over a perfectly electric conducting (PEC) ground. The computation
158 is performed according to (Thottappillil et al., 1997), where the contributions of current dipoles along
159 the channel are summed up to obtain the fields at an observation point located on the ground surface.



160 To account for the attenuation and dispersion that affects the lightning EM fields while
161 propagating above a lossy ground, the fields computed assuming a PEC ground can be corrected by
162 applying specific filters as described, e.g., in (Norton, 1937), (Wait, 1953), (Shoory et al., 2010) (see
163 also (Wait, 1970) for a thorough compendium on wave propagation effects along stratified media).
164 Cross-validations using cylindrically symmetric 2D-FDTD (finite-difference time-domain) simulations
165 have shown the best agreement using the Wait's attenuation function for a stratified ground, as described
166 in details in (Shoory et al., 2010). While the use of Wait's function allows to straightforwardly consider
167 a horizontally stratified ground, the case of a homogeneous (single-layer) ground was considered
168 assuming a very thick upper layer (e.g., 10 km) to account accurately for the attenuation for arbitrary
169 distances without spurious reflection phenomena from the lower layer boundary (for poor ground
170 conductivities). The according equations (see (Shoory et al., 2010) for details) can be readily
171 implemented by typical numerical computational libraries. The Wait's expression for the attenuation
172 function is given by

$$F_{str}(p_{str}) = 1 - j\sqrt{\pi p_{str}} e^{-p_{str}} \operatorname{erfc}(j\sqrt{p_{str}}), \quad (1)$$

in which

$$p_{str} = -0.5 \gamma_0 d \Delta_{str}^2$$

$$\gamma_0 = j\omega\sqrt{\mu_0\epsilon_0}$$

$$\Delta_{str} = \sqrt{\frac{\epsilon_0}{\mu_0}} K_1 \frac{K_2 + K_1 \tanh(u_1 h_1)}{K_1 + K_2 \tanh(u_1 h_1)}$$

$$K_i = u_i / (\sigma_i + j\omega\epsilon_0\epsilon_{ri}), \quad i \in 1,2$$

$$u_i = \sqrt{\gamma_i^2 - \gamma_0^2}, \quad i \in 1,2$$

$$\gamma_i = \sqrt{j\omega\mu_0(\sigma_i + j\omega\epsilon_0\epsilon_{ri})}, \quad i \in 1,2$$

173 where d is the propagation distance, h_1 is the thickness of the upper layer of the stratified ground. The
174 sub-index i denote the parameters of the respective layer, $i \in 1,2$, and sub-index 0 denotes parameters
175 of the free space.

176 The next step is to determine the radial E-field (E_r) in the direction of propagation, at the ground
177 level and below the ground surface. To achieve this, the procedure described in (Rubinstein, 1996) is
178 implemented, in which the so-called “wave-tilt” formula (Rubinstein, 1996) is used to obtain the radial
179 E-field at the ground surface from the horizontal H-field as determined in the previous step, through the
180 surface impedance of the air-ground interface:

$$E_r(z=0) = -H_\phi(z=0) \sqrt{\frac{\mu_0}{(\epsilon_g + \frac{\sigma_{ag}}{j\omega})}} \quad (2)$$



181 with ϵ_g being the ground permittivity, σ_{loc} the local ground conductivity, μ_0 the magnetic permeability of
182 free space and ω the angular frequency. Finally, the electric field below the ground level is found using
183 Weyl's formulation, which reads

$$E_r(z = -d) = E_r(z = 0) \cdot e^{j\omega \sqrt{\mu_0(\epsilon_g + \frac{\sigma_{loc}}{j\omega})}} \quad (3)$$

184 These three formulations (1), (2) and (3) lead to very accurate results, as confirmed by comparisons with
185 full wave numerical simulations using the FDTD method (see also Section 3.1). Finally, the radial E-
186 field E_r is projected onto the direction of the cable by multiplying with the cosine of the angle between
187 the cable's direction and E_r . These techniques provide us with the means to accurately compute the
188 impinging horizontal electric fields on the cable, which serve as inputs for the field-to-cable coupling
189 equations.

190 2.2 Field-to-cable coupling

191 An essential component of the angle and amplitude site error investigations is the field-to-cable
192 coupling model, which uses frequency-domain solutions based on Green's functions. These functions
193 incorporate the coupling equations, while horizontal electric fields act as distributed sources along the
194 cable length, as detailed in (Aguet et al., 1980) and (Teschke et al., 1997). Various approaches for coupling
195 models have been explored, such as the treatment of bare and insulated wires for infinitely long lines in
196 (Bridges, 1995), shielded cables with multiple layers and terminations in (Aguet et al., 1980), and finite-
197 difference time-domain methods for buried conductors and cable shields subjected to lightning strikes
198 in (Petrache et al., 2005). Further discussions on generated electric and magnetic fields in buried cables
199 can be found in (Bridges, 1992). Bridges derived exact solutions for the induced current on an infinite
200 bare or insulated cable buried in soil due to a transient plane wave (Bridges, 1995) and demonstrated
201 that the transmission line theory provides accurate results for a wide range of cases. An experimental
202 validation for the accuracy of the transmission line theory for field-to-cable coupling computations is
203 presented in (Paolone et al., 2005).

204 The relation describing the induced currents at a point x along a finite-length buried cable of length
205 L , using Green's function $G_I(x, x_s)$, reads (see for example (Petrache et al., 2005)):

$$I(x) = \int_0^L G_I(x, x_s) E_x^e(x_s, z = -d) dx_s \quad (4)$$

206 where E_x^e is obtained from Eq. (3) presented earlier and the Green's function reads:

$$G_I(x, x_s) = \begin{cases} \frac{e^{-\gamma L}}{2Z_c(1 - \rho_1\rho_2e^{-2\gamma L})} [e^{-\gamma(x_s-L)} - \rho_2e^{\gamma(x_s-L)}] \cdot [e^{\gamma x} - \rho_1e^{-\gamma x}] & \text{for } x < x_s \\ \frac{e^{-\gamma L}}{2Z_c(1 - \rho_1\rho_2e^{-2\gamma L})} [e^{-\gamma(x-L)} - \rho_2e^{\gamma(x-L)}] \cdot [e^{\gamma x_s} - \rho_1e^{-\gamma x_s}] & \text{for } x > x_s \end{cases} \quad (5)$$



207 The equations involve the complex propagation constant $\gamma = \sqrt{Z'Y'}$, the cable characteristic
208 impedance $Z_c = \sqrt{Z'/Y'}$, the line length L and the voltage reflection coefficients
209 $\rho_i = (Z_i - Z_c)/(Z_i + Z_c)$, with $i \in 1,2$, at the line ends, whereby Z_1 and Z_2 are the source and load
210 impedances of the cable respectively. Due to the different expressions for $x < x_s$ and $x > x_s$ in Eq. (5),
211 the integral in Eq. (4) needs to be partially integrated: $I(x) = \int_0^{L-x} \dots dx_s + \int_{L-x}^L \dots dx_s$, resulting in an
212 analytical expression that can be straightforwardly calculated.

213 The longitudinal impedance Z' and transverse admittance Y' involve the calculation of the per-
214 unit-length ground impedance Z'_g and ground impedance Y'_g (see in Fig. 5). In the present study,
215 Theethayi's ground impedance formulation was used (see Equation (9) in (Theethayi et al., 2007)).
216 Detailed discussions on ground impedance models are available in (Petrache et al., 2005) and (Theethayi
217 et al., 2007), while advanced formulations that account for the soil parameter frequency dependence are
218 found in (Visacro and Alipio, 2012) and (Duarte et al., 2021). For the convenience of the reader, the
219 expressions for the distributed parameters (Fig. 5), as described in (Theethayi et al., 2007), are
220 reproduced as follows.

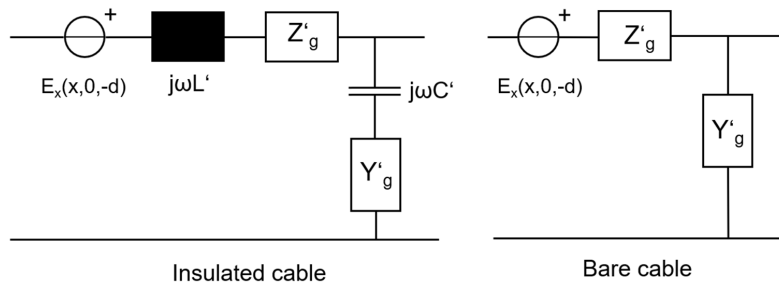


Fig. 5: Equivalent circuit based on the Transmission Line model of an infinitesimal element of the cable (left: insulated, right: bare) in presence of an external electromagnetic field (tangential E-field, E_x)

$$Z'_g = \frac{j\omega\mu_0}{2\pi} \left\{ \ln \left(\frac{1+\gamma_g R_{ab}}{\gamma_g R_{ab}} \right) + \left[\frac{2e^{-2d|\gamma_g|}}{4+\gamma_g^2 R_{ab}^2} \right] \right\} \quad (6)$$

$$Y'_g \approx \frac{\gamma_g^2}{Z'_g} \quad (7)$$

in which $\gamma_g = \sqrt{j\omega\mu_0(\sigma_g + j\omega\epsilon_g)}$ is the complex propagation constant in the ground.

$$L' = \frac{\mu_0}{2\pi} \cdot \ln \left(\frac{\rho_b}{\rho_a} \right) \quad (\text{for an insulated wire}) \quad (8)$$

$$C' = 2\pi\epsilon_{ins} / \ln \left(\frac{\rho_b}{\rho_a} \right) \quad (\text{for an insulated wire}) \quad (9)$$

221 where,

- 222 • for insulated wires, ρ_a is the inner wire radius, ρ_b is the outer radius (including the dielectric
223 jacket of permittivity ϵ_{ins}), $R_{ab} = \rho_b$;
- 224 • for bare wires, $R_{ab} = \rho_a$.



For insulated wires, the total per-unit cable series impedance is $Z' = j\omega L' + Z'_g$, and the total per-unit shunt admittance is $Y' = \frac{CY'_g}{j\omega C + Y'_g}$, while for bare wires, the total per-unit series impedance is $Z' = Z'_g$, and the total per-unit shunt admittance is $Y' = Y'_g$ (see Fig. 5).

All the equations were implemented in the frequency domain and applied under the assumption that both the input signal and the resulting outputs are real, causal signals. The input signal (horizontal magnetic field H_ϕ above PEC) spectrum was obtained using Fast Fourier Transform (FFT), with frequencies considered up to half the Nyquist frequency. To reconstruct real, causal signals, the upper half of the frequency spectrum (from half the Nyquist frequency to the Nyquist frequency) was completed by appending the complex conjugate of the computed results from the lower half of the spectrum. The final time-domain signal was then obtained by performing an inverse FFT on the completed frequency-domain data.

2.3 Scattered magnetic field generated by the induced current on the cable shield and the resulting angle/amplitude site-errors

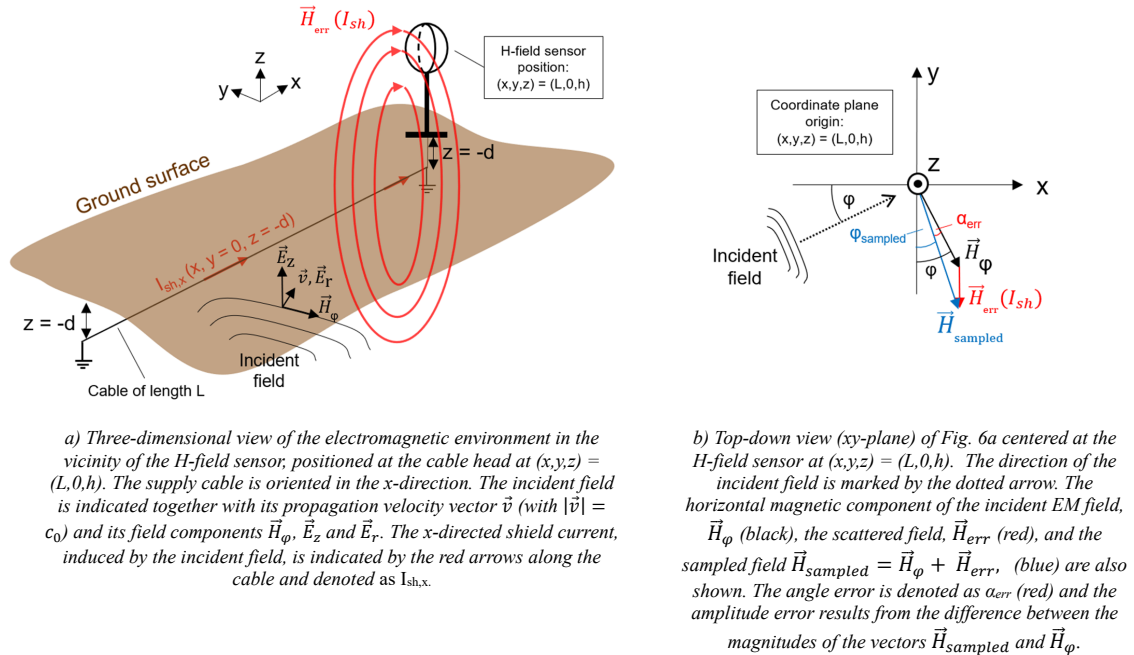


Fig. 6: Illustrations of LLS angle and amplitude site errors caused by the induced shield currents in the power supply cable.

As shown in (Bridges, 1992), the calculated currents along the line can be straight-forwardly used to compute the nearby magnetic fields, as they are not strongly impacted by the air-earth interface. Consequently, the scattered magnetic field is computed using Biot-Savart's law, integrating the contributions of the current elements, obtained in the previous step (Section 2.2), along the nearest 50



242 m to the magnetic field sensor for each time instant. To solve the spatial integral by summing the
243 contributions of the small current elements, it is important to have an accurate spatial current function.
244 This can be readily achieved using a quadratic interpolation function. Moreover, due to the $1/r^2$ distance
245 dependency, contributions beyond 50 m are assumed to be negligible. Fig. 6 illustrates the mechanism
246 of site errors. The scattered field is denoted as $\vec{H}_{err}(I_{sh,x})$, exhibiting y- and z-components in the given
247 geometrical arrangement. The cable is oriented in the x-direction, as the field depends on the induced
248 shield, respectively conductor current $I_{sh,x}$, which is aligned with the cable's direction.

249 This field introduces an error to the incident field $\vec{H}_\varphi$ (horizontal, thus purely in the xy-plane) resulting
250 in a sampled field $\vec{H}_{sampled}$ that exhibits an altered angle and magnitude in comparison to the true
251 incident field $\vec{H}_\varphi$. As a consequence of Ampere's law, the error magnetic field vector $\vec{H}_{err}$ is azimuthal
252 around the power supply cable, which is assumed to be straight. The vertical component of the head of
253 the supply cable below the sensor, and thus the corresponding H-field, is not considered, as it is aligned
254 axially with the sensor and is assumed to have a negligible impact. For more complex shapes of the
255 power supply cable paths, including corners and bends), the scattered (error) field vector $\vec{H}_{err}$ may have
256 arbitrary orientations. Further, it must be noted that Fig. 6 must be understood as a snapshot at a specific
257 sampling instant, where all vector lengths and angles are time-dependent according to the incident EM
258 wave and the induced currents. A typical LLS sensor samples the field when the vector $\vec{H}_{sampled}$ reaches
259 its maximum value, referred to as maximum sampled magnitude in the present study. At this instant, the
260 difference between the true incident field angle φ and the sampled angle, $\varphi_{sampled}$, computed using the
261 arc tangent of the output voltage ratio of the x- and y-component of the crossed loop antenna, is defined
262 as the angle error α_{err} . The amplitude error (sometimes also called "signal error") is denoted as s_{err} and
263 defined as $s_{err} = \frac{|\vec{H}_{sampled}|}{|\vec{H}_\varphi|}$.

264 3 Simulation results

265 3.1 Lightning EM fields and ground penetration

266 This section presents the simulation results of lightning incident electric fields following the procedure
267 described in Section 2.1, considering a channel-base current typical of subsequent return strokes, as
268 depicted in Fig. 4. All results are obtained for a distance to the lightning discharge of 100 km.

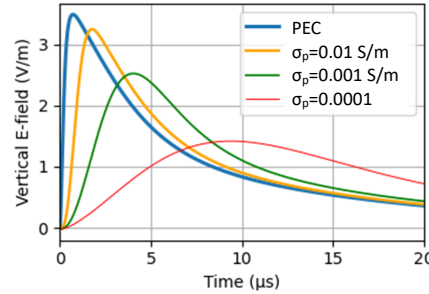


Fig. 7: Distant vertical electric field (100 km) at the ground level as a function of the ground electric conductivity (PEC, $\sigma_p = 10^{-2}$ S/m, 10^{-3} S/m and 10^{-4} S/m).

269 The vertical electric field above the PEC is shown in Fig. 7 (blue curve). Note that the vertical E-field
 270 (E_z) and the horizontal H-field ($\vec{H}_\varphi$, which is sensed by an MDF sensor) above a PEC ground are related
 271 through $H_\varphi = c_0/\mu_0 \cdot E_z$. Simulation results using Wait's formalism, accounting for the attenuation of
 272 the fields due to the propagation over a lossy ground, are also shown in Fig. 7 for different ground
 273 conductivities. As can be seen, the higher the conductivity, the lower the attenuation and dispersion.
 274 Lower values for the ground conductivity lead to more attenuated and dispersed fields with longer rise
 275 times (about 2 μ s, 4 μ s and 10 μ s for the orange, green and red curve, respectively).

276 The radial E-fields at the ground level and below ground are obtained after applying Eq. (2),
 277 and Eq. (3) to the horizontal magnetic field above lossy ground. The results are shown in Fig. 8.

278 The impact of the burial depth for higher ground conductivities, namely $\sigma_{loc} = 10$ mS/m and
 279 $\sigma_{loc} = 50$ mS/m, for an incident field according to Fig. 7 ($\sigma_p = 10^{-2}$ S/m, orange curve), is shown in Fig.
 280 8a and Fig. 8b. As can be seen, a significant reduction in the amplitude is already observed within the
 281 first 5 meters below the ground level.

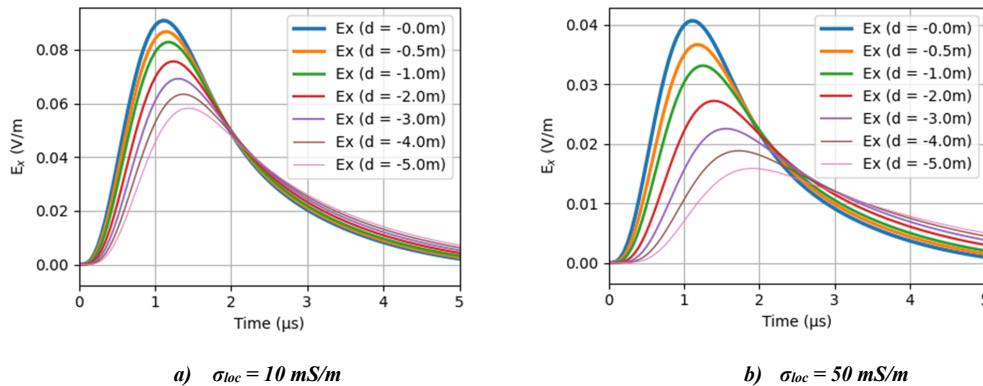


Fig. 8: Variation of the horizontal electric field as a function of the burial depth for higher values of the ground conductivity σ_{loc} with $\sigma_p = 10^{-2}$ S/m. Compare to Fig. 9.



282 Since these horizontal E-fields serve as the input for the cable current coupling model, ensuring
283 their validity is crucial for achieving accurate results in subsequent computational steps. Thus, the fields
284 were validated through cross-comparison with results obtained from a cylindrical-symmetry 2D-FDTD
285 solver ((Oskooi et al., 2010), (Anon, 2024)). The results for both approaches are found to be in excellent
286 agreement. A validation example is shown in Fig. 9. The depicted scenario represents very low ground
287 conductivities (10^{-4} S/m and 10^{-5} S/m) and large depths ($d = -50$ m) below ground. The difference in the
288 amplitudes between the fields at ground level and those below ground is very small, indicating that
289 attenuation in low-conductivity ground is negligible for typical burial depths of power supply cables,
290 which range from a few tens of centimeters to about 1 m.

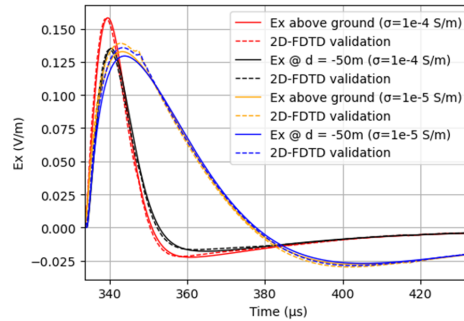


Fig. 9: Validation of the proposed approach with respect to FDTD simulations. Shown are the E-fields at the ground level and 50 m below ground for very low ground conductivities $\sigma_{loc} = 10^{-4}$ S/m and $\sigma_{loc} = 10^{-5}$ S/m, with $\sigma_p = 10^{-4}$ S/m.

291 Fig. 10 illustrates the general dependencies of the horizontal electric fields, both at the ground level and
292 below ground, on the ground parameters. Fig. 10a shows the ratio of the peak value of the horizontal
293 electric field at ground level, $E_{x,peak}(d=0)$ to that of the vertical E-field, $E_{z,peak}$, at the ground level. This
294 ratio can be as high as 30% for very low conductivity ($\sigma_{loc} = 10^{-5}$ S/m) and drops to 2.5% for high ground
295 conductivity ($\sigma_{loc} = 50$ mS/m). Due to the frequency dependence of the physical mechanisms governing
296 the local field configuration, these ratios depend on the frequency content of the incident field. To
297 account for attenuation and dispersion along the 100-km propagation path, different lossy grounds with
298 ground conductivity values ranging from $\sigma_p = 10^{-1}$ S/m to $\sigma_p = 10^{-4}$ mS/m have been investigated.

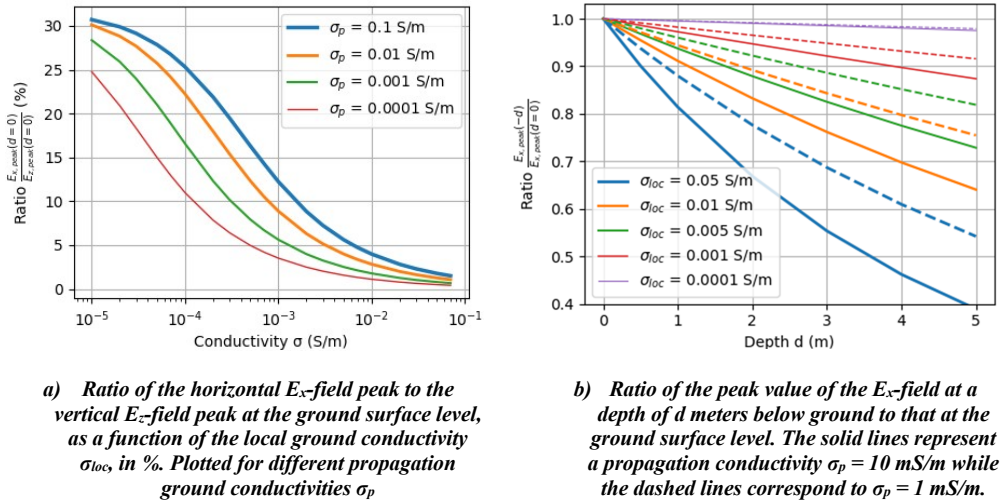


Fig. 10: Field peak ratios for various local ground conductivities σ_{loc} and the impact of the propagation ground conductivity σ_p

299 The examination of Fig. 10a reveals two key aspects. (1) fields with shorter rise times (fast transients)
 300 tend to create larger E_x -field peaks, and (2) fast transients are better preserved over propagation path
 301 with high conductivity σ_p (e.g., see the blue curve in Fig. 10a). A realistic scenario for a lightning EM
 302 field involves propagation over lossy ground with conductivity values σ_p between
 303 10^{-4} S/m and 10^{-2} S/m over 100 km, resulting in incident fields similar to those shown in Fig. 7.

304 The penetration of the horizontal E-field, E_x ($d=0$) into various depths below ground level is shown in
 305 Fig. 10b, illustrating the ratio of the E_x -field peaks at the surface and below ground. The figure clearly
 306 shows that low ground conductivities allow for deep penetration, with minimal attenuation of the field
 307 peak over depth. Conversely, for high conductivities such as $\sigma_{loc} = 50$ mS/m, the attenuation with depth
 308 below ground is more significant. Notably, e.g., at $d=1$ m below ground, attenuation ranges between 13%
 309 (dashed, thickest blue line) to 20% (solid, thickest blue line) – a critical observation discussed further in
 310 Section 4. The discrepancy between the solid and dashed lines again stems from the fact that waveforms
 311 with a higher frequency spectrum (i.e., fast transients) are better preserved during propagation along a
 312 medium with high conductivity σ_p during propagation, are more significantly attenuated through the
 313 ground at the sensor site, irrespective of the local ground conductivity σ_{loc} .

314 3.2 Field-to-cable current coupling

315 Hereafter in this section, the field-to-cable coupling computations described in Section 2.2 are
 316 performed assuming a propagation ground conductivity $\sigma_p = 10$ mS/m and a local ground conductivity
 317 $\sigma_{loc} = 10$ mS/m, unless stated otherwise. The conductor radius is $\rho_a = 10$ mm, while the cable jacket is 5
 318 mm thick, resulting in an outer radius of $\rho_b = 15$ mm. The cable jacket has a relative permittivity of $\epsilon_{r,d}$
 319 $= 3$. The conductor can be regarded as a cable shield with an equivalent outer radius typical of power



supply cables. The impact of a slightly higher electrical resistance due to a thin screen, as opposed to a solid conductor, is negligible in the coupling analyses that follow.

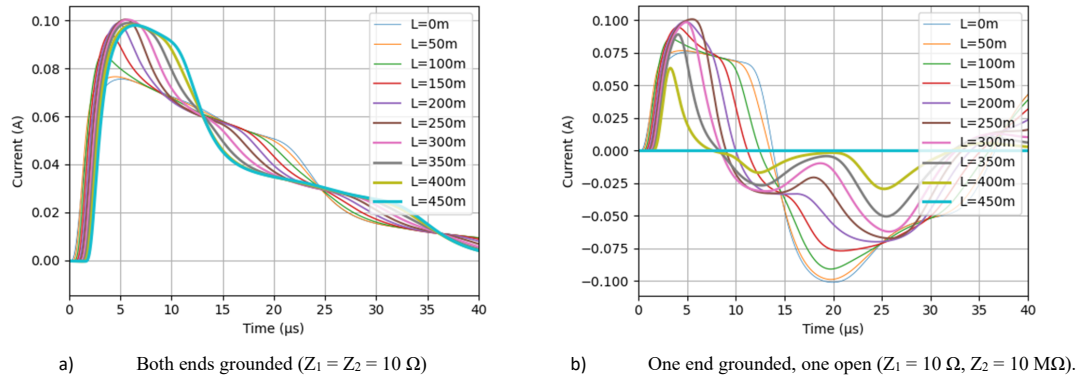


Fig. 11: Shield currents of an insulated cable responding to a distant (100 km) lightning-incident field, as shown in Fig. 7 for $\sigma_p = 10$ mS/m, $\sigma_{loc} = 10$ mS/m for a line length of 450 m buried at $d = 1$ m below ground.

Two examples considering different termination impedance scenarios are considered. The results for the induced currents are shown in Fig. 11. Fig. 11a presents the results for a cable terminated at its ends with an impedance of $Z_1 = Z_2 = 10 \Omega$, a value typically achievable at sites with $\sigma_{loc} = 10$ mS/m. The peak current at the line end (at $x = 450$ m, thickest lines in Fig. 11) reaches a value of 82 mA. Fig. 11b shows

the simulation results assuming that the line end is disconnected (shield not connected to ground modeled through a large impedance value of $Z_2 = 10 M\Omega$). In this configuration, the current at $x = 450$ m is naturally zero. However, within the first 50 m away from the cable end (see $x = 400$ m), the current gradually increases, reaching a peak value of about 75% of the overall peak current, which occurs near the middle of the line. Since a completely ungrounded shield (with no connection to ground on either side of the cable) is an uncommon installation practice for shielded cable, this scenario is not considered. Such a setup would also result in zero current at the start of the line ($x = 0$ m).

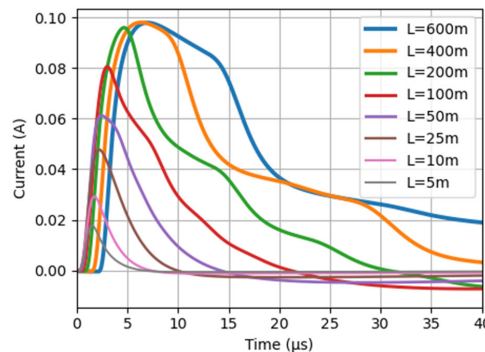


Fig. 12: Currents at the line end with $Z_1 = Z_2 = 10 \Omega$ (same parameters as in Fig. 11a), considering different line lengths L and a local site conductivity of $\sigma_{loc} = 10$ mS/m. Burial depth $d = 1$ m.



334 The effect of the line length on the induced current is shown in Fig. 12. As the line length
335 decreases, a corresponding reduction in the induced shield current peak is observed. A significant
336 increase of the induced shield current peak current would be observed when the local conductivity on
337 site reduces. This observation is aligned with the findings of Section 3.1, where a reduction in the local
338 conductivity results in an increased horizontal electric field and decreased attenuation during ground
339 penetration. However, in scenarios with reduced conductivities, the smallest achievable line termination
340 impedance also increases, compensating the current increase and reducing the currents near the line
341 ends. The largest shield current amplitudes would in turn be closer to the midsections of the cable.

342 The induced currents also depend on the conductor radius, generally exhibiting higher
343 amplitudes for larger radii. However, these differences are in the order of +10% when the conductor
344 radius is doubled. Therefore, the influence of conductor radius on the results is not further analyzed in
345 this study.

346

347 3.3 Scattered magnetic field and angle/amplitude site errors

348 In this section, the waveshape of the scattered magnetic field $\vec{H}_{err}$ (se Fig. 6) is examined more in detail,
349 and the related angle and amplitude site errors that would, at least theoretically, be observed by the MDF
350 sensor used in LLSs is explored. The impact of several parameters, such as line length, ground
351 conductivity, burial depth and distance of the MDF to the supply cables is investigated.

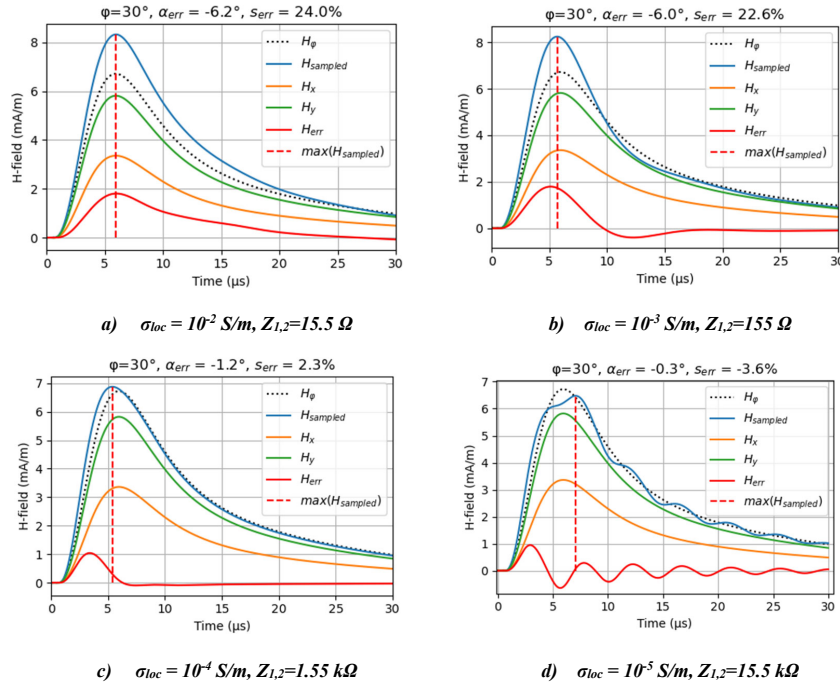


Fig. 13: Dependence of the site errors on the local ground conductivity. All magnetic field components are shown for an incident EM field (compare Fig. 6, whereby this graph has been adapted to depict all fields with positive polarity). The incident angle is $\varphi = 30^\circ$ relative to the cable (aligned in x-direction). The resulting angle errors α_{err} and amplitude errors s_{err} are presented on the top of each figure. The insulated cable length is $L = 200 \text{ m}$ and buried 1 m below ground. Conductor radius: $\rho_a = 10 \text{ mm}$, outer radius $\rho_b = 15 \text{ mm}$, cable jacket permittivity $\epsilon_{rd} = 3$. Sensor position (line end) at $z = +2 \text{ km}$ distance to the lightning strike. σ_p was set to 1 mS/m . The DC termination impedances $Z_{1,2}$ correspond to a vertical grounding rod of 10 m length and 3 cm thickness for the considered ground conductivity σ_{loc} .

352

353 As explained in Section 2.2, once the spatiotemporal behavior of the conductor currents is
 354 determined, the scattered magnetic field can be computed by applying Biot-Savart's law. Given the short
 355 distances of the relevant current elements impacting the sensed magnetic field (less than 50 m), time
 356 retardation can be disregarded. Fig. 13 presents the results for four local ground conductivity (σ_{loc})
 357 scenarios as described in the figure caption. To get reasonable results, the termination impedances Z_1
 358 and Z_2 were assumed to be conductivity- and frequency-dependent, considering a 10-m long, 3-cm
 359 thickness vertical grounding rod (see for example (Grcev, 2009)). Their DC-values are given in the sub-
 360 figures of Fig. 13. Otherwise, the shield currents would reach unrealistically high amplitudes due to a
 361 grounding impedance value which would be unattainable at a site with, for example, a very low
 362 conductivity. The graphs show the main lightning field to be detected, $\vec{H}_\varphi$ (black dotted line), incident
 363 with an angle of $\varphi = 30^\circ$ relative to the cable (aligned with the x-axis). They also show the x- and y-
 364 components (orange and green) of $\vec{H}_\varphi$, the scattered field $\vec{H}_{err}$, which contributes to H_y , due to a current



365 I_{sh} oriented along the x-axis, and finally the resulting total field $\vec{H}_{sampled}$, which the sensor samples at
 366 the instant of its maximum magnitude. The scattered field $\vec{H}_{err}$ is responsible for a distortion of the H-
 367 field vector of the true incident lighting EM field, resulting in angle and amplitude errors, α_{err} and S_{err} ,
 368 respectively.

369 The differences in site errors shown in Fig. 13 a-d is attributed to the characteristics of the
 370 scattered fields ($\vec{H}_{err}$) impacting the site errors at the sampling instant. For high conductivity values
 371 (e.g., $\sigma_{loc} = 10^{-2}$ and 10^{-3} S/m, Fig. 13a Fig. 13b), the maximum of the scattered field $\vec{H}_{err}$ nearly
 372 coincides with the maximum of $\vec{H}_{sampled}$. However, for very low ground conductivity (10^{-4} S/m and
 373 smaller, see Fig. 13c Fig. 13d), wave propagation effects with minimal attenuation result to pronounced
 374 reflections and resonances along long lines. As a consequence, oscillations arise in the induced currents,
 375 producing fast ringing effects in the scattered field $\vec{H}_{err}$. These oscillations exhibit a frequency that
 376 depends on the line length and can arbitrarily impact the sampling instant - and thus the site errors
 377 potentially causing sampling to occur when $\vec{H}_{err}$ is zero or even of opposite polarity. It must be noted
 378 that the sharper the impinging transients, the more pronounced the oscillations of the induced current
 379 response. While incident fields with very high-frequency content (i.e., short rise times), combined with
 380 very low local ground conductivity σ_{loc} and long cables, may occur in reality, such scenarios are rare.
 381 Nevertheless, this possibility should not be overlooked.

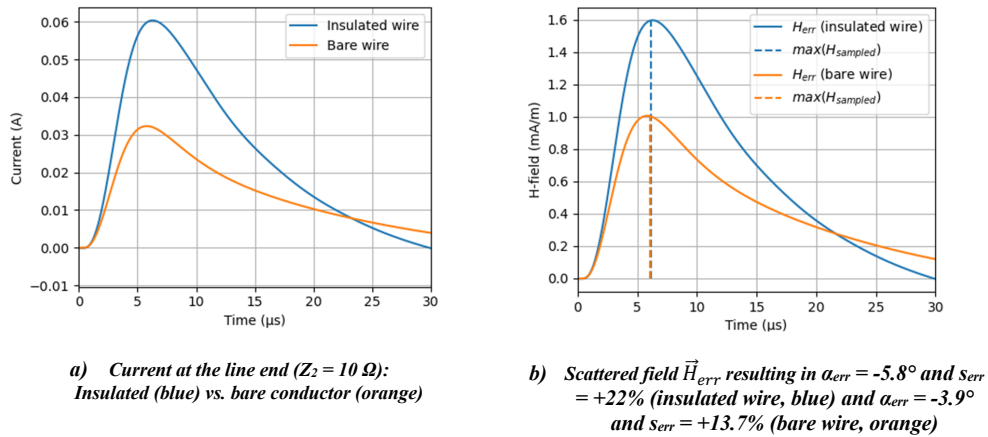


Fig. 14: induced currents and scattered magnetic fields: insulated vs. bare conductors ($L = 100$ m).

382 If an insulated power supply cable were hypothetically replaced by a bare conductor of the same
 383 length, parameters, and termination conditions, simulations indicate that the induced conductor currents
 384 near the line end would be moderately reduced. The associated scattered field $\vec{H}_{err}$ would, in turn, be
 385 reduced as well, resulting in reductions in site errors. Specifically, the angle site error decreases by 33%,
 386 and the amplitude site error by 37%, as shown in the example presented in Fig. 14a and Fig. 14b. The
 387 results of currents and H-fields were compared to results obtained from fully integrated 3D-FDTD



simulations, using the open source FDTD-solver Elecode (see (Kuklin, 2022)), which supports modeling insulated conductors. The comparisons showed very good agreement.

Next, after thoroughly addressing the physical quantities, such as the electric fields below ground, coupled currents and scattered magnetic fields, that contribute to the LLS sensor site errors, it remains to finally investigate the angle and amplitude site errors over a full 360° azimuth rotation. Hereby, key parameters impacting the results will be highlighted. The H-field sensor is assumed to be located at a height of $z = 2$ m above ground.

First of all, we investigate the impact of the burial depth of the power supply cable on the site errors. The simulation results are presented in Fig. 15. It is important to note that this simulation is conducted for two scenarios:

a) In the first case, the distance between the cable to the sensor head increases as the burial depth increases, which is the most realistic scenario. Consequently, the site error reduction is influenced by a combined effect of increasing distance from the cable to the H-field sensor and ground attenuation.

b) In the second case, the relative distance between the H-field sensor and the cable remains constant at 2 meters. This scenario isolates the effect of ground attenuation from the distance effect, highlighting their distinct contribution.

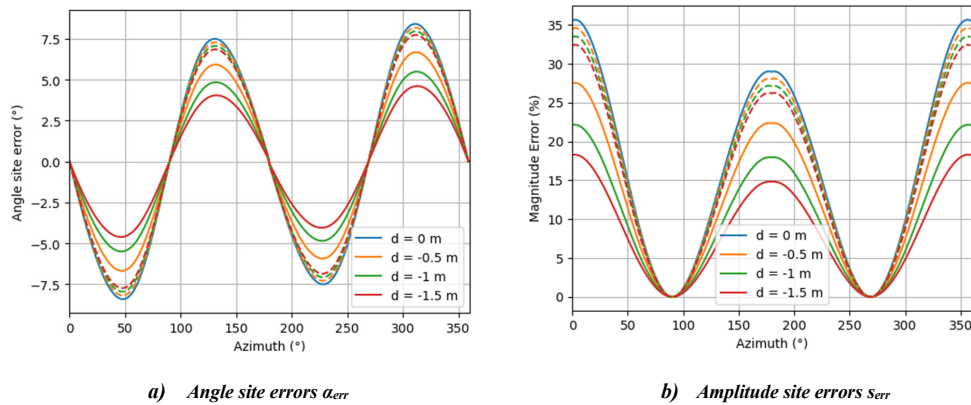


Fig. 15: Impact of the burial depth on LLS sensor site errors for $\sigma_p = 10$ mS/m and $\sigma_{loc} = 10$ mS/m. Solid lines: Combined effect of field attenuation and increased distance to the sensor. Dashed line: Impact of field attenuation below ground, while the distance of the cable to the sensor is kept constant ($z = 2$ m).

The results shown in Fig. 15 reveal a significant finding: The site errors are very strongly impacted by the (vertical) distance of the cable to the H-field sensor, as indicated by the solid-line graphs. In contrast, the dashed-line graphs exhibit a very small reduction with increasing burial depth. For a burial depth of 1.5 m, the angle site errors α_{err} decrease by only -8.5%, while the total reduction reaches -46%. This is not too surprising, since the inspection of Fig. 10b suggests exactly this result based on the attenuation caused by the ground penetration of the E_x -field for the assumed parameters.



410 The amplitude site errors s_{err} decrease by roughly the same amounts. For a very high local ground
411 conductivity of $\sigma_{loc} = 50$ mS/m, the reduction in the angle site errors at $d = 1.5$ m is higher and reaches
412 -20% (from $\pm 3.86^\circ$ down to $\pm 3.07^\circ$) for the pure ground attenuation effect and -54% (from $\pm 3.86^\circ$
413 down to $\pm 1.78^\circ$) for the combined effect of distance and ground attenuation. Thus, the contribution of
414 the cable distance to the sensor remained practically the same, as expected.

415 At this point, one further investigation naturally presents itself: examining the impact of the
416 increasing sensor's distance to the cable, as the sensor may be mounted on top of a high mast. This
417 installation type has been employed at some LLS sensor sites, where it has been associated with minimal
418 site errors. It is expected that the observed behavior will approximate an inverse relationship to the
419 distance $1/r$, with r being the vertical distance from the cable to the sensor. This expectation aligns with
420 Ampere's law, according to which the magnetic field of an infinitely long cable is $H = I/(2\pi r)$. This
421 relationship is confirmed in Fig. 16, which shows the angle site error decreasing from $\pm 6.7^\circ$ at a
422 vertical distance of 2.5 m from the cable to $\pm 1.8^\circ$. Although slightly less pronounced, this reduction
423 closely follows the expected $1/r$ relationship. One important comment must be added: further induction
424 phenomena are expected due to the prevailing vertical E-field (E_z) impinging on the high mast and the
425 cable. This could lead to potential additional induced currents in the grounding of the mast, which could
426 impact the shield currents of the supply cable and, consequently, the behavior illustrated in Fig. 16. The
427 results match well with real-world experience of this type of installation, which was previously
428 employed by the Austrian LLS operator ALDIS (Austrian Lightning Detection and Information System).

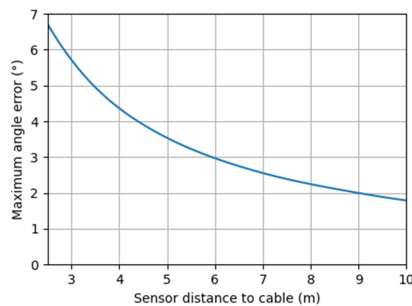


Fig. 16: Dependence of the angle site error on the vertical distance of the H-field sensor to the supply cable. Cable burial depth $d = 0.5$ m, $\sigma_{loc} = 10$ mS/m. The maximum angle errors should be interpreted as $\pm$ the values shown on the y-axis, according to the two-cycle error characteristic.

429 The final evaluation aims to provide a comprehensive overview of the influence of the local ground
430 conductivity, length and termination impedance - factors previously illustrated by means of time-domain
431 graphs of coupled currents and magnetic fields - on the site errors, summarized in a single figure. All
432 simulations were performed considering an insulated wire with an outer cable radius of $\rho_b = 7.5$ mm and
433 core radius of $\rho_a = 5.8$ mm, a jacket dielectric relative permittivity of $\epsilon_{r,d} = 3$, buried at a depth of 50 cm.
434 Two different sets of ground parameters are considered to examine the impact of different (short vs.

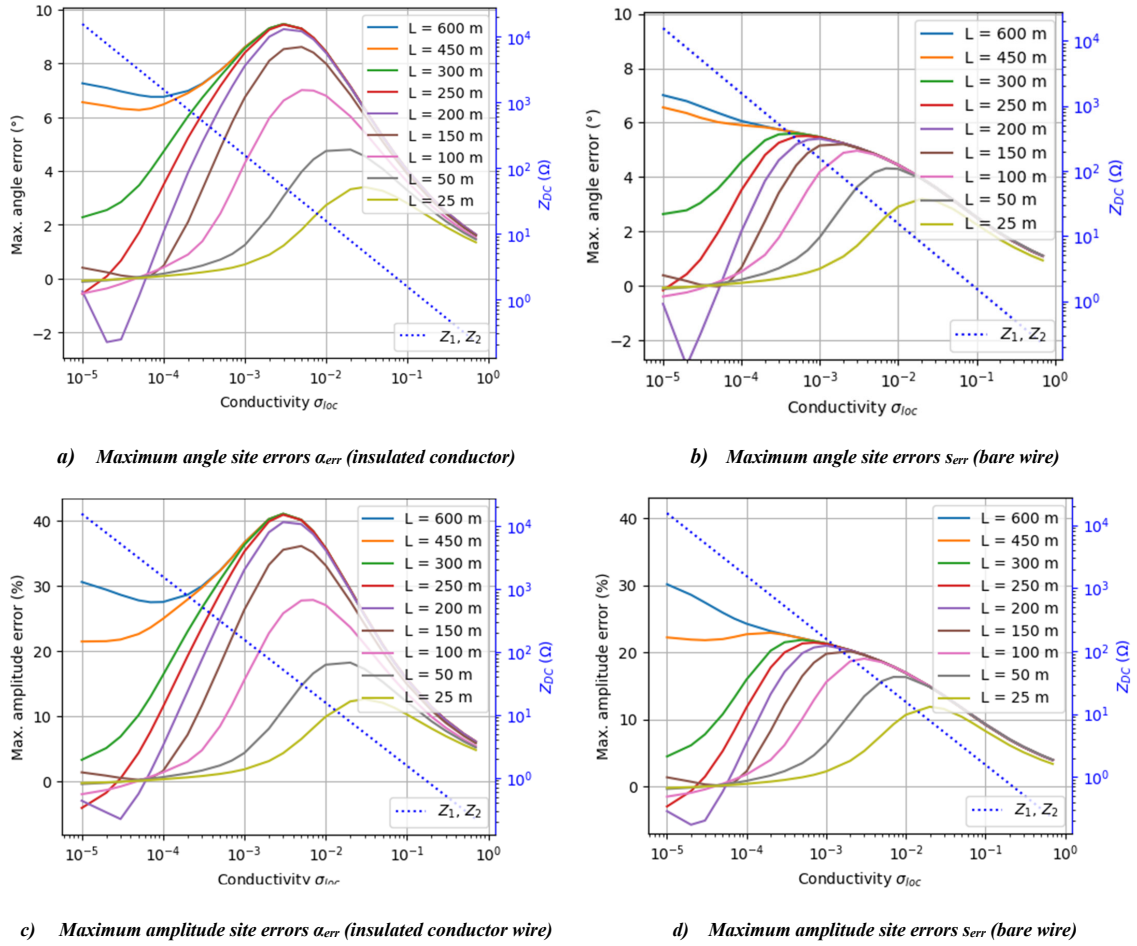


Fig. 17: Influence of local ground conductivity σ_{loc} , cable length L and (conductivity- and frequency-dependent) termination impedances Z_1 and Z_2 on site errors. $\sigma_p = 10^{-3}$ S/m, burial depth $d = 0.5$ m. Sensor position at $z = 2$ m. The order of the color legend agrees with the maximum site errors according to the line lengths: largest site errors for $L = 600$ m on top of the legend, monotonically decreasing to the smallest site errors for $L = 25$ m at the bottom of the legend.

435

436 long) rise times of the incident lightning fields: (i) $\epsilon_{r,g} = 10$, $\sigma_p = 10^{-3}$ S/m, and,
 437 (ii) $\epsilon_{r,g} = 10$, $\sigma_p = 10^{-4}$ S/m. The distance to the lightning strike was assumed to be 100 km. Both line
 438 ends are assumed to be grounded with a vertical grounding rod (see (Greev, 2009)), resulting in
 439 termination impedances which are frequency- and conductivity-dependent. This approach provides a
 440 more realistic representation compared to a constant grounding impedance, which may not appropriately
 441 represent the prevailing local grounding conditions. Having both ends grounded, where the largest
 442 currents can flow, represents the worst-case scenario for LLS sensor site errors.

443 This analysis deliberately focuses on the worst-case scenario where both ends of the shield are
 444 grounded, with the aim of understanding the primary interrelation between the influencing parameters.
 445 In this context, the interpretation of this significant, yet very isolated and hypothetical scenario should
 446 be taken with care and regarded solely as a rough estimate for the maximum expected site errors. The

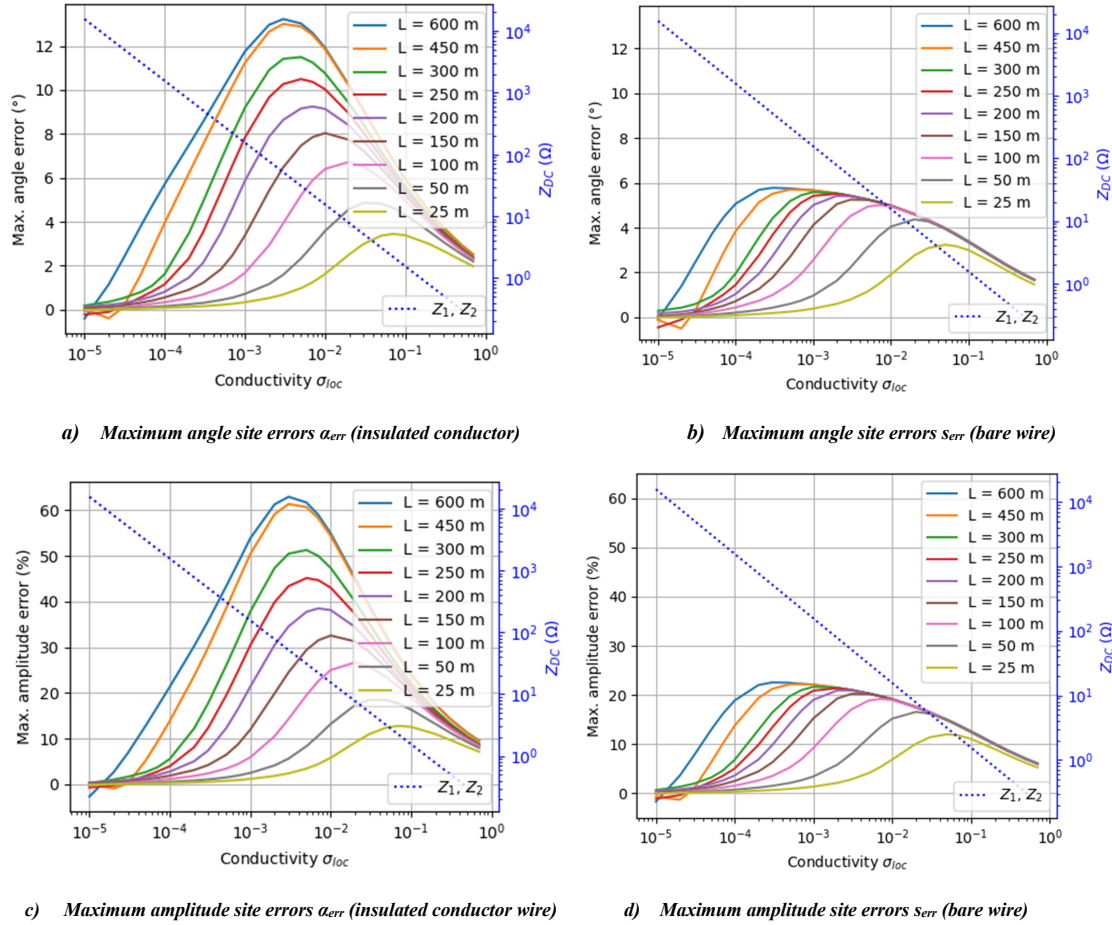


Fig. 18: Same as Fig. 17, but with $\sigma_p = 10^{-4}$ S/m, further reducing the frequency content of the incident lightning EM field and thereby increasing the rise time (see Fig. 7). The order of the color legend agrees with maximum site errors according to the line lengths: largest site errors for $L = 600$ m on top of the legend, monotonically decreasing to the smallest site errors for $L = 25$ m at the bottom of the legend.

discussion section is dedicated to further considerations and to the elaboration on more special or real-world case studies.

Fig. 17 a-d ($\sigma_p = 10^{-3}$ S/m) and Fig. 18 a-d ($\sigma_p = 10^{-4}$ S/m) depict the maximum observable angle site errors α_{err} (occurring at an incident angle of about $\varphi = 45^\circ$, refer to Fig. 6 for comparison) and amplitude site errors s_{err} (occurring at $\varphi = 0^\circ$) for different incident fields – one with faster rise times (Fig. 17) and one with slower rise times (Fig-18). Fig. 17 and Fig. 18 a & c show α_{err} and s_{err} for insulated cables, b & d for bare wires. The graphic also shows the direct current impedance value Z_{DC} , which the line termination impedances Z_1 and Z_2 assume for a vertical grounding rod of 10 m length, on the right ordinate. The frequency dependence was taken into account as well, although the impact is negligible for the frequency range of the induced currents.



457 The most important observations are summarized in the following bullet points, based on the use of
458 “fast fields” for the case with $\sigma_p = 10^{-3}$ mS/m (i.e., Fig. 17), with rise times of about 4 μ s (Fig. 7, green
459 curve), due to the higher frequency components present in the incident field, and “slow fields” for
460 $\sigma_p = 10^{-4}$ S/m (i.e., Fig. 18) with rise times of about 10 μ s (Fig. 7, red curve), reflecting the low frequency
461 content of the incident field:

- 462 • Decreasing the supply cable length decreases site errors but causes a slight shift in the maximum
463 site errors towards higher conductivity values. This trend is consistent for incident fields of
464 different rise times.
- 465 • The effects visible for fast fields and low local ground conductivities σ_{loc} in conjunction with
466 long lines are attributed to resonance effects (see Fig. 13d). In this case, small four-cycle site
467 errors and a slight underestimation of the lightning peak current can be observed. However,
468 these phenomena are not observed for incident fields with longer rise times (see Fig. 18
469 corresponding with rise times of about 10 μ s).
- 470 • For long power supply cables, slow fields result in greater site errors than fast fields. This might
471 be attributed to the extended time available for wave propagation effects to impact the total
472 current at the line’s endpoint, leading to higher current values and, consequently, larger site
473 errors. In contrast, for short lines ($L < 100$ m), the simulation results exhibit no dependence on
474 rise time.
- 475 • The almost identical values of the maximum site errors for different line lengths in the case of
476 fast fields (Fig. 17) becomes noticeable for line lengths exceeding approximately 200 m. This
477 is because for high local ground conductivity, dissipation prevents significant (unattenuated)
478 wave propagation until the time that the sampling is performed by the sensor. Henceforth,
479 remote current induction effects are not detected by the sensor. For lower values of σ_{loc} , wave
480 propagation effects are present, leading to a divergence of the curves below $\sigma_{loc} = 10^{-3}$ S/m. For
481 slow fields (Fig. 18), the saturation is observed for longer cables (exceeding 450 m). This is
482 because, by the time of the sampling of H_{sample} at about 10 μ s, the wave can, in contrast to fast
483 fields, propagate farther and build up currents close enough to the sensor.
- 484 • The decrease in amplitude and angle errors on the right-hand side of the bell-shape site error
485 curves, where the grounding impedance Z_{DC} (resulting from high σ_{loc}) is very low, is constrained
486 by the diminishing E_x -field components caused by the high ground conductivity (see Fig. 10a).
487 To the left to the peak site errors, the site errors decrease due to the high grounding impedance
488 Z_{DC} . However, as conductivity decreases, reducing current dissipation along the line, wave
489 propagation effects become more pronounced. Consequently, long lines are more susceptible to
490 higher site errors, and even polarity reversal effects for α_{err} and amplitude attenuation (negative
491 s_{err}) may occur for low ground conductivity. Within the considered range of conductivities σ_{loc} ,



these effects are observed only for fast fields with sharper transients (Fig. 17), but not for slow fields (Fig. 18).

- Bare wires, being in contact with the ground, dissipate propagating currents much more efficiently. This can explain, why the site error shows no significant dependence on the rise time of the field (compare Fig. 17 b & d, with Fig. 18 b & d).

The graphs in Fig. 17 a & c and Fig. 18 a & c have practical application. For a given site provisioned for sensor installation, the LLS operator can easily estimate the expected maximum site errors. These graphs represent the worst-case scenario, where the cable shield of a supply cable (insulated conductor scenario) is grounded at both sensor ends. For a given cable length L and a vertical cable-to-sensor distance of 2.5 m, the maximum angle error α_{err} or amplitude error s_{err} can be determined based on the local ground conductivity σ_{loc} , and the sensor grounding impedance Z_{DC} . If the sensor grounding impedance is lower than the Z_{DC} value (blue ordinate in Fig. 17 and Fig. 18) for the given conductivity, the maximum site errors will exceed those shown in the graphs (due to higher currents at lower impedance). Conversely, if the grounding impedance is higher, the site errors will be smaller.

4 Discussion

This section serves as the ground to discuss the phenomenon of LLS sensor site errors, both in general and in relation to how they align with the practical experience of LLS operators.

While supply cable-related LLS sensor site errors exhibit a two-cycle periodicity, they are not fully symmetric, as suggested in (Schulz et al., 1998) and shown in Fig. 2. Specifically, they are not two-cycle sinusoidal. This asymmetry is more pronounced for long insulated supply cables, in which, when the angles of incidence align with the cable's orientation, the E-field interacts with a larger segment of the cable, and the resulting induced current propagates along the cable, reaching close proximity to the sensor, significantly affecting the site error. Conversely, if the EM wave approaches from the opposite direction, reaching the sensor first, currents are gradually induced, and the current elements along the cable take effect later in time, resulting in a lesser impact on the site errors. Consequently, both angle and amplitude site errors, α_{err} and s_{err} , are slightly lower for angles of incidence $90^\circ < \varphi < 270^\circ$, compared to $0^\circ < \varphi < 90^\circ$, respectively $270^\circ < \varphi < 360^\circ$ (see Fig. 6 and Fig. 15).

A real-world example for the angle site error is presented in Fig. 19, corresponding to a 600-m long power supply cable oriented at approximately $\varphi = 290^\circ$. The ground parameters are given in the figure caption. The simulation results align well with the observed site error levels, of about ± 10 -12°, and replicate the increased peak errors at angles around 290° , where the incident field direction is aligned with the supply cable. The x-axis depicts the azimuth measured by the sensor, which includes the angle error α_{err} . In Fig. 19b this was taken into account by plotting $\varphi_{\text{sampled}} = \varphi + \alpha_{\text{err}}$ (see also Fig. 6). While this study does not include detailed results, preliminary analyses indicate a positive correlation between site errors α_{err} and s_{err} and the rise time of the incident field, suggesting that larger rise times tend to



527 produce higher site errors. This observation is consistent with simulation results (see Fig. 17 and Fig.
528 18). However, further in-depth investigations involving a larger number of sensors are still needed to
529 corroborate this trend.

530 An important effect is observed, when the insulated wire is replaced with a bare conductor. This
531 change leads to an effective reduction in the angle and amplitude site errors, as illustrated in Fig. 14. In
532 (Theethayi and Thottappillil, 2007), the interaction between a horizontal grounding electrode and
533 parallel power supply and communication cables is discussed. This interaction may help explain why

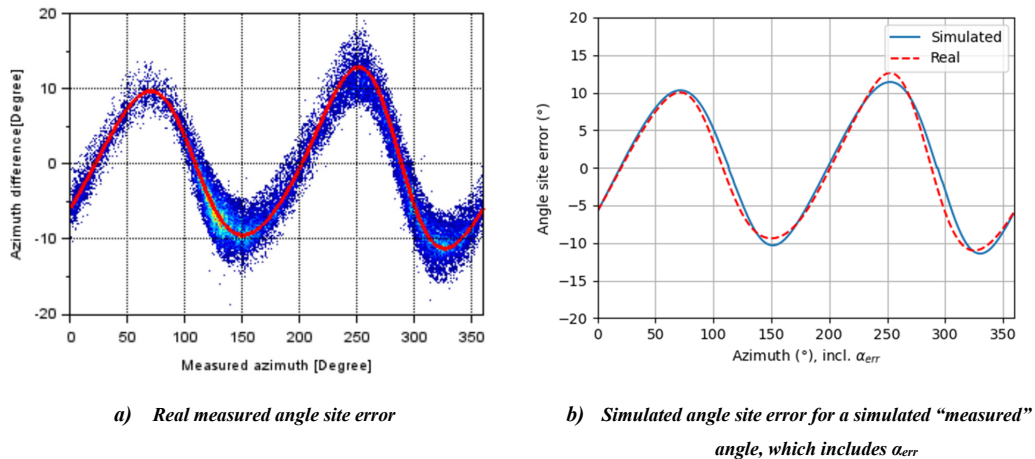


Fig. 19: Real measured site error (a) compared to simulation results (b) based on the methodology presented in this paper. Parameters: $\sigma_p = 2 \cdot 10^{-4}$ S/m to obtain incidents fields of about 8-9 μ s rise time (according to the median measured rise time at the sensor site). $\sigma_{loc} = 10$ mS/m, $\epsilon_{rg} = 10$. A 600-m long insulated power supply cable, oriented at $\varphi = 290^\circ$, is assumed to be buried 20 cm below ground in flat, swampy open terrain. The cable shield ends are both connected to ground.

534 the measured shield currents in (Schulz et al., 1998) were significantly lower than those predicted by
535 the present study. In (Schulz et al., 1998), the measured shield current magnitude was about 28 mA,
536 while the incident (vertical) E_z -field was approximately 6 V/m – twice the magnitude considered here.
537 This implies that for a field strength of 3 V/m (as shown in Fig. 7), the shield current would be
538 approximately 14 mA. This value is substantially lower than the simulated results presented in Fig. 12,
539 which assume a line length of 200 m and a local conductivity of $\sigma_{loc} = 10$ mS/m. Even with an
540 unrealistically high value of $\sigma_{loc} = 50$ mS/m, the computed shield current would still be much higher
541 than the measured value. The findings in (Theethayi and Thottappillil, 2007) suggest that a horizontal
542 ground electrode of about 10 m length and aligned with the power supply and communication cable,
543 may have favorably influenced the results by reducing site errors observed in (Schulz et al., 1998).
544 Future studies should consider the impact of a follow-on bare wire, such as horizontal electrode placed
545 10 cm from the cable. In (Theethayi and Thottappillil, 2007), this configuration was shown to
546 significantly reduce the internal voltages between the core and the cable shield.



547 In addition to the ideas presented in the preceding paragraph, further investigations are necessary
548 to analyze the impact of the sensor's precise electrical wiring, as this is likely to influence the results in
549 practice. Although not explicitly demonstrated in this study, the simulated shield currents - with the
550 cable shield being disconnected from the ground - yield angle and amplitude site error results that
551 significantly underestimate those occasionally observed in reality when shields are left open-ended. In
552 practice, disconnecting the shield often results in angle site errors reduced to half their original value.
553 This real-world observation could not be fully explained within the scope of the present study. It is
554 hypothesized that, in such cases, additional coupling mechanisms are at play, impacting the site error
555 behavior.

556 In areas with low conductivities, achieving grounding resistances often recommended by the
557 electrical equipment manufacturers, such as $10\ \Omega$, is nearly impossible. Instead, grounding resistance
558 tends to increase as the local ground conductivity σ_{loc} decreases. Taking this into account in the site error
559 simulations of the present study yielded results (see Fig. 17 and Fig. 18) that align more closely with the
560 overall behavior of LLS sensor site errors observed by LLS network operators. Interestingly, the most
561 problematic range of ground conductivities in terms of angle and amplitude site errors lies between σ_{loc}
562 $= 10^{-3}\ \text{S/m}$ and $\sigma_{loc} = 10^{-2}\ \text{S/m}$, which are commonly found at sensor sites. Thus, a shield connected to
563 ground is typically associated with high site errors, precisely as predicted in Fig. 17 and Fig. 18.

564 The complex interplay between σ_p , σ_{loc} , peak value ratios E_x/E_z , $E_x(z=-d)/E_x(z=0)$, and their
565 impact on the induced current has been demonstrated theoretically in this study (see Fig. 10a and Fig.
566 10b). However, it was also emphasized that, even for high local ground conductivity σ_{loc} , burial depth
567 alone does not significantly influence overall site errors. Instead, the increasing distance to the H-field
568 sensor with greater burial depth becomes the dominant factor in reducing the observed site errors.
569 Notably, the exact sensor location plays a crucial role, exhibiting inversely proportional ($1/r$) site error
570 levels. The higher the sensor is positioned above ground and farther from horizontally buried cable
571 segments, the smaller the sensor site errors. This observation aligns with the experience of LLS network
572 operators.

573 The seasonal contrast between dry and wet soil due to variations in precipitation and humidity,
574 likely plays a significant role in site errors, as it causes substantial changes in ground conductivity. It is
575 well established that the soil conductivity reaches its lowest values during seasons with little rainfall and
576 its highest during periods of frequent rainfall, particularly in the uppermost soil layer ($< 1\text{m}$). This
577 phenomenon is discussed in details in (Coelho et al., 2015).

578 Moreover, the present study assumes a one-layer ground model. In reality, scenarios are far more
579 complex, often involving stratified ground, inhomogeneous soil (particularly in terms of conductivity),
580 various cables, cable paths, installation circuitry, and diverse grounding methods. Consequently, the
581 theoretical considerations presented in this work, while providing insight into the fundamental principles



behind site errors, cannot fully capture the complexity of real-life scenarios. More in-depth investigations, both empirical and theoretical, are left for future research.

5 Conclusion

The presented study constitutes the first attempt to explain the physical mechanisms underlying angle and amplitude site errors when magnetic direction finders (MDF) are employed in lightning location systems (LLSs). From the outset, these errors have been attributed to shield currents in the sensor power supply cable, driven by the horizontal E-field component of the incident lightning EM field, resulting from ground losses. The objective was to present a modeling approach allowing to simulate LLS sensor's angle and amplitude site errors. Specifically, the computational model took into account the whole chain of physical interactions between the lightning EM field and the EM environment during propagation and detection at the sensor site. This process started from the computation of typical lightning EM field incident at the sensor site after propagating over lossy ground. It was followed by determining the horizontal E-fields responsible for driving coupled currents in the sensor power supply cable shield. After theoretically calculating cable shield currents, the resulting scattered magnetic fields, which cause LLS sensor site errors by altering the true incident H-field of interest, were computed using Biot-Savart's law. This involved considering current elements up to 50 meters from the sensor head. By computing the scattered H-fields ($\vec{H}_{err}$), it became possible to evaluate the theoretically expected site errors given for a given set of parameters, including the ground conductivity along the propagation path σ_p , the local ground conductivity at the sensor site σ_{loc} , the power supply cable length L , the burial depth d , and the grounding resistance of the shield connected to ground. The applicability and adequacy of each step are supported by a substantial body of literature, cited in this work and providing valuable resources for similar investigations.

The simulations of theoretical scenarios such as insulated and bare single-conductor cables or wires (representing cable shields or grounding electrodes), successfully reproduced angle and amplitude site errors across the entire azimuth range (0° - 360°), with satisfactory agreement to real-life observations from operational sensors. The impact of various parameters on the resulting sensor site errors was thoroughly discussed, and key graphs in Fig. 17 and Fig. 18 highlight the influence of the local ground conductivity σ_{loc} - and accordingly the grounding resistance - on the maximum expected site errors. These results provide LLS network operators with a straightforward tool to estimate expected site errors at provisioned sensor locations, or, retrospectively, to evaluate whether observed site errors align with expectations.

For optimal behavior, it is recommended that the shield always remain disconnected from ground at the sensor-side end, as this minimizes coupled shield currents near the MDF sensor. The observed reduction of site errors to approximately half when the shield is disconnected from ground could not be fully explained within the scope of this study, requiring further in-depth investigations. Furthermore, bare



wires (e.g., horizontal ground electrodes) exhibit smaller site errors and show a significantly reduced dependency on wire length. Thus, they can be beneficial as follow-on electrodes parallel to the supply cables to reduce site errors.

The simulations also replicated subtle deviations from a perfectly symmetric double-cycle sinusoidal site error behavior. These nuances, apparent when comparing Fig. 2 and Fig. 15, and further confirmed by Fig. 19, corroborate the reliability of the study's results.

The presented methodology provides a solid foundation for further studies related to supply cable-induced LLS sensor site errors. Subsequent investigations should aim to identify optimal configuration for LLS sensors at specific sensor site locations.

Code/Data availability:

The code and data can be provided upon request at OVE Service GmbH (corresponding author)

Competing interests:

The authors declare that they have no conflict of interest

Author contribution:

HK: Methodology, Simulation, Results, manuscript writing

WS: LLS sensor data extraction and preparation, manuscript writing, proof reading

FR: Proof reading, contextual and editorial corrections and suggestions

ND: Assistance and validation in ground modeling and field-to-cable coupling

DK: Result validation of the simulations, assistance w.r.t. the FDTD-method

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