We thank the reviewer for the thorough reading of the manuscript and the insightful comments, which clearly helped to improve the manuscript. We addressed all points in the revised version as described below. Reviewer comments are in normal font, answers in italics. A summary of the main changes in the revised version is given in the bullet point list here (with more details given in the replies to the specific comments below).

- The title of the manuscript has been revised to more accurately reflect its content, including the requested modifications.
- In response to Reviewer 2's suggestion, we refined the scale-based separation to distinguish between planetary, synoptic-scale, and mesoscale waves. The separation is now presented in sect. 2.2, and discussed with the new Fig. 3 in sect. 3.
- A decomposition into more physical atmospheric wave phenomena was requested by the reviewer 1. In response, the relation between wave scales and physical wave types is discussed at the end of sect. 2.2: planetary waves are associated with quasi-stationary Rossby waves, synoptic-scale waves represent a mixture between Rossby and gravity waves, and mesoscale waves are associated with gravity waves.
- As requested by Reviewer 3, we conducted additional analysis of upwelling trends to examine whether separating the ozone recovery period from the post-ozone recovery period improves the statistical significance of the trends.
- More information from reanalyses other than ERA5 was added to the sect. 3 and 4 of the manuscript as requested by Reviewer 2
- As requested by Reviewer 2 and 3 rewriting and restructuring of several text parts throughout the entire manuscript was done to improve clarity and readability eg. sect. 2.2, sect. 3 and sect. 4.
- The comparison of resampled ERA5 data with original fine resolution ERA5 data was expanded (see Sect. 5), as requested by reviewer 2.

This paper uses reanalysis data primarily ERA-5 data to determine if stratospheric circulation branches may be defined based on wave-forcing. Three science questions are posed: (1) Can different branches in the stratospheric circulation be dynamically defined based on the wave driving? (2) Did different branches of the stratospheric circulation change differently over the past decades? (3) How robust is the representation of the circulation structure and its changes in different reanalyses? Results identify two robust branches: a deep branch predominantly driven by waves with wavenumbers 1 to 3 and a shallow branch predominantly driven by waves with wavenumbers 4 to 180. Results also find a long-term trend in these branches.

My main issue is that the paper performed the calculations based on a definition of atmospheric waves solely in terms of wavenumber. This paper quantifies the contributions of atmospheric waves on these deep and shallow circulation branches. The waves are separated based on their wavenumbers. There's a category of waves comprising those with wavenumbers 1 to 3 and another for waves with wavenumber 4 to 180. There is no mention of phase and periodicities. Taking this approach, the paper doesn't properly decompose the dynamical variables into real physical atmospheric wave phenomena. Hence, there is questionable physical meaning/significance to the results.

The deep and shallow circulation branches that are reported in the introduction seem to allude to winter circulation. Hence, a proper analysis should've focused on real physical planetary-scale waves (e.g. wave-1 VS wave-2 and/or Stationary vs travelling waves) that are known to occur when these branches form. The analysis should have involved processing daily reanalysis data. The paper actually doesn't clearly state how the problematic decompositions are made. Some sections indicate the decomposition was done on monthly-means which isn't sufficient. Either way, defining waves solely in terms of wavenumber was a major flaw. Consequently, the analysis of long-term changes are rendered wrong. Owing to this, I cannot recommend this paper for publication. I suggest the authors take guidance from papers like Yasui et al [2021] and Koval et al [2023] on how to properly quantify the role of atmospheric waves on any middle atmospheric phenomena. You'll see all of them properly acknowledging different kinds of physical planetary-scale waves (e.g. Rossby waves, kelvin waves, etc).

We thank the reviewer for the thorough reading and evaluation of the manuscript and the critical comments.

In principle, wave types can be distinguished using a variety of criteria, including spatial and temporal scales, underlying restoring mechanism, or variable of interest. There is no universally correct classification scheme; the appropriate choice depends on the specific context and the aspects of wave behavior one aims to interpret. The analysis in this study is performed on a data record of 38 years length and for four different reanalysis data sets. This requires a diagnostic approach which keeps the computational effort of the analysis affordable and motivates our decision for a solely-longitudinal scale separation. In the stratosphere, such an approach is well justified and commonly used (e.g. Abalos et al., 2024; McLandress and Shepherd, 2009; Kim et al., 2016). We agree that in the mesosphere and at even higher altitudes, issues can arise when applying simple scale separation as the reviewer has pointed out. However, the main focus of our paper is on the region near the separation level in the lower stratosphere (below approximately 30 km). Here, Kelvin waves are typically confined to within approximately $\pm 10^o$ *of the equator, away from the turn-around latitudes. As the tropical upwelling of the stratospheric circulation at these altitudes is mainly driven by wave drag around the turn-around latitudes in the subtropics, Kelvin waves therefore have a very limited impact on the circulation. Furthermore,* *some insight into the physical wave types can be inferred based on their spatial scales: planetary waves (wavenumbers 1–3) are most likely stationary or quasi-stationary Rossby waves; synoptic-scale waves (wavenumbers 4–20) are dominated by Rossby waves with increasing secondary contributions from gravity waves for higher wavenumbers; and mesoscale waves (wavenumbers greater than 20) are likely dominated by gravity waves. A related discussion linking wave scales to physical wave types has been incorporated in the revised version of the manuscript at the end of Sect. 2.2 and is further elaborated in Sect. 5.*

We agree that a physical separation of the waves would be interesting, but this would be another, complementary study. Using reanalysis data, which do not have an inherent separation of wave modes, this would require a space-time decomposition which is costly. Hence, a physical separation is immensely complex, would need thorough validation and thus is unfortunately out of scope of the paper.

We further would like to clarify that the decomposition was performed on instantaneous, regularly sampled global reanalysis data (6-hourly basis, even 1-hourly for the high-resolution ERA5 test case). All averaging in the paper is performed after evaluation of these snapshots. Thus, the effects of all wave modes on the EP-flux are kept and no information is lost by averaging. To avoid confusion, this is described in a much clearer way now at the end of the methods Sect. 2.2.

Other issues:

- This paper is centered on the deep and shallow circulation branches that are centered on a TAL between 20 N/S and 40 N/S. In the introduction, the paper acknowledges previous studies identifying shallow and deep branches in the stratospheric circulation. However, the science questions made it seem as though this paper aimed to find other very distinct kinds of circulation. The science questions need to be re-written to clarify that this paper still focuses on these deep and shallow circulation branches.

  We thank the reviewer for pointing out this potential for confusion regarding the research questions. We have clarified the related text to present the scientific questions more clearly, specifically emphasizing that the deep and shallow circulation branches are the focus of this paper. The revised questions can be found at the end of the introduction in Sect. 1.

- Add a paragraph at the end of section 2.2 that explains how each science question can be answered using these parameters (and any accompanying methodology)? This will help readers immediately know what to look out for.

The paragraph is added at the end of Sect. 2.2. To address the research questions outlined in the Sect. 1, a Fourier transformation is applied to 6-hourly snapshots of the fluctuation components, and $\nabla \cdot F(s)$ is calculated (see Eq. 4). After that, the monthly mean is computed from the $\nabla \cdot F(s)$ snapshots. Utilizing the DWCP framework (Eq. 5), the monthly mean $\nabla \cdot F(s)$ related to specific wavenumbers is used to reconstruct the circulation driven by individual waves or sets of waves. Subsequently, the associated upwelling and outflow are computed from the reconstructed circulation using Eq. 7, and 8. The analysis is then conducted on the resulting upwelling and outflow fields.

- The authors need to introduce figures first before reporting on results found in the figures.

  We agree that at several points the figures could be introduced and described in a clearer way to make it easier to the readers to follow the line of argumentation. We carefully worked through the entire manuscript to make sure that figures are always explained clearly.

- The authors need to make use of the mathematical symbols of terms (that they already introduced). Some sections are too wordy because instead of using the short-cut symbols, the authors still write the complete name/phrase of the parameter.

  Thanks for the comment. It is always a balance between conciseness and readability when using mathematical symbols. To maintain readability also for readers who are not familiar with the mathematical language of the TEM formalism and stratospheric circulation we find it beneficial not to use mathematical symbols too excessively. Nevertheless, we followed the reviewer's suggestion partly and carefully worked through the entire manuscript to shorten the text by use of symbols wherever we found that reasonable.

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