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July 18, 2025

To: Dr. Heng Dai, Editor of Hydrology and Earth System Sciences
Subject: Revision of Paper # EGUSPHERE-2024-4145

Dear Editor:

Upon the recommendation, we have carefully revised Paper # EGUSPHERE-2024-4145 entitled "Improving heat transfer predictions in heterogeneous riparian zones using transfer learning techniques" after considering all the comments made by the reviewers. The following is the point-point response to all the comments.

Response to Reviewer #1:

Overview:

Thank you for submitting your manuscript addressing the comments previously made. This new version of the manuscript is in better shape. I only have a few comments and one technical correction that are presented below.

Reply: Thanks for your constructive comments. We have carefully revised Paper # EGUSPHERE-2024-4145.

Specific Comments:

(1) It is still not clear how the authors find the prescribed values for Darcy's fluxes (q_x and q_z) in the analytical solution. The authors argue in their response (lines 91 to 96 in the response document) that the groundwater flow and heat transfer models are coupled through q_x and q_z , which they prescribed in the analytical solution. The numerical model has two boundary conditions where a constant head is set (Figure S2). Did the authors calculate the Darcy's fluxes using the difference between these heads throughout the domain and the given hydraulic conductivity, or did they use the Darcy's flux simulated results for a given cell within the domain? I recommend providing further clarification on this.

Reply: Implemented. In the analytical model of Shi et al. (2023), the thermal front velocities v_x and v_z are directly prescribed, please see Eqs.(1)-(10).

$$T = A(x, z, t) \left[\int_0^\infty \int_0^L E(\lambda, \eta) B(x - \lambda, z - \eta, t) d\lambda d\eta + \int_0^t g(\tau) C(x, z, t - \tau) d\tau \right] \quad (1)$$

$$A(x, z, t) = \frac{1}{4} \exp \left[-\frac{v_z(v_z t - 2z)}{4D_z} - \frac{v_x(v_x t - 2x)}{4D_x} \right] \quad (2)$$

$$B(x - \lambda, z - \eta, t) = \frac{1}{\pi t \sqrt{D_x D_z}} \left\{ \exp \left[-\frac{(z - \eta)^2}{4D_z t} \right] - \exp \left[-\frac{(z - \eta)^2}{4D_z t} \right] \right\} \\ \left\{ \exp \left[-\frac{(x - \lambda)^2}{4D_x t} \right] + \exp \left[-\frac{(x - \lambda)^2}{4D_x t} \right] \right\} \quad (3)$$

$$C(x, z, t - \tau) = \frac{z}{\sqrt{\pi D_z(t-\tau)^3}} \exp \left[-\frac{z^2}{4D_z(t-\tau)} \right] \left\{ \operatorname{erf} \left[\frac{L-x}{\sqrt{4D_x(t-\tau)}} \right] + \operatorname{erf} \left[\frac{L+x}{\sqrt{4D_x(t-\tau)}} \right] \right\} \quad (4)$$

$$E(\lambda, \eta) = h(\lambda, \eta) \exp \left[-\frac{2\eta v_z}{4D_z} - \frac{2\lambda v_x}{4D_x} \right] \quad (5)$$

$$g(\tau) = f(\tau) \exp \left[\frac{v_z(v_z\tau)}{4D_z} + \frac{v_x(v_x\tau-2x)}{4D_x} \right] \quad (6)$$

$$q_x = -K_x \frac{\partial H}{\partial x} \quad (7)$$

$$q_z = -K_z \frac{\partial H}{\partial z} \quad (8)$$

$$v_x = \frac{c_w}{c_s} q_x = -K_x \frac{c_w}{c_s} \frac{\partial H}{\partial x} \quad (9)$$

$$v_z = \frac{c_w}{c_s} q_z = -K_z \frac{c_w}{c_s} \frac{\partial H}{\partial z} \quad (10)$$

One can find that v_x and v_z are directly prescribed in the analytical model, which can be transformed into the Darcy's fluxes through Eqs.(9)-(10).

For heterogeneous scenarios (numerical model), the boundary conditions of groundwater flow model are applied to generate two-dimensional nonuniform flow fields. The Darcy's fluxes are calculated using the difference between these heads throughout the domain. The settings of initial and boundary conditions are shown in Figure S2.

(2) The main objective of the manuscript is to propose a novel physics-informed deep transfer learning (PDTL) approach to improve the accuracy of spatiotemporal temperature distribution predictions. The manuscript, as it is, works towards that goal and presents a promising methodology. My only concern is the use of the location in the domain as the only input variable for the machine learning model. This limits the transferability of the model to other potential locations and disconnects the association of the model to measurable physical variables, such as the temperature at the surface and the fluxes of water from the river to the underlying aquifer. Some of these limitations are already presented in the discussion, but I encourage the authors also to discuss the limitations of their selected input variables.

Reply: We agree that using locations as inputs may limit the model's transferability to other sites and weaken its direct connection to measurable physical variables. We have added further discussion on this limitation in the revised manuscript. Future work will incorporate additional physically measurable parameters, such as surface temperature, river-aquifer fluxes, or hydraulic gradients, to enhance the model's generalizability and physical relevance. Please see Lines 335-339.

Technical Correction:

(1). The text in the axes of Figures 5, 6, and 9 is difficult to read. Consider increasing the fonts. Also, include the units of the variables plotted.

Reply: Implemented. We have increased the fonts and added the units of the variables in Figures 5, 6, and 9.

Response to Reviewer #3:

Overview:

Thank you for the opportunity to review this thoroughly revised manuscript. The authors present a novel hybrid framework that combines simple analytical solutions for homogeneous systems with transfer learning techniques to address heterogeneous heat transfer problems in riparian zones. The topic is timely and of broad interest, and the methodology, which leverages well understood analytical models to inform data driven learning, represents a creative and potentially widely applicable approach. The revised manuscript is

significantly improved in organization, clarity, and technical depth compared to the previous version, and I recommend acceptance after a few clarifications and minor edits.

Reply: Thanks for your constructive comments. We have carefully revised Paper # EGUSPHERE-2024-4145.

General Comments:

(1) First, the rationale for using both analytical and numerical models to generate training data and benchmarks would benefit from a more explicit explanation. Although the hybrid strategy intuitively combines the interpretability and low-cost evaluation of analytical solutions with the flexibility of numerical simulations, the manuscript would be stronger if it contrasted this approach against purely numerical or purely statistical alternatives. For example, how does the introduction of analytical physics reduce the required volume of high-fidelity simulations and how does it improve generalization to untested heterogeneous conditions? A brief discussion of these tradeoffs and any computational savings or improved convergence properties observed would help readers appreciate the advantages of the proposed framework.

Reply: Implemented. We have added explicit rationale for employing both analytical and numerical models to generate training data and benchmarks. Please see Lines 171-176.

In this study, we focus on the physical principles transferred from homogeneous to heterogeneous streambeds using transfer learning techniques. Accordingly, the source and target datasets are spatiotemporal distributions of temperature fields in homogeneous and heterogeneous streambeds, respectively. The analytical model developed by Shi et al. (2023) is employed to provide training dataset for pre-training. However, for the heterogeneous streambed, the analytical model is not available; therefore, numerical models are employed to generate the fine-tuning dataset and serve as the benchmarks to evaluate the performance of the proposed PDTL model.

We also conducted the comparative discussion between the hybrid approach (PDTL model) and or pure data-driven method (DNN model). Results indicate that the PDTL model significantly outperforms the DNN model in scenarios with scarce training data. Furthermore, increasing streambed heterogeneity and observation noise levels raises the mean *MSE* values of the PDTL and DNN models, with the PDTL model exhibiting greater robustness than the DNN model, highlighting its potential for practical applications in riparian zone management. Please see Section 3.2-3.4.

(2) Second, the choice of Shi et al. (2023) as the benchmark analytical model deserves further justification. The literature contains numerous analytical expressions for subsurface heat transport in riparian contexts; explaining why this formulation was selected (for example, because of its balance of simplicity and fidelity, its treatment of boundary conditions, or its previous validation against field data) would clarify its role in the study. If other models were considered but found less suitable, a sentence or two outlining those comparisons would reinforce confidence in the benchmark's relevance.

Reply: Implemented. We have added a justification for selecting the analytical model of Shi et al. (2023) as the benchmark in our revised manuscript. Specifically, we clarified that this model offers a balanced trade-off between analytical simplicity and accuracy, particularly in representing boundary conditions relevant to heat transport between surface water and groundwater. Moreover, this model has been validated against field data, further enhancing its credibility. Please see S1-"Analytical solution of the 2D heat transfer process in homogeneous streambed" in the Supplement.

(3) Third, the description of the transfer learning workflow indicates that some network layers were frozen while others remained trainable, but the manuscript does not specify which layers were selected nor the criteria guiding these decisions. Since layer freezing can critically affect the retention of low-level physical features versus high level adaptation to heterogeneous data, please detail which layers were frozen, which

were fine-tuned, and why. Were early convolutional filters preserved to encode general diffusion patterns while later dense layers were adapted to capture heterogeneity? Clarifying this architecture will help readers reproduce the experiments and understand how transfer learning choices influence model performance.

Reply: Implemented. The difference in weights and biases between the two pre-training models with different hydraulic conductivities is evaluated using the relative change rate (*RCR*):

$$RCR = \frac{1}{I} \sum_i^I \frac{|\theta_{1i} - \theta_{2i}|}{\theta_{1i}} \quad (11)$$

where θ_{1i} and θ_{2i} are parameter matrixes in two pre-training models, respectively, I is the number of elements in the parameter matrix. For enhanced comparability and credibility, each of the two pre-training models undergoes 20 training processes. Figure 1 presents the average *RCR* of weights and biases across all layers for two pre-training models over 20 trials. The *RCR* of biases shows consistent stability across all layers, except for layer 3. In contrast, variations in weights are more prominent, particularly in layers 1, 2, and 3, which underscores the heightened sensitivity of these layers to hydraulic conductivity. Consequently, layers 1, 2, and 3 of the pre-training models are marked as trainable, while the remaining layers are held frozen in the following analysis. Please see Section 3.1.

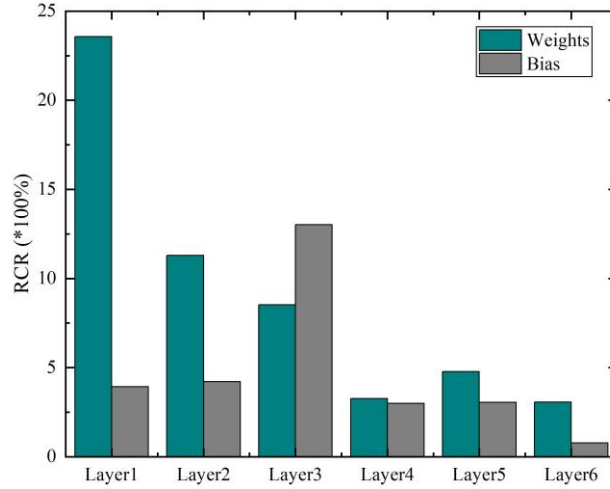


Figure 1. The average relative change rate (*RCR*) of weight between pre-training neural network with different K values.

(4) Fourth, the results demonstrate that transfer learning informed by analytical solutions performs remarkably well, even in heterogeneous systems that deviate from the homogeneous assumptions underlying the analytical form. It would be valuable to discuss potential reasons for this robustness: for instance, do the analytical solutions capture the dominant modes of heat propagation that persist under moderate heterogeneity? Is there a particular physical principle or scaling law embedded in the base model that remains valid across a range of conductivity contrasts? A short analysis of what features the network retains from the analytical initialization and how those guide learning in more complex settings would deepen insight into why the approach succeeds.

Reply: Implemented. The proposed PDTL model integrates advantages of analytical solutions, deep learning models, and transfer learning techniques. The analytical model is used to efficiently produce physically consistent heat distribution patterns and data in homogeneous riparian zones, which serve as the training data for the pre-training deep learning model. Subsequently, the weights and biases learned from the pre-training model are transferred to a new deep learning model under heterogeneous scenarios through transfer learning. Furthermore, we have discussed how the transferred physical knowledge impacts the parameters of deep learning model. Results indicate that the hydraulic conductivity primarily influences the parameters

of the shallow layers (Layers 1-3) in the DNN model. Therefore, layers 1, 2, and 3 of the pre-training models are marked as trainable, while the remaining layers are held frozen in the following analysis. Please see Lines 192-215.

Specific Suggestions:

(1) In Lines 65-73 “DL” should be “deep learning”

Reply: Implemented. We have revised “DL” to “deep learning”. Please see Lines 65-73.

(2) In Line 87, “transfer learning” should be “transfer learning techniques”.

Reply: Implemented. We have revised “transfer learning” to “deep learning techniques”. Please see Line 87.

(3) In Line 91, “power” should be “capability”.

Reply: Implemented. We have revised “power” to “capability”. Please see Line 92.

(4) In Line 106, $L=0.32m$ is used in Section 2.1 or in all cases?

Reply: Implemented. $L = 0.32m$ is used in all cases.

(5) In Lines 138, 151 and 152, “physical information” should be “physical principles”.

Reply: Implemented. We have revised “physical information” to “physical principles”. Please see Lines 138-152.

(6) In Lines 207-208, add references to support this point.

Reply: Implemented. We have added some references to support our point to restrict the number of epochs in the model training process.

Reference:

[1] Wang, N., Chang, H., and Zhang, D. (2021). Deep-learning-based inverse modeling approaches: A subsurface flow example [J]. Journal of Geophysical Research: Solid Earth, 126, e2020JB020549.

[2] Zhang J, Liang X, Zeng L, et al. Deep transfer learning for groundwater flow in heterogeneous aquifers using a simple analytical model [J]. Journal of Hydrology, 2023, 626: 130293.

If you have any further questions about this revision, please contact me.

Sincerely Yours,

Quanrong Wang, PhD, PG.

Professor and

Holder of Endowed CUG Scholar in Hydrogeology

