17 April 2025

We appreciate the comments raised by the two reviewers, and thank them for the time and energy they spent on our paper. Their comments below are in black text*, and our replies to each comment are in blue italics. Any text directly added to the manuscript will be shown in green italics in our reply below.*

Reviewer #1:

This manuscript describes how to propagating uncertainties and determine the information content of quantities derived from multiple instruments, and it illustrates this technique with an example. It is well written and will be a valuable addition to this topic. I recommend that it be published after minor revisions. I have listed comments/suggestions below that I hope the authors consider to make the manuscript a bit more accessible to those not proficient in these topics.

Thank you for taking the time to read our paper and provide these comments!

As a general comment coming from a reader who is not as familiar in this way of thinking as the authors, terms such information content and DFS are not in my daily vocabulary, and some of the results are not intuitive. I would urge the authors to assist us in making these concepts, which I believe is important ones, more accessible, and to provide more physical explanations where possible. For instance, the definition of information content is not given until line 55, and DFS not until line 110. Only on line 194 did I see it clearly stated that the variability in these quantities is due to instrument differences (also stated on line 324). For instance, is it possible to show or discuss the contributions of the various measurements to DFS? These comments should not be construed as criticisms of the manuscript, which is well written and reads nicely, but as an appeal to make challenging and non-intuitive concepts easier to comprehend.

Information content in retrievals can be a challenging concept to grasp. We point out that the first paragraph of the abstract does indicate what perfect information content would be in an observation. However, as we know many people struggle to really understand the concept of information content, the first 50 lines of the paper are set up to bring the reader into the idea. In the body of the paper, the first time we introduce the phrase "information content" is line 53, and then we immediately define it in lines 54-55.

*Similarly, the first time we define degrees of freedom for signal (DFS) is at line 110; it was not important to understanding the material that appeared earlier in the paper. We did add a new sentence at line 116, after we have defined DFS, that states: "*In other words, the DFS quantifies

the information content in the retrieval for each variable that is being retrieved in the vector *x*."
We are not sure how to reorganize the paper to help the reader more easily understand either information content or DFS beyond

Specific comments:

Line 130: This would be a good place to start a new section.

We agree – and created a new subsection 2.2 Case study: 13 June 2019

Line 145: It is not obvious from the figures that the retrieved winds are larger above 1 km.

We believe that the reviewer mis-read the line: we state that "the uncertainties in the retrieved winds from the E37 DL…two DLs, especially above 1 km." We believe that this is easily seen in Fig 3 c1,d1, which is what we are pointing out here. (Note we will slightly adjust this explanation, after addressing another comment from this reviewer down below).

Line 177: Addition of another equation or a bit more explanation of how Eq. 5 becomes Eq. 6 would help the reader who is not as familiar with this topic as the authors are.

We have added another sentence to help address this after Eq 6: "Note the translation of Eq 5 to Eq 6 uses the fact that $K_x = \partial F/\partial x$ and that the covariance of x can be written both as σ_x^2 or as the matrix S_x."

Line 179: The phrase "where the superscript T in this content represents matrix transpose" should be place near Eq. 1 where it first appears.

Good point. We added that phrase to the sentence right after equation 1. We also left this phrase after Eq 6 also (just to be clear).

Lines 180-195: These would be better places before line 150.

Excellent suggestion, and we agree. We moved that text that describes the instrument-level uncertainties to (the new) section 2.2, just after Figure 3. This aids the flow of the information content derivation from the 6 instruments after Equation 6.

Line 194: The explanation "due to the differences in the instrument uncertainties at the different locations" should appear earlier.

We agree. This was done when we addressed the suggestion immediately above.

Line 200: The statement "the cold air advection … has a lot of uncertainty" is vague; is it the magnitude of the cold air advection, the timing, the direction, or what? Perhaps (likely?) that is my lack of understanding, but "cold air advection" sounds like a process, not a quantity (such as

temperature advection or water vapor advection); thus, it is unclear what the uncertainty would refer to.

Ah, that is a good point. We are referring to the magnitude of the cold air advection here. We have clarified that point in the text.

Line 203: Show the location of the other two sites in Fig. 4.

On the spatial scale of this map, which we chose so that the larger synoptic conditions could be examined, if we used yellow dots of the same size then the three sites would be virtually on top of each other. This brings us to a comment raised by the other reviewer, who asked that we reconfigure Figure 1 to be more useful with distance information.

Line 210: The statement that the information content is approximately 5 seems impossible from the statement on line 112 that the information content is between 0 and 1, and those shown in Fig. 5 are less than unity. I may be (likely am?) confusing different quantities, but that merely demonstrates that a typical reader may be confused here, and that a bit more explanation would be useful.

We agree; our language was not clear. We have modified that sentence to say: "the sum of the total information content from the surface to 3 km (i.e., $\sum_{z=0\ \mathrm{km}}^{z=3\ \mathrm{km}} \mathrm{DFS(z)}$) is approximately *5 for T(z) and between 3-to-5 for q(z)."*

Line 234: Perhaps move the title of the section to line 245, as that seems to be where the example actually starts.

The reviewer is talking about the start of Section 4. The second sentence is that section is discussing the DFS of the temperature and water vapor advection, and pointing it out in Figure 5, which is part of this example. So we think that the section is starting in the correct place.

Line 244: The statement that the information content on one quantity can be near unity even though the information content of an instrument can be low is crucial and should be more strongly emphasized.

We agree. We have added the statement: "this is because the advection is essentially an evaluation of spatial gradients, which the AERI is able to determine even with its limited information content in the vertical."

Line 244: "in information content" should be "an information content"

Yes. Good catch

Line 244: Panels b1, b2, and b3 of Fig. 5 show DFS of water vapor exceeding 0.05 at heights greater than 50 m.

We updated that sentence to say: "AERI's information content is very limited with DFS < 0.05 at any height above 50 m for T and DFS < 0.3 for q"

Line 245: The end of this sentence is a great location to remind the reader that this is due to the instrument.

Good suggestion: we added "due to the larger instrument noise level in the E37 DL".

Line 259: I had a "why?" after the statement that doubling the covariance matrices by a factor of 2 had little effect on the DFS, and found myself desiring a more physical explanation of this result.

It is a curious result. We have added these sentences to offer some thoughts: "Presumably, this is because advection is a spatial calculation, and that the uncertainties at the vertices has relatively little impact on the derived advection. However, this result likely would depend on the size of the polygon used for the calculation; Wagner et al. (2022) demonstrated that the current locations of the ARM site is close to optimal in minimizing both the random and sampling error in the calculation."

Figure 5: In the caption, perhaps label that columns 1, 2, and 3 refer to E37, C1, and E39 so they don't have to look back to Figure 2 to find this information. The rest of the information on this panel (e.g., T, WV, U, V) are labeled, but the locations aren't.

Done.

Line 260: "diagonal" or "diagonal elements"?

We have adopted your suggestion of diagonal elements.

Line 298: A bit more discussion would be helpful here. What happens at those altitudes, where the standard deviation is greater than the mean?

We have added the phrase "implying that there is marked variability in the DFS above this height from case-to-case" to the discussion in that paragraph to discuss the implications.

Line 324: This statement should appear much earlier in the manuscript.

We agree, and added a new sentence directly after Figure 3 that states: "Note that differences in the noise characteristics among the AERIs will result in differences in the retrieval uncertainties; this is also true for the Doppler lidar systems."

Line 331: "strong function height" should be "strong function of height"

Done

Line 336: “derived from remote sensors” or “retrieved from remote sensors” or “obtained from remote sensors”

We restructured the sentence to make it clearer: “This work demonstrates how to derive the information content of an observation that is derived from multiple remote sensing datasets.”