The global ocean mixed layer depth derived from an energy approach

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Abstract

The mixed layer depth (MLD) is critical for understanding ocean-atmosphere interactions and internal ocean dynamics. Traditional methods for determining the MLD ~~often rely on hydrographic thresholds that vary spatially and temporally with local oceanographic conditions~~, commonly relying on constant temperature and density thresholds, may not adequately address spatial and temporal variations in local oceanographic conditions, limiting their global consistency and applicability. ~~To address this, we propose an energy-based methodology that defines the MLD as the depth at which the work done by buoyancy (WB) reaches 20 Jm^{-3}. This approach provides a robust, globally consistent, and easy-to-implement criterion grounded in physical principles. Our methodology captures the upper ocean's well-mixed layer in energetic terms, aligning with turbulent boundary layer dynamics while maintaining quasi-homogeneity in density and temperature for most of the global ocean. A global monthly MLD climatology derived from this method demonstrates its reliability across diverse oceanic conditions and its accuracy in regions and seasons where conventional methods struggle. This energy-based approach~~ An energy-based definition of the mixed layer could be a more physically consistent alternative to address this. We propose a physically derived and energy-based methodology that defines the mixed layer as the energetically homogeneous upper ocean layer in which water parcels can move with little or no buoyancy work. The threshold in buoyancy work determining the mixed layer globally throughout the year was carefully investigated. This approach provides a robust criterion that is globally and temporally consistent and easy to implement. An energy-based global monthly MLD climatology demonstrated the reliability of the methodology across diverse ocean conditions and its usefulness for seasonal to climate time scale studies, from regional to large spatial scales. Our methodology aligns with turbulent boundary layer dynamics while maintaining quasi-homogeneity in energy, density, and temperature year-round for most of the global ocean. This study advances the development of MLD energy-based methodologies that could offer significant potential for advancing the study of dynamic, and thermodynamic processes, including heat content and vertical exchanges. ~~It~~ Our methodology could also serve as a robust tool for validating ocean circulation models and to support intercomparison studies in initiatives such as the Ocean Model Intercomparison Project (OMIP) and the International Coupled Model Intercomparison Project (CMIP). Future research will explore its applicability to high-frequency processes and regional variability, further enhancing its utility for understanding and modeling oceanic phenomena.

Short Summary (500 characters, non-technical)

The surface mixed layer depth (MLD), where ocean properties are uniform, is key to ~~ocean-atmosphere interactions and ocean dynamics. We propose an energy-based method that defines the MLD using a constant value of buoyancy work that accurately captures the upper ocean's well-mixed layer. This approach is globally and temporally consistent and reliable, improving MLD estimates, aiding ocean model validation, and advancing studies of ocean heat content and vertical exchanges.~~ ocean dynamics and ocean-atmosphere interactions. We propose an alternative definition of the mixed layer as the layer in which water parcels can move with little or no work. This approach provides realistic mixed layer depth estimates across space and time under diverse ocean conditions. It has potential implications for improving our understanding of various ocean-atmosphere phenomena, including dynamic and thermodynamic ones.

1. Introduction

The ocean mixed layer is the ocean's surface layer in direct contact with the atmosphere, whose properties (potential density, temperature, salinity, and other tracers) are relatively homogeneous in the vertical. Such relatively vertical homogeneity is due to turbulence caused by wind effects, buoyancy-driven fluxes, and waves (D'Asaro, 2014; Sallée et al., 2021; Reichl et al., 2022). The mixed layer concept allows for diagnosing vertical exchanges within the ocean and those between the ocean and the atmosphere without a detailed analysis of the associated turbulent processes (D'Asaro, 2014; Sutherland et al., 2014; Franks, 2014). The mixed layer plays a crucial role in the Earth's weather and climate since it determines the energy, mass, and momentum exchanges between the ocean and the atmosphere (Gill, 1982). It largely determines different aspects of the climate system: ocean surface temperature (Deser et al., 2010), formation and properties of water masses (Hanawa and Talley, 2001; Groeskamp et al., 2019), thermal energy available to a tropical cyclone (Shen and Ginis, 2003), biological productivity (Franks, 2014; Bouman et al., 2020), chlorophyll content (Briseño-Avena et al., 2020; Carvalho et al., 2017) and carbon subduction (Bopp et al., 2015; Omand et al., 2015).

The mixed layer depth (MLD) is a key variable in understanding the past, present, and future variability of Earth's weather and climate (Sallee et al., 2021; Reichl et al., 2022; Treguier et al., 2023). However, the definition of the mixed layer as a relatively homogeneous layer is very vague (de Boyer Montégut et al., 2004), which has led to numerous definitions and estimates of the MLD (e.g., de Boyer Montégut et al., 2004; Holte and Talley, 2009; Schofield et al., 2015; Reichl et al., 2022; Romero et al., 2023). The above has resulted in high uncertainty in estimating the MLD, mainly in deep and intermediate water formation regions, polar seas, and barrier layer regions (Treguier et al., 2023). That also has affected the analysis of the relationship between the MLD and various ecological processes, such as chlorophyll-a content, phytoplankton dynamics, and primary production (Carvalho et al., 2017; Bouman et al., 2020). No MLD definition provides accurate estimates for all world regions under different ocean conditions throughout the year.

Different authors have suggested that the physically relevant definitions of MLD should be density-based since this approach captures both temperature—and salinity-driven stratifications at the mixed layer (Griffies et al., 2016; Sallee et al., 2021; Treguier et al., 2023). The protocol used to compute the MLD in the Ocean Model Intercomparison Project (OMIP) and the International Coupled Model Intercomparison Project (CMIP) considers a constant density threshold; however, in regions with vertically compensated layers, the density threshold may overestimate the MLD (de Boyer Montégut et al., 2004). No physical reason sustains the choice of a specific density threshold; instead, it is heuristically obtained. Reichl et al. (2022) proposed a MLD definition considering the gravitational potential energy of the water column; ~~while~~ their work is promising because it is based on physical principles~~, further research is required to select the energy value that defines the MLD and investigate if such value is globally applicable during all seasons~~, but they did not provide specific energy values to define the MLD globally during all seasons. Consequently, MLD methodologies, physically derived and energy-based, need to be developed and investigated in more detail to provide accurate estimates for all world regions under different ocean conditions (Treguier et al., 2023).

~~Consequently, further work is needed to develop a methodology based on physical principles for calculating the MLD that provides accurate estimates for all world regions under different ocean conditions. Energy-based approximations for calculating the MLD need to be investigated in more detail to determine if a single energy value can determine the MLD globally during all seasons.~~

This study aims to develop a physics-based methodology, based on energy considerations, for calculating the MLD under different ocean conditions ~~in all world regions. The methodology's global applicability is one of its crucial contributions. Based on the work done by the buoyancy force to vertically displace a water parcel, the homogeneity of the ocean's upper layer was quantified. Then, the value of this work, which determines the well-mixed upper layer and the MLD globally during all months, was carefully investigated. Finally, an observation-based global MLD climatology was constructed, which could be a reference to validate numerical solutions and perform MLD model intercomparison studies. The energy-based definition of the MLD is consistent with physics and is easy to implement numerically;~~ The mixed layer was defined from an energy measure of the vertical homogeneity of the water column, which quantifies the work done by the buoyancy force to displace a water parcel vertically. The methodology's global applicability is one of its crucial contributions; therefore, long-term observational data providing extensive global coverage were used to construct a gridded MLD climatology. The resulting global monthly MLD climatology and the value of the buoyancy work determining the mixed layer during each month were carefully investigated and compared with other MLD methodologies. The energy-based definition of the MLD is consistent with physics; it has the potential to provide further insights into various dynamic (e.g., vertical exchanges within the ocean and between the ocean and the atmosphere), thermodynamic (upper ocean heat content), and ecological (e.g., chlorophyll-a content and phytoplankton dynamics) processes. The observation-based global MLD climatology could be a reference to validate numerical solutions, perform MLD model intercomparison studies, and provide new insights into understanding the mixed layer.

2. Methodology and data

2.1. An energy measure of the mixed layer depth

~~The relatively vertical homogeneity of the mixed layer is due to turbulence caused by wind effects, buoyancy-driven fluxes, and waves (D'Asaro, 2014; Sallee et al., 2021; Reichl et al., 2022). The mixed layer concept allows for diagnosing vertical exchanges within the ocean and those between the ocean and the atmosphere without a detailed analysis of the associated turbulent processes (D'Asaro, 2014; Sutherland et al., 2014; Franks, 2015; Reichl et al., 2022). Therefore, a quantitative measure of the vertical homogeneity of the upper ocean layer is needed to calculate the MLD.~~ Previous research has established the mixed layer as the upper ocean layer that is relatively homogeneous in the vertical. Several approaches to quantify the upper ocean layer's vertical homogeneity have been proposed from this premise. This work proposes a quantitative measure of the vertical homogeneity of the upper ocean layer, derived from physical principles and based on energy considerations, from which the MLD can be defined.

Since vertical changes in density hinder turbulence and subsequent mixing, a physically relevant ~~definition~~ metric of the water column's vertical homogeneity should be density-based ~~and derived from physical principles~~. ~~The approach proposed here to quantify the water column's vertical homogeneity~~

is based on energy considerations. It quantifies the work done by the buoyancy force to vertically displace a water parcel under static instability conditions, Thus, we propose quantifying the water column's vertical homogeneity in terms of the work done by the buoyancy force in vertically displacing a water parcel under static instability conditions, herein referred to as the work done by buoyancy (WB). By When considering vertical displacements from the ocean's interior to the surface, WB can constitute a proxy for the vertical homogeneity of the water column: the higher the WB, the lower the vertical homogeneity of the water column, and vice versa.

The following shows the mathematical development to define WB. Consider a fluid in hydrostatic balance in which a water parcel is adiabatically displaced along the vertical from level z to $z+\Delta z$; in such a displacement, the potential density is materially conserved. According to Vallis (2017), the parcel experiences a force (per unit volume) given by

$$\delta F = g\frac{d\rho^{\theta}(z)}{dz}\delta z, \tag{1}$$

where g is the acceleration of gravity and ρ^{θ} is the potential density referred to the pressure at level $z+\Delta z$. For vertical displacements not exceeding a few thousand meters, it is adequate to use the potential density referred to 0 dbar (Stewart, 2008). From the above equation, the buoyancy force experienced by a parcel, initially at equilibrium at z_{eq}, when displaced from z_{eq} to any depth z is given by

$$F(z) = g\left[\rho^{\theta}(z) - \rho^{\theta}(z_{\text{eq}})\right]. \tag{2}$$

The force is null at z_{eq}, positive if $\rho^{\theta}(z)>\rho^{\theta}(z_{eq})$ and negative if $\rho^{\theta}(z) <\rho^{\theta}(z_{eq})$. By calculating the line integral of the buoyancy force along such a displacement, WB is obtained,

$$\text{WB}_{z_{\text{eq}}\rightarrow z} = \text{WB}(z) = \int_{z_{\text{eq}}}^{z} F(\gamma)d\gamma = -g\left(z - z_{\text{eq}}\right)\rho^{\theta}(z_{\text{eq}}) + g\int_{z_{\text{ref}}}^{z}\rho^{\theta}(\gamma)d\gamma. \tag{3}$$

WB depends on the cumulative effect along the vertical of the buoyancy force, which in turn depends on the difference between ρ^{θ} at z_{eq} and any depth z. In stable density profiles, where ρ^{θ} increases with depth, WB is negative for upward and downward displacements of the water parcel; the displaced water parcel tends to return to its original depth z_{eq} where it was at equilibrium. For an upward displacement (positive displacement), the force is downward; for a downward displacement (negative displacement), the force is upward. The opposite behavior is obtained for unstable density profiles, where ρ^{θ} decreases with depth. Since WB quantifies the energy required for a water parcel to displace vertically from its equilibrium level to any level, it represents the potential energy barrier to its displacement. Consequently, WB is better than density for diagnosing the water column section in direct contact with the atmosphere and its vertical homogeneity in energetic terms.

An alternative expression of WB that allows us to appreciate its connection with the turbulent kinetic energy budget can be obtained by expressing the delta of force in Eq. (1) in terms of the square of the buoyancy frequency $N^2(z)$,

$$\delta F = -\rho^{\theta}(z)N^2(z)\delta z, \ \text{where} \ N^2(z) = -\frac{g}{\rho^{\theta}(z)}\frac{d\rho^{\theta}(z)}{dz}, \tag{4}$$

such that the buoyancy force is given by

$$F(z) = -\int_{z_{\mathrm{eq}}}^{z} \rho^{\theta}(\beta) N^2(\beta) d\beta, \tag{5}$$

where β refers to the vertical coordinate. Again, WB is calculated by the line integral of the buoyancy force along such a displacement,

$$\mathrm{WB}_{z_{\mathrm{eq}} \to z} = \mathrm{WB}(z) = \int_{z_{\mathrm{eq}}}^{z} F(\gamma) d\gamma = \int_{z_{\mathrm{eq}}}^{z} \rho^{\theta}(\beta) \beta N^2(\beta) d\beta - z \int_{z_{\mathrm{eq}}}^{z} \rho^{\theta}(\beta) N^2(\beta) d\beta. \tag{6}$$

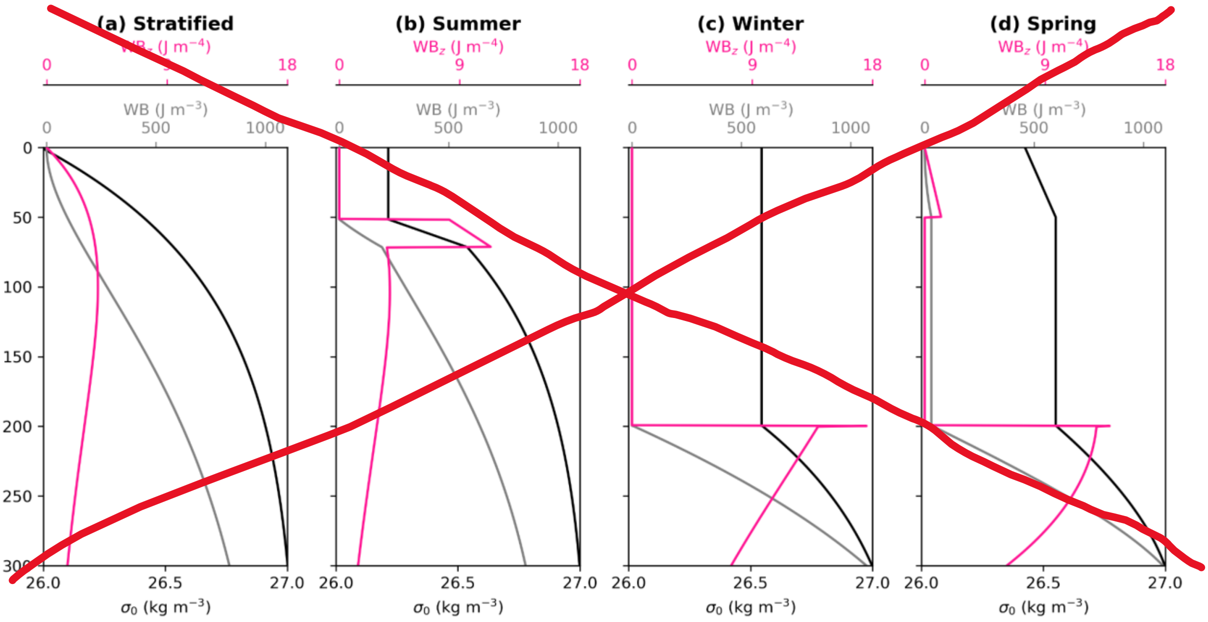
Considering the displacement h-> 0 in the above equation, WB reduces to

$$\mathrm{WB}_{h \to 0} = \int_{h}^{0} \rho^{\theta}(z) z N^2(z) dz = -\int_{0}^{h} \rho^{\theta}(z) z N^2(z) dz, \tag{7}$$

which is very similar to the expression of the columnar buoyancy given by Herrmann et al. (2008) in their Eq. (3), except that in our case, it considers the potential density $\rho^{\theta}(z)$ in the integrand. According to Herrmann et al. (2008), the columnar buoyancy represents the time integral of the buoyancy flux required to mix the water column from the surface down to *h*. The buoyancy flux is a major driver of vertical processes; given a water column stratification, it determines the depth of vertical exchanges within the ocean and, consequently, the MLD (Gill, 1982; Sutherland et al., 2013).

Equation (6) shows that WB is composed of (i) the time integral of the buoyancy flux required to mix the water column from z_{eq} to z and (ii) an integrated measure of the local stratification (given by N^2 weighted by the potential density ρ^{θ} along the same vertical section. The existence of a relationship between WB and the buoyancy flux allows us to connect WB with the turbulent kinetic energy budget (Zippel et al., 2022) and, thus, with the physics of boundary layer turbulence; further analysis of such a connection is beyond the scope of this paper and is proposed for future research. The above suggests that our energy-based methodology to define the MLD will be consistent with the turbulence approach of the mixed layer formation.

The use of WB as a proxy for the vertical homogeneity of the water column can be exemplified by considering typical density profiles in different seasons (Fig. 1). The WB required to displace each water parcel in the water column from its equilibrium level to the surface is also shown for each density profile; to better appreciate the relationship between density and WB, the negative value of WB is plotted in all the figures. ~~There is~~ a strong correspondence exists between the density profile and its associated WB profile: ~~a water column homogeneous in density has zero or small WB values~~ a perfectly homogeneous water column has zero WB values, whereas a stratified water column has density and WB increasing with depth. However, because WB is a nonlinear function of density, the density variation between two depths is not proportional to the corresponding increase in WB. ~~The stratified and winter profiles in Fig. 1 show that~~ For example, the stratified profile (Fig. 1d) has a larger density variation than the winter profile (Fig. 1b), but the WB variation is larger in the winter than in the stratified profile: large density variations do not always correspond to large WB values.



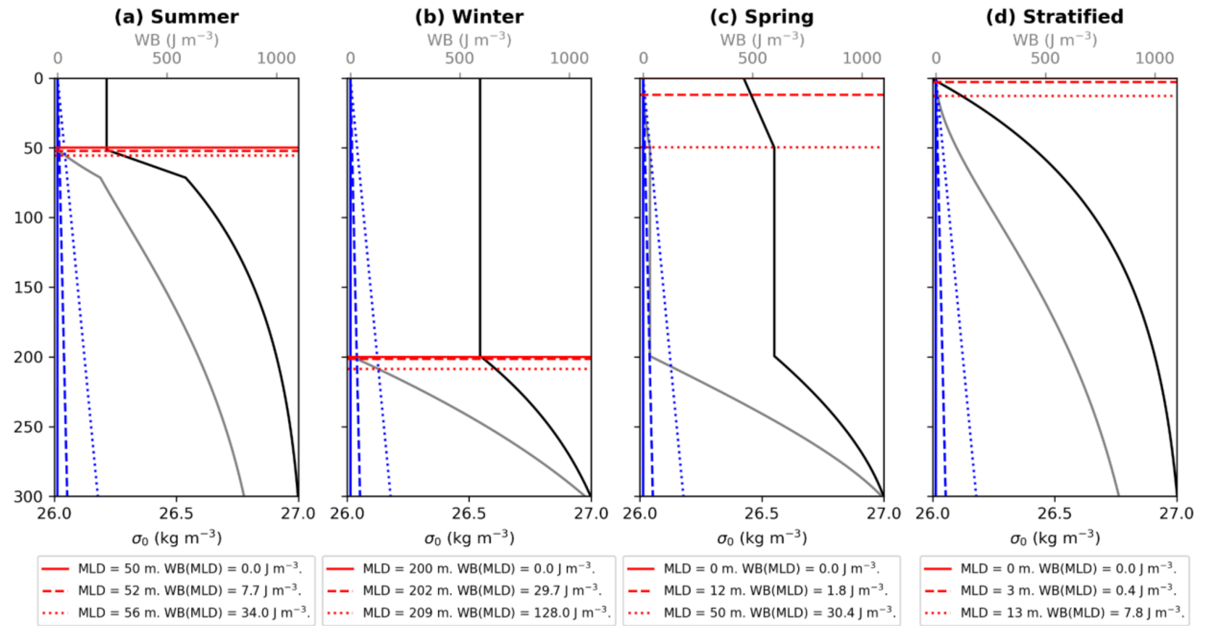


Figure 1. ~~Typical density profiles in different conditions and seasons (black curves), the corresponding WB required to displace a water parcel from any depth to the surface (gray curves), and its associated vertical gradient WB_z (pink curves). The density profiles are the same as those used by Treguier et al. (2023) in their Fig. 2. The plots show the potential density anomaly σ_{0}, assuming that the potential density referred to 0 dbar was used.~~ Typical density profiles for (a) summer, (b) winter, (c) spring, and (d) a stratified condition (black curves) and the corresponding WB (gray curves). The density profiles, from Treguier et al. (2023) in their Fig. 2, show the potential density anomaly referred to 0 dbar (σ_0). Three $\Delta\overline{\rho}^{\theta}$ values (0, 0.0150, and 0.0625 kg m^{-3}) were considered; the larger the $\Delta\overline{\rho}^{\theta}$, the larger the slope of $WB_{ref}(z)$ (blue curves). For each $\Delta\overline{\rho}^{\theta}$, the corresponding MLD (red curves) and WB value at the MLD are also shown.

~~Since the mixed layer is defined as the upper ocean layer in direct contact with the atmosphere, it is expected that the water parcels can move with little or no energy within it. Consequently, WB is better than density for diagnosing the water column section in direct contact with the atmosphere and its~~

~~vertical homogeneity in energetic terms. In this study, the mixed layer is defined as the upper ocean layer relatively homogeneous in WB (with small WB values), which is well-mixed in energetic terms and, therefore, in contact with the atmosphere. From the mixed layer definition, the MLD is thus the depth at which WB changes from zero or small values near the surface to values that considerably increase downward (the depth at which WB has a structural change). Different mathematical methods can be used to find the structural change in WB; in this study, we used the vertical gradient of WB (WB_z). However, for strongly stratified or very smooth density profiles, a clear structural change in WB is not easy to find; in these cases, it is useful to consider a specific WB threshold characterizing a homogeneous layer.~~

~~The depth of the structural change in WB (Fig. 1) depends on the corresponding density profile. For idealized density profiles with a strong and unique pycnocline (summer and winter profiles in Fig. 1), the MLD is clearly defined: the mixed layer corresponds to the upper section of the water column homogeneous in WB, in which no work is required to displace any water parcel from its equilibrium level to the ocean surface. For strongly stratified density profiles, profiles with several pycnoclines, or very smooth density profiles, the mixed layer can be shallow or deep depending on a chosen threshold characterizing homogeneous WB values. For the strongly stratified density profile in Fig. 1a, the mixed layer could be very shallow or non-existent. For the density profile with near-surface restratification in Fig. 1d, the mixed layer can be as shallow as 50 m, as deep as 200 m, or has other depths depending on a WB threshold characterizing a homogeneous section.~~

~~This new MLD definition is based on energy considerations and derived from physical principles. Thus, it can be considered adequate and applicable in different regions and oceanic conditions, such as polar seas, intermediate and deep water formation regions, and barrier and compensated layers. However, it is necessary to investigate the WB threshold that determines the MLD globally during all seasons, if there is one.~~

2.2. Defining the mixed layer

The mixed layer definition taken in this study is based on that of Brainerd and Gregg (1995), who defined it as the zone in which surface fluxes have been mixed through timescales longer than several days. The mixed layer does not address the diurnal mixing cycle or the timescales in which mixing is currently active, that is, the timescales relevant to the mixing layer. The mixed layer considered in this study is representative of seasonal timescales; it is the relatively homogeneous upper ocean layer formed by the history of mixing when the ocean is nearly in thermal equilibrium with the atmosphere on timescales of a few days or more (Brainerd and Gregg, 1995). This study defines the mixed layer as the energetically homogeneous upper ocean layer, in which water parcels can move with little or no work. This energetic homogeneity is the key and distinctive property of our mixed layer definition. Since WB is a proxy for the vertical homogeneity of the water column, the mixed layer is thus the upper ocean layer with small WB values, which is well-mixed in energetic terms and, therefore, in contact with the atmosphere.

How small should the WB values be to characterize a well-mixed layer? To answer this question, we will continue the mathematical development of WB. From Eq. (3), when considering the vertical section from any depth *h* to the free surface η and using the first mean value theorem of calculus for definite integrals, we can express Eq. (3) as a relationship between the degree of density inhomogeneity $\Delta\overline{\rho^{\theta}}$ of any upper section of the water column and the corresponding WB required to displace a water parcel from its base to its top,

$$\mathrm{WB}_{h\to\eta} = -g(\eta - h)\left[\rho^{\theta}(h) - \overline{\rho^{\theta}}\right] = -g(\eta - h)\Delta\overline{\rho^{\theta}}, \quad (8)$$

where

$$\overline{\rho^{\theta}} = \frac{1}{\eta - h}\int_{h}^{\eta} \rho^{\theta}(z)dz \quad \text{and} \quad \Delta\overline{\rho^{\theta}} \equiv \rho^{\theta}(h) - \overline{\rho^{\theta}}.$$

The intrinsic relationship between WB and the density variations along the water column can be explored via Eq. (8). We can explore the density structure of an energy-homogeneous layer and, reciprocally, the energy behavior of a density-homogeneous layer. Below, we describe these cases:

- For a layer with a unique, non-zero WB value, Eq. (8) establishes a nonlinear decrease of $\Delta\overline{\rho^{\theta}}$ with depth; the density variation should decrease with depth to maintain the same work value (Fig. 2a). The degree of density inhomogeneity $\Delta\overline{\rho^{\theta}}$ along the water column changes according to the intensity of turbulence and mixing in the ocean: the greater the turbulence and mixing, the greater the vertical homogeneity of the water column (the density variation and the WB value are small at large depths) and the larger the MLD. For example, WB=10 J m^{-3} implies $\Delta\overline{\rho^{\theta}}$=0.015 kg m^{-3} from the surface to 68 m depth and WB=20 Jm^{-3} implies $\Delta\overline{\rho^{\theta}}$=0.015 kg m^{-3} from the surface to 136 m depth. The opposite behavior is obtained for low turbulence and mixing, which produces low vertical homogeneity of the water column (the density variation and the WB value are large at shallow depths) and a shallow MLD.
- A layer with a unique, non-zero $\Delta\overline{\rho^{\theta}}$ value is associated with a linear increase of WB with depth; more work is required to raise a parcel from a larger depth (Fig. 2b).

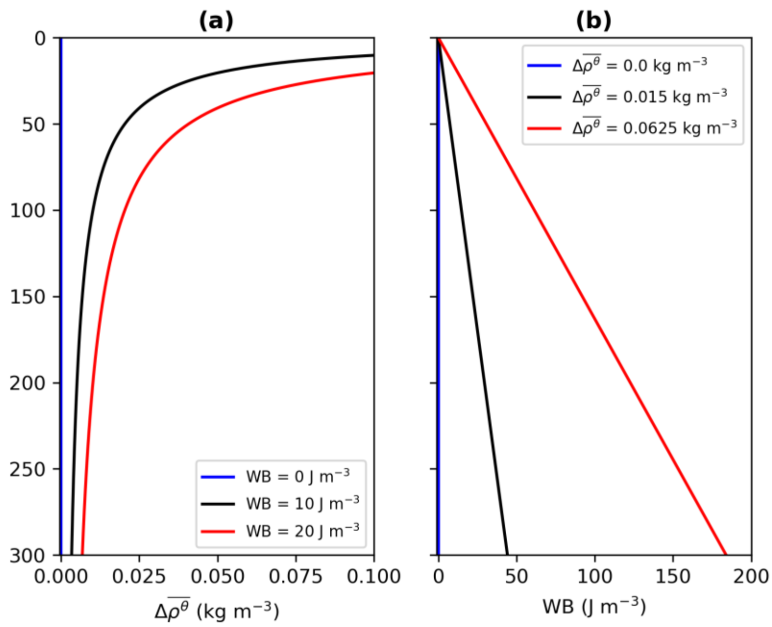


Figure 2. Relationship between the degree of density inhomogeneity ($\Delta\overline{\rho^{\theta}}$) along the water column and the corresponding work done by buoyancy (WB) required to displace a water parcel from any depth to the surface. Profiles are shown for various single values of (a) WB and (b) $\Delta\overline{\rho^{\theta}}$.

Determining a well-mixed layer in energetic terms is not direct; finding the WB threshold requires a density-based reference value, i.e., the density variations along the mixed layer. An approach to finding the WB threshold that defines the MLD is to use Eq. (8), considering a specific degree of density inhomogeneity $\Delta\overline{\rho^{\theta}}$ along the mixed layer. The procedure is as follows:

1. Select a $\Delta\overline{\rho^{\theta}}$ value characteristic of a well-mixed layer in density; some of the density criteria suggested in the literature can be used (Levitus, 1982; Kara et al., 2000; de Boyer Montégut et al., 2004). A $\Delta\overline{\rho^{\theta}}$ homogeneous in space and time will lead to spatially and temporally variable WB thresholds.
2. For the selected $\Delta\overline{\rho^{\theta}}$ value, use Eq. (8) to construct the associated reference curve of buoyancy work, $WB_{ref}(z)$. WB values smaller than WB_{ref} indicate a layer quasi-homogeneous in density; in contrast, WB values larger than WB_{ref} indicate a layer not quasi-homogeneous in density.
3. The intersection depth between the WB profile of interest and WB_{ref} determines the vertical extension of the well-mixed layer in energetic terms for the profile of interest, that is, its MLD. The WB value at the MLD thus represents the WB threshold characterizing the well-mixed layer in energetic terms, according to the $\Delta\overline{\rho^{\theta}}$ value. Naturally, the WB threshold depends on the choice of the reference depth at which WB_{ref}=0.
4. The resulting WB threshold should be analyzed to determine whether it identifies the entire vertical extent of the energetically homogeneous upper ocean layer with small WB values. If necessary, the WB threshold should be adjusted. The above determines its adequacy in producing a physically realistic MLD in energetic terms.

The above represents our approach to defining the MLD via a physics-derived, energy-based methodology herein referred to as EBM. This new MLD methodology is applicable in different regions and ocean conditions, such as polar seas, intermediate and deep water formation regions, and barrier and compensated layers. Figure 1 also exemplifies the application of EBM in determining the MLD and the corresponding WB threshold, considering three $\Delta\overline{\rho^{\theta}}$ values. For idealized density profiles with a strong and single pycnocline (summer and winter profiles in Fig. 1), the MLD is clearly defined (even by eye), but their MLD and corresponding WB threshold vary depending on the $\Delta\overline{\rho^{\theta}}$ considered. The MLD varies slightly (a few meters), but the WB threshold can vary significantly: it varies from 0 to 34 J m^{-3} for the summer profile (Fig. 1a) and from 0 to 128 J m^{-3} for the winter profile (Fig. 1b). For strongly stratified density profiles, profiles with several pycnoclines, or very smooth density profiles, the mixed layer can be shallow or deep depending on the chosen threshold characterizing quasi-homogeneous WB values. For the spring profile (Fig. 1c) with near-surface restratification, the mixed layer can be as shallow as 0 m, as deep as 50 m, or have intermediate depths depending on the WB threshold characterizing a quasi-homogeneous section. For the strongly stratified density profile (Fig. 1d), the mixed layer could be very shallow or non-existent. The variability in the WB threshold underscores the complexity of estimating the MLD.

2.~~2~~.3. Data

~~Two types of global-scale in situ data were used to analyze the MLD. The first dataset, from the World Ocean Circulation Experiment (WOCE), considered transects along the Pacific Ocean. These transects were used to investigate a WB value that could determine the MLD during all seasons. The second dataset, comprising Argo profiles for the global ocean (Wong et al., 2020), was used to construct an energy-based global monthly MLD climatology, further enhancing the applicability of this study.~~

~~2.2.1. WOCE data~~

~~Five transects in the Pacific Ocean were obtained from the WOCE (https://cchdo.ucsd.edu/, accessed in March 2024). The chosen transects cover the seasonal variation, except spring, of the hydrography throughout the Pacific Ocean:~~

- ~~The meridional section transects P14S (67°S-10°S along the meridian ~170°W, during January-March 1996) and P15N (10°S-54°N along the meridian ~167°W, during September-November 1994). These transects correspond to the austral summer and the boreal autumn, respectively.~~
- ~~The zonal section transect P02T (133°E-54°W along the parallel ~30°N, during January-February 1994). This transect corresponds to the boreal winter.~~
- ~~The zonal section transects P06E (153°E-148°W along the parallel ~31°S, during July-August 2017) and P06W (147°W-71°W along the parallel ~32°S, during August-September 2014). These transects correspond to the austral winter.~~

~~Using the Thermodynamic Equation of SeaWater 2010 (McDougall and Barker, 2011), the Conservative Temperature (Θ), Absolute Salinity (S_A), and surface-referenced potential density (ρ^{θ}_{0}) along these transects were calculated. All these variables were interpolated vertically with a resolution of 1 m, starting at 10 m depth.~~

~~2.2.2. Argo data~~

~~Argo in situ profiles for the world ocean from January 2005 to December 2023 were used; the profiles were obtained from the Argo snapshot of June 2024 (Argo, 2024). Delayed mode profiles deemed good and probably good (quality flags 1 and 2) were selected, obtaining ~2 million in situ profiles. As for the WOCE data, Θ, S_A, and ρ^{θ}_{0} were calculated using the Thermodynamic Equation of SeaWater 2010. The original vertical resolution of Argo profiles was retained to construct an observation-based global monthly MLD climatology; however, the different variables were interpolated to 10 m if there were no measurements at that depth. The spatial and temporal distribution of the Argo profiles used in this study are shown in Fig. S1 of the Supplement.~~ The Argo profiles for the global ocean (Wong et al., 2020) were used to compute the MLD. The profiles were obtained from the Argo snapshot of June 2024 and comprise data from January 2005 to December 2023 (Argo, 2024). Delayed mode profiles deemed good and probably good (quality flags 1 and 2) were selected, obtaining ~2 million in situ profiles. Using the Thermodynamic Equation of SeaWater 2010 (McDougall and Barker, 2011), the conservative temperature (Θ), absolute salinity (S_A), and surface-referenced potential density (ρ_0^{θ}) were calculated for all the profiles, retaining their original vertical resolution. The spatial and temporal distribution of the Argo profiles used in this study is shown in Fig. S1 of the Supplement. The spatial coverage of the Argo data does not completely map the entire ocean (neritic and oceanic zones): coastal zones are consistently not mapped, and the data are somewhat scattered south of 60°S and scarce north of 60°N, mainly in the Pacific Ocean. Beyond these limitations, Argo data provide extensive global coverage and can be considered representative of the world ocean (Wong et al., 2020).

2.~~3.~~4. ~~Common MLD methodologies~~ Construction of energy-based global monthly MLD climatologies

~~The performance of the energy-based MLD methodology described in section 2.1 will be contrasted with three commonly used methodologies: (i) the 0.03 $kg\ m^{-3}$ and (ii) the 0.2°C thresholds of de Boyer Montégut et al. (2004), and (iii) the multi-criteria method of Holte and Talley (2009), and the recent~~

~~(iv) sigmoid function fitting method by Romero et al. (2023). Hereinafter, we refer to these methodologies as B04D, B04T, HT09, and R23, respectively. To compute WB and the density and temperature thresholds, we used a reference depth of 10 m, in agreement with de Boyer Montégut et al. (2004) and Treguier et al. (2023). WB was interpolated every meter from 10 m to greater depths, and WB_z was calculated accordingly using central differences. That made it possible to compare MLD methodologies.~~ In this study, we constructed two energy-based global monthly MLD climatologies considering two $\Delta\overline{\rho^{\theta}}$ values, 0.0150 and 0.0625 kg m^{-3}, characteristic of density variations of approximately 0.030 and 0.125 kg m^{-3} along the mixed layer, respectively (Levitus, 1982; Kara et al., 2000; de Boyer Montégut et al., 2004). The MLD was computed for each Argo profile within a 2°x2° grid cell for each month in the long-term record; the resulting MLD values were then averaged to obtain a representative value of the MLD for that grid cell and month. Since the original vertical resolution of the Argo profiles was retained, the resulting gridded MLD climatologies are observation-based. In determining the MLD, we used a reference depth of 10~m to avoid diurnal influences, in agreement with de Boyer Montégut et al. (2004) and Treguier et al. (2023). Therefore, we identified the energetically homogeneous layer below 10~m depth, considering the WB required to displace a water parcel from any depth to 10 m so that WB(10 m)=0. The ocean variables were interpolated to 10 m if no measurements were at that depth.

The characteristics of the energy-based MLD methodologies were analyzed, and their performance was contrasted with three commonly used methodologies and a recent one, further expanding the applicability of this study. The first two common MLD methodologies are the \$0.03 kg m^{-3} and the 0.2°C thresholds of de Boyer Montégut et al. (2004). The third common MLD methodology is the multi-criteria method of Holte and Talley (2009), which calculates possible MLDs derived from threshold and gradient methods to select a final MLD estimate based on physical features in the profile. The recent MLD methodology is the sigmoid function fitting method of Romero et al. (2023), which computes the MLD and the maximum thermocline depth by evaluating the fit of the sigmoid function to the temperature profile. We will refer to these methodologies as B04D, B04T, HT09, and R23, respectively (and collectively as the common MLD methodologies). All the MLD climatologies were calculated using the same Argo dataset, with the same temporal and spatial scales and reference depth, making them comparable.

3. Results

~~The results are presented in three parts. First, we analyze a WB value that could determine the MLD in the Pacific Ocean transects during various seasons. Then, we analyze the performance of the energy-based MLD methodology in regions where common MLD methodologies do not agree on the MLD calculation. This analysis will enable us to compare the energy-based MLD methodology with others. Finally, we present the energy-based monthly climatology of the global MLD, along with the associated mixed-layer hydrography.~~ The results are presented in three parts. In the first part, we analyzed two energy-based global monthly MLD climatologies and the WB threshold that defines the mixed layer. We then evaluated the EBM performance using a quality index and explored the degree of homogeneity of the mixed layer in density and temperature. In the second part, we compared the EBM with other methodologies, considering their MLD magnitude and energy consistency in determining the MLD. The above represents an important contribution to previous studies on MLD climatologies by offering new insights into understanding the mixed layer. In the third part, we very preliminarily explored what WB values can define the MLD globally throughout the year.

3.1. A WB value that defines the MLD

This section investigates if a single WB value can determine the MLD throughout the Pacific Ocean during various seasons. WB_z was calculated to find the depth of the structural change in WB that defines the MLD. Figure 2 shows the WB required to displace a water parcel from any depth to 10 m along each Pacific Ocean transect (upper panels) and the corresponding WB_z (lower panels) in grayscale. The location of the WOCE transects is shown in the left panels.

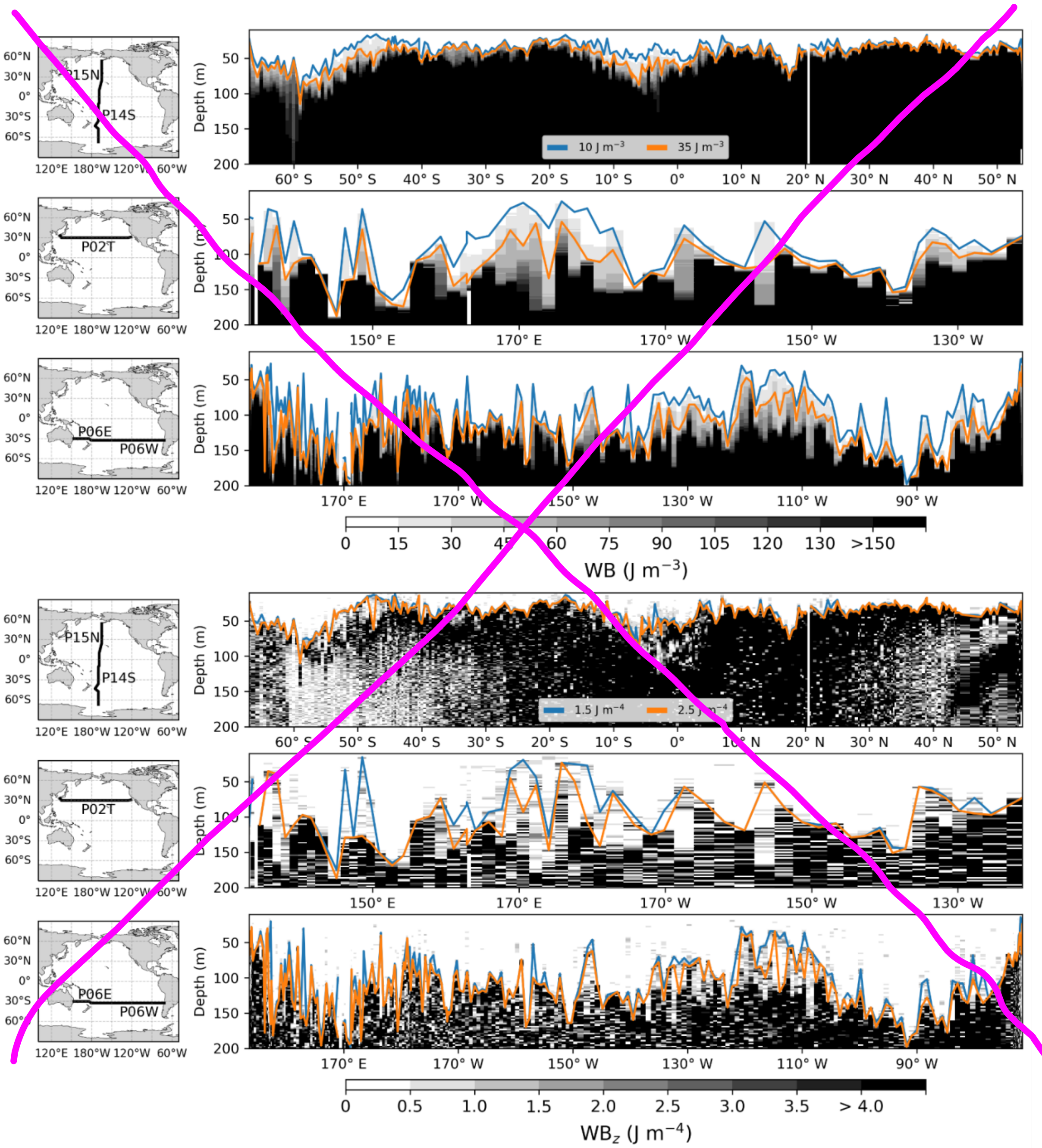


Figure 2. WB (upper panels) and WB_z (lower panels) along each Pacific Ocean transect (grayscale). For the WB plots, the 10 and 35 $J m^{-3}$ WB isolines are shown; for the WB_z plots, the 1.5 and 2.5 $J m^{-4}$ WB_z isolines are shown. Note that only the first 200 m of the water column are shown.

An upper section of the water column with relatively low WB values is shown in Fig. 2; this section increases non-linearly with greater depths. The WB and WB_z plots reflect the characteristics of vertical stratification across the Pacific Ocean during different seasons; a comprehensive discussion of the relationship between WB and stratification is beyond the scope of this study and is proposed for future research. More importantly, the WB and WB_z plots clearly show the depth of the structural change in WB, the depth at which a considerable increase in WB occurs. The question is whether this depth corresponds to a single WB value. A single WB_z value does not consistently locate the structural change in WB; it is located between the isolines 1.5 and 2.5 of WB_z. The average difference in depth between the isolines 1.5 and 2.5 of WB_z is:

- 2.4 m in transects P14S and P15N,
- 14.1 m in transect P02T, and
- 9.9 m in transects P06E and P06W.

The differences in depth between the WB_z isolines reflect the seasonal stratification characteristics in each transect: the vertical gradients of stratification and WB are strong during summer-autumn (transects P14S and P15N) and weak during winter (transects P02T, P06E, and P06W). Given the small differences in depth between the isolines 1.5 and 2.5 of WB_z, it is concluded that the depth of the structural change in WB is truly located within the depths of the 1.5-2.5 WB_z isolines.

The depth of the 1.5-2.5 WB_z interval roughly corresponds to the depth of the 10-35 WB interval (Fig. 2). To find the WB value that best locates the depth of the structural change in WB, a statistical analysis of the differences in depth between specific WB_z and WB isolines was performed. We calculated boxplots of the differences in depth between a specific WB_z isoline and various WB isolines (see Fig. S2 in the Supplement). For each WB_z isoline, we selected the WB isolines with the smallest differences in depth with respect to those in the WB_z interval, according to the following criteria: the median is lower than ± 5 m, and the difference between the third and first quartiles is lower than 10 m. Table 1 shows the results.

Table 1. Results from a statistical analysis of the differences in depth between specific WB_z and WB isolines, considering the WOCE transects. Only the WB isolines with the smallest differences in depth with respect to the 1.5, 2.0, and 2.5 J m-4 WB_z isolines are shown.

WB_z isoline (Jm^{-4})	Transects P14S and P15N	Transect P02T	Transects P06E and P06W
1.5	WB = 10, 15, 20, 25, 30, and 35 Jm^{-3}	WB = 10, 15, and 20 Jm^{-3}	WB = 10, 15, and 20 Jm^{-3}
2.0	WB = 10, 15, 20, 25, 30, and 35 Jm^{-3}	WB = 10, 15, 20, and 25 Jm^{-3}	WB = 15, 20, 25, 30, and 35 Jm^{-3}
2.5	WB = 10, 15, 20, 25, 30, and 35 Jm^{-3}	WB = 20, 25, and 30 Jm^{-3}	WB = 15, 20, 25, 30, and 35 Jm^{-3}

The 20 Jm^{-3} WB isoline is the only one common to the three WB_z isolines in all transects (Table 1); the depth of such WB isoline is the one that best locates the depth of the structural change in WB. Thus, the depth of the 20 Jm^{-3} WB isoline delineates an upper layer of the ocean that is well-mixed in energetic terms, which defines the MLD. This analysis shows that a single WB value can be used to define the MLD throughout the Pacific Ocean, which is a remarkable result.

We propose using inductive reasoning to define the MLD globally. Assuming that WB and WB_z behave similarly in the world ocean as in the Pacific, we suggest that the depth of the 20 Jm^{-3} WB isoline can reliably define the MLD globally during all seasons. The results for the Pacific Ocean shown in the previous analyses are thus consistent with that general assertion. The above sustains the proposal of our energy-based MLD methodology applicable to the global ocean, which will be referred to as EBM.

3.2. Performance of the energy-based MLD methodology in challenging regions

One crucial contribution of this study is the global applicability of the energy-based MLD methodology. This methodology should provide a realistic description of the MLD, consistent with the seasonal variation of local hydrography, even in challenging regions. The regions where common MLD methodologies do not agree on the MLD calculation can be considered challenging; due to the dynamics and particular hydrography of these regions, the different methodologies do not coincide when evaluating their mixing conditions. This subsection compares the performance of the energy-based methodology with that of other common methodologies in challenging regions.

3.2.1. Challenging regions for the MLD calculation

To identify the challenging regions, the global monthly MLD climatologies were first computed considering each common methodology; the Argo data were used to calculate global climatologies. The MLD was computed for each Argo profile. Within a 2°x2° grid for each month in the long-term record, the resulting MLD values were then averaged to obtain a representative value of the MLD for each cell on each given month. For each 2°x2° grid cell in a month, the absolute difference between the minimum and maximum MLD values from the four common methodologies was calculated (the MLD range). Figure 3 shows the global monthly climatology of the MLD range.

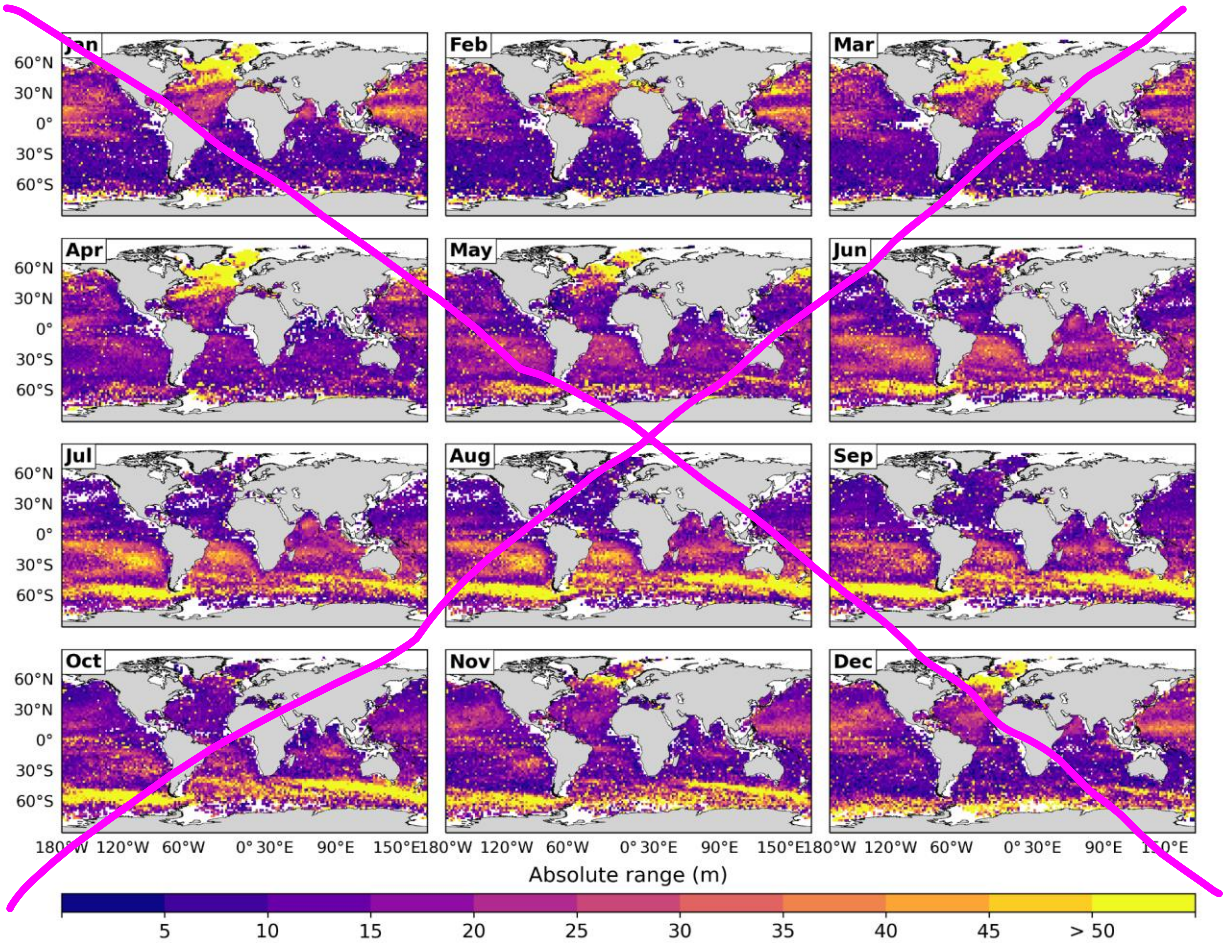


Figure 3. Global monthly climatology of the MLD range, i.e., the absolute difference between the minimum and maximum MLD values from the four common methodologies.

The MLD range has relatively high values (> 10 m) across most of the world ocean (Fig. 3). The above reflects the subjective nature of the mixed layer definition and the resulting lack of consistency among the four common methodologies for determining the MLD. The most significant disagreements occur during winter and early spring when mixing is more active, eroding sharp density and temperature gradients in winter and creating near-surface restratification in spring. The common MLD methodologies also show low consistency in regions where salinity significantly influences density, such as polar seas, intermediate and deep water formation regions, and barrier and compensated layers. Profiles with these characteristics present a challenge for these methodologies, which are mainly based on density and temperature thresholds, in finding the MLD. The smallest values of the MLD range occur during summer when the pycnocline and thermocline are shallow with sharp density and temperature gradients; in these conditions, the common MLD methodologies are in agreement.

The challenging regions, thus, correspond to the regions with high MLD range values. The threshold value associated with those high MLD range values was determined using a statistical outlier detection method. A probability distribution was constructed from the monthly climatology of the MLD range, and its upper limit (or maximum non-atypical) was calculated according to Q3 + [1.5(Q3 – Q1)], where Q1 and Q3 are the first and third quartiles, respectively. The resulting upper limit value was 50 m (the 91st percentile of the probability distribution); thus, the challenging regions correspond to the regions with MLD range values higher than 50 m. Three regions resulted (regions colored yellow on the map in Fig. 3): (i) the Southern Ocean (~65-40°S) during austral winter (July, August, September) and part of spring (October and November), (ii) the North Atlantic (>30°N) during boreal winter (January, February, March) and early spring (April), and (iii) the subtropical eastern Pacific (~0-30°S) during austral winter (July and August).

3.2.2. MLD methodologies intercomparison

The performance of the energy-based methodology was compared with that of the common methodologies by analyzing random profiles in the three identified challenging regions. Figure 4 shows the vertical profiles of WB, σ_{0}, Θ, and S_A and the corresponding MLD calculated with each methodology in five locations for each challenging region. For each profile shown in Fig. 4, Tables S1-S3 in the Supplement show the MLD and the differences in Θ, σ_0, and WB from the reference depth of 10 m to the MLD calculated with each methodology.

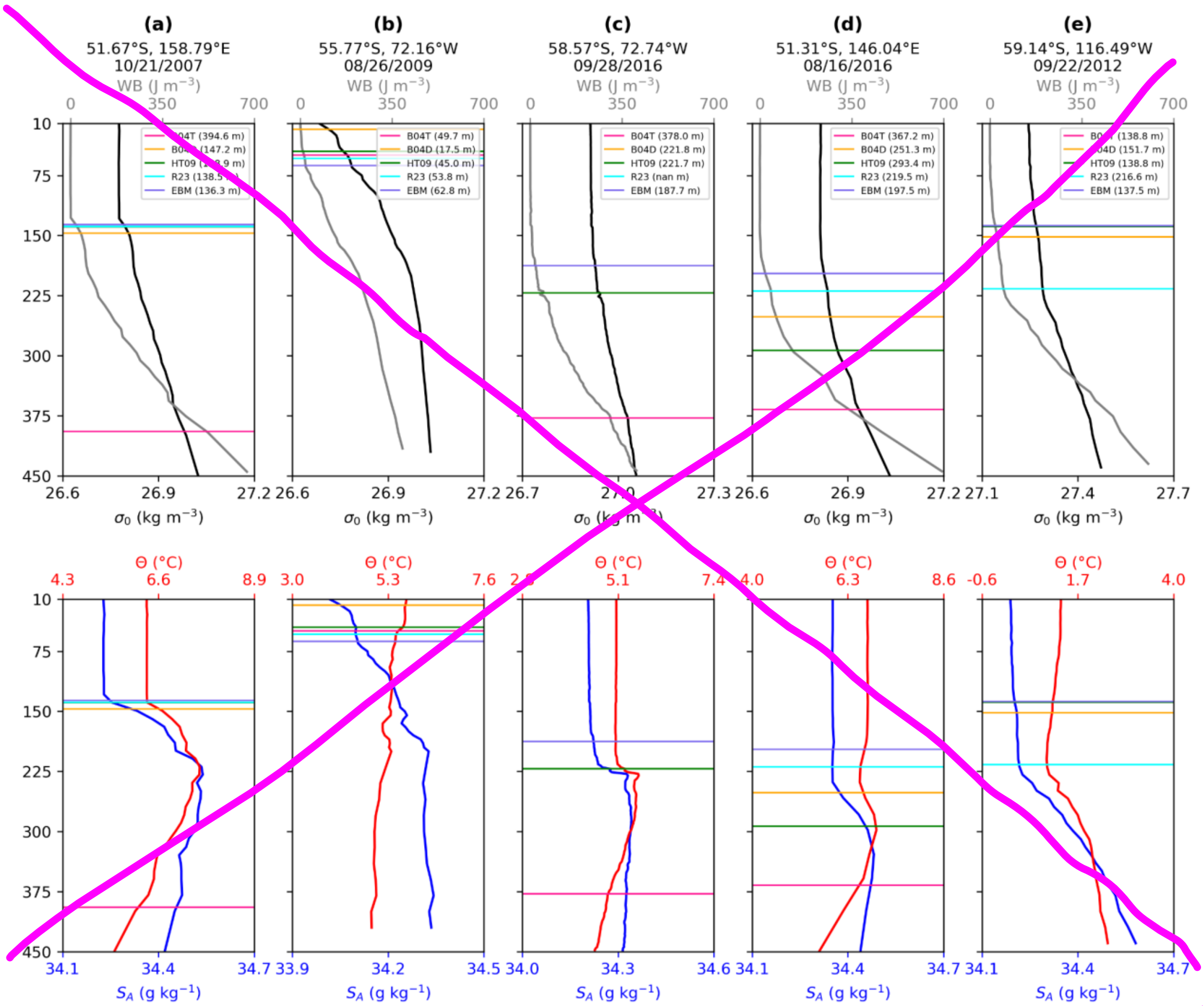


Figure 4 (part one). Random profiles in the Southern Ocean. Vertical profiles of WB, σ_{0}, Θ, and S_A and the corresponding MLD calculated with each methodology are shown.

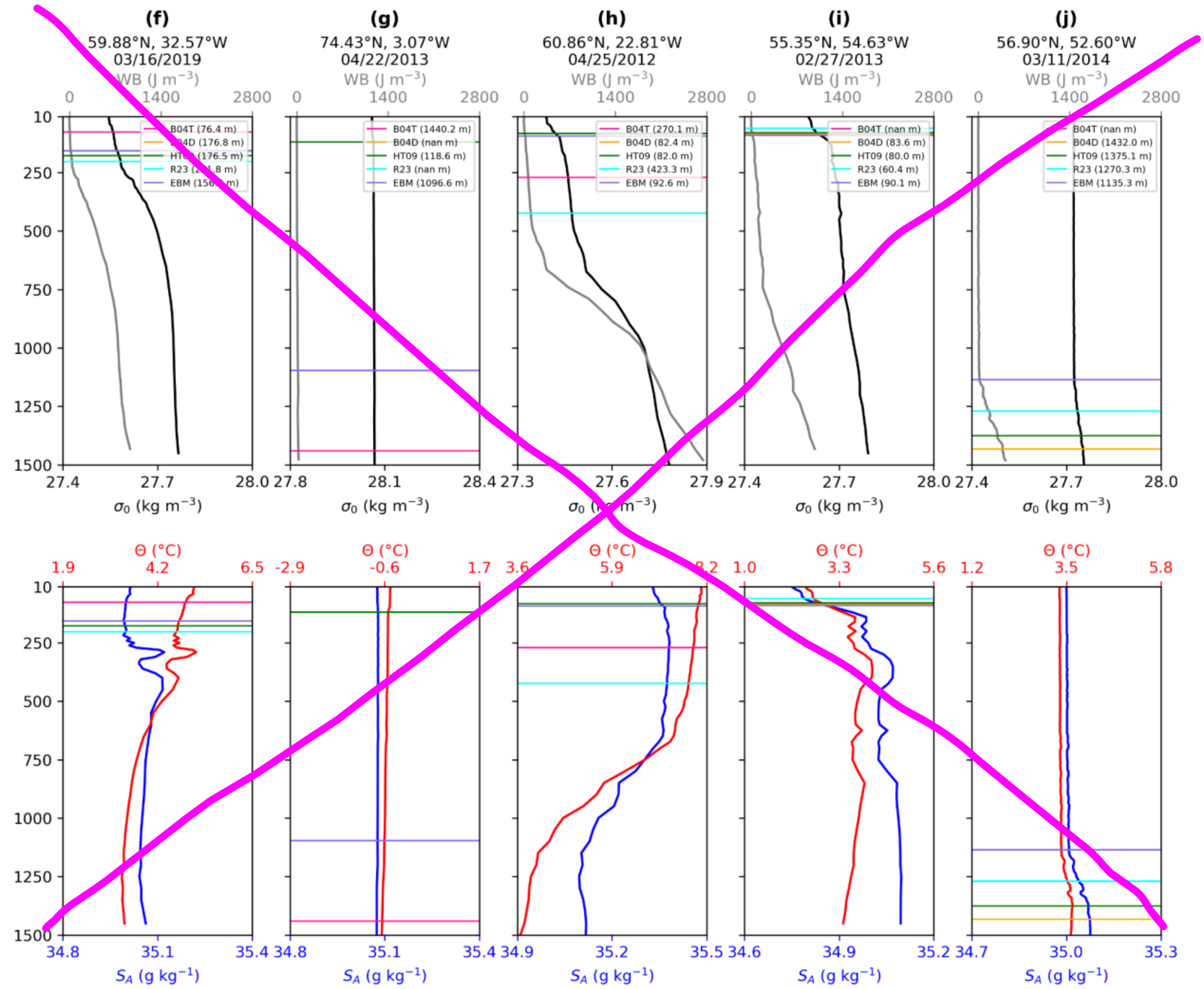


Figure 4 (part two). Random profiles in the North Atlantic.

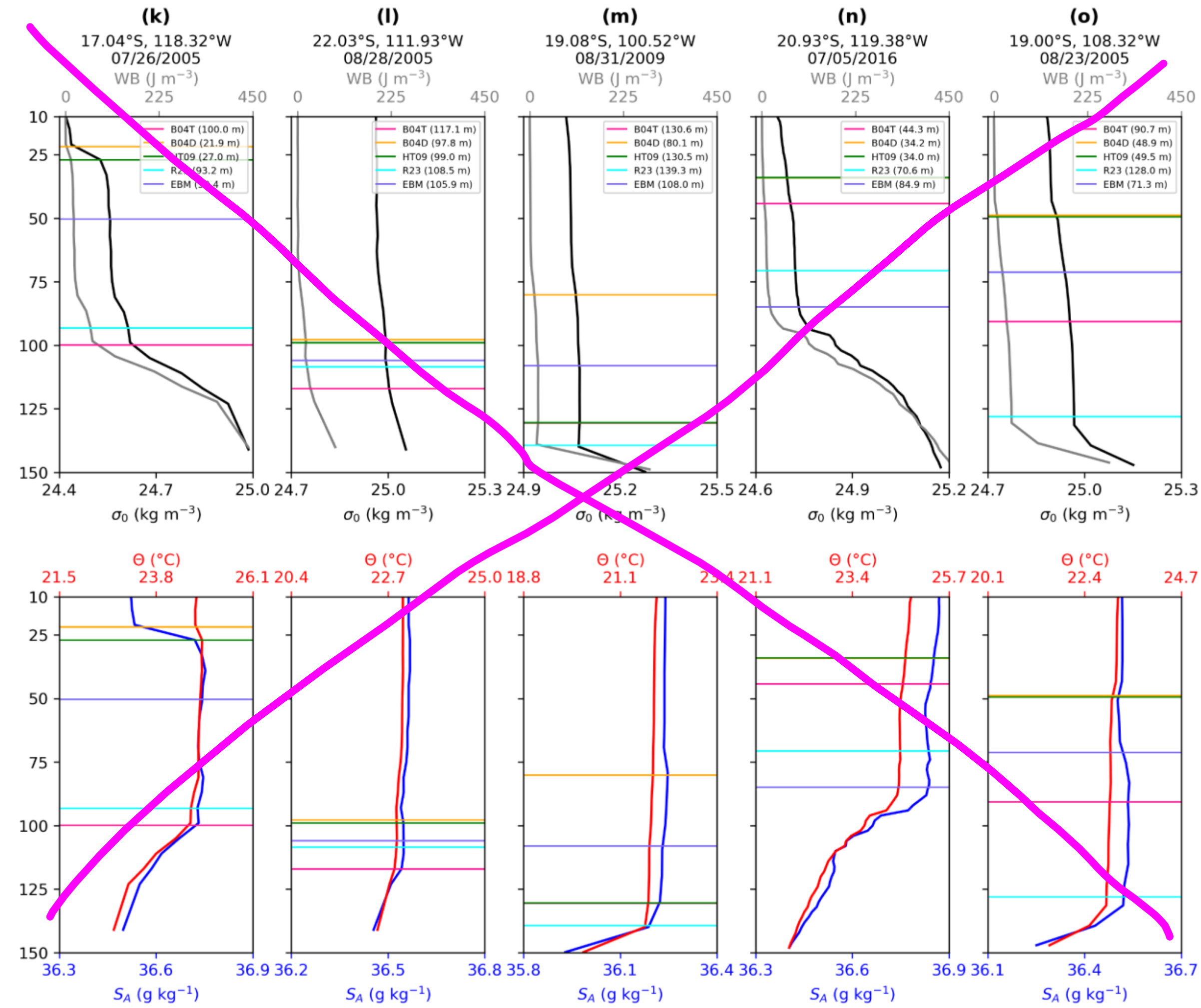


Figure 4 (part three). Random profiles in the subtropical eastern Pacific.

In the challenging regions, there is no consensus on the MLD calculation; in principle, no single methodology should be preferred over the others. The following is observed regarding the ability of each methodology to identify a relatively homogeneous upper ocean layer in the challenging regions (Fig. 4). All the methodologies approximately coincide in the MLD calculation for ideal profiles, that is, for hydrographic profiles with a clear homogeneous upper section and sharp density and temperature gradients below it, without temperature inversions. For profiles with increasing density and decreasing temperature from the near-surface, all the methodologies give very shallow MLD values, which do not necessarily coincide. The most significant disparity in the MLD values is obtained for profiles with a quasi-homogeneous upper section and smooth gradients below it; these profiles are ubiquitous in the challenging regions and are primarily responsible for the significant disagreement on the MLD calculation. In some cases, some common methodologies fail to provide a MLD value.

The common methodologies provide an ensemble of the possible extent of the mixed layer, in which the smallest and largest values of the MLD account for the uncertainty in calculating it. In this ensemble approach, the energy-based methodology gives MLD values close to the shallowest or between the extreme ones calculated with the common methodologies (Fig. 4). Furthermore, due to its construction, the energy-based methodology performs without failure under every ocean condition. In contrast, the common methodologies are prone to fail when analyzing hydrographic profiles differing from the ideal ones. Therefore, it is concluded that the energy-based methodology is robust under different ocean conditions and provides realistic estimates of the MLD in challenging regions. Next, we provide some specificities concerning the performance of the MLD methodologies for each challenging region.

The profiles in the Southern Ocean (Figs. 4a-e) exhibit notorious temperature inversions, which cause the homogeneous density layer to not coincide with the homogeneous temperature layer. Generally, the homogeneous temperature layer is deeper than the homogeneous density layer; therefore, B04T generally identifies inhomogeneous upper sections in every hydrographic variable. For the profiles in Figs. 4a-b, all the methodologies perform well according to their criteria and give similar MLD values. The profiles in Figs. 4c-d have the highest disparity in the MLD calculation; compared to the common methodologies, EBM better identifies the relatively homogeneous upper ocean layer by providing the shallowest MLD values. For the profile in Fig. 4e, R23 performs best in identifying the MLD; however, EBM identifies the upper homogeneous layer associated with near-surface restratification.

The profiles in the North Atlantic (Figs. 4f-j) exhibit a high disparity in the MLD calculation, except for the profile in Fig. 4i, in which all the methodologies provide shallow and similar MLD values. All the methodologies perform well according to their criteria for the profiles in Figs. 4f and 4h, and no methodology is preferred over any other. Temperature inversions also occur in this region (profiles in Figs. 4i-j); B04T fails to estimate the MLD, whereas EBM performs very well in identifying the relatively homogeneous upper ocean layer. The quasi-homogeneous profile in Fig. 4g represents a real challenge for calculating the MLD; B04D and R23 fail, whereas EBM provides a value between HT09 and B04T.

The hydrography in the subtropical eastern Pacific (Figs. 4k-o) has the largest variations compared to the other challenging regions; consequently, the MLD is the shallowest. In that region, profiles with a quasi-homogeneous upper section and smooth gradients below it predominate, besides profiles with near-surface restratification and slight temperature inversions. The calculated MLD is very dissimilar among all the methodologies, except for the profile in Fig. 4l, in which all the methodologies provide similar values. Again, all the methodologies perform well according to their criteria, and no methodology is preferred over any other. EBM is supported because it provides MLD values between the extremes calculated with the common methodologies. In an ensemble approach, such values represent realistic estimates of the MLD. Generally, B04D and HT09 provide the smallest MLD values, whereas B04T and R23 provide the largest MLD values.

3.3. Global monthly MLD climatology and mixed layer hydrography

The global monthly MLD climatology calculated with EBM is shown in Fig. 5. The MLD is highly heterogeneous in space. The tropical oceans have relatively shallow mixed layers throughout the year, with moderate seasonal changes; the MLD varies in a few tens of meters range. Semiannual cycles can be discerned in the region of barrier layer formation, approximately located in [15°S, 15°N] x [150°E, 150°W], and in the northern Indian Ocean, mainly in the Arabian Sea. In contrast to the tropical oceans, the regions from midlatitudes to high latitudes have deeper mixed layers with strong seasonal behavior, with the MLD ranging from several tens of meters during summer and early autumn to a few hundred meters during winter and early spring. The largest MLD values occur during wintertime in deep and intermediate water formation regions and polar seas in the North Atlantic (south of Iceland and the Labrador, Greenland, Iceland, and Norway Seas) and the southern Pacific and Indian Oceans between 65°S and 45°S. The MLD can reach depths of up to 650 m in the south of Iceland and the Labrador Sea, 750 m in the Greenland, Iceland, and Norway Seas, and 600 m in the southern Pacific and Indian

Oceans. The MLD values in the northern Pacific are asymmetric during wintertime; they are larger in the northwest than in the northeast.

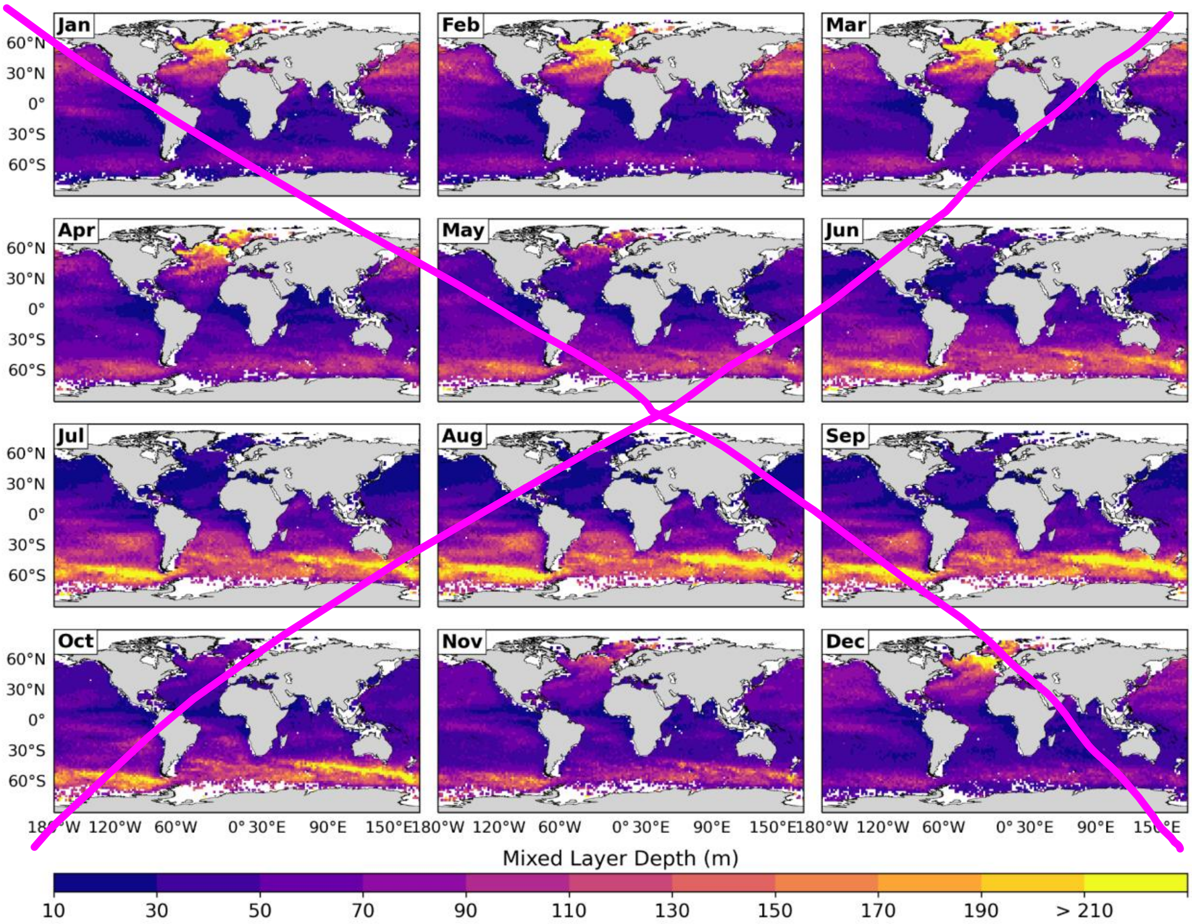


Figure 5. Energy-based global monthly MLD climatology, calculated with the Argo data.

A map of the energy-based MLD in the Southern Ocean is shown in Fig. 6; a typical month for each austral season is shown: February (summer), May (autumn), August (winter), and November (spring). In agreement with the expected physical behavior, the MLD exhibits clear seasonality, shallow during summer and deep during winter. During each season, the mixed layer is shallow in the continental shelves, deepens in the Antarctic Circumpolar Current region, and becomes shallower towards the north of the Antarctic Circumpolar Current. The largest MLD values are located south of the Pacific and Indian Oceans, where they can reach 600 m depth. The largest MLD values are not located between the same latitudes; they are located between 60°S-50°S in the Pacific, between 50°S-40°S in the Indian Ocean, and between 60°S-50°S in the Atlantic Ocean.

To extend our understanding of the Southern Ocean, histograms (expressed in densities) of the monthly MLD for this region are also shown in Fig. 6. The histograms are not symmetrical and exhibit an appreciable bimodal distribution during summer, which tends to diminish over time until it disappears during winter. The MLD is concentrated on values of around 30-80 m during summer and around 110-150 m during winter; such behavior is accompanied by MLD variances increasing from about 300 m during summer to 3000 m during winter, in agreement with Johnson and Lyman (2022). The MLD distribution has a persistent positive skewness throughout the year, with the shortest tail during austral summer (skewness=0.42) and the largest during austral winter (skewness=1.33). This behavior does not agree with Johnson and Lyman (2022), who found that the MLD skewness changes from negative in May to positive in November using the HT09 methodology.

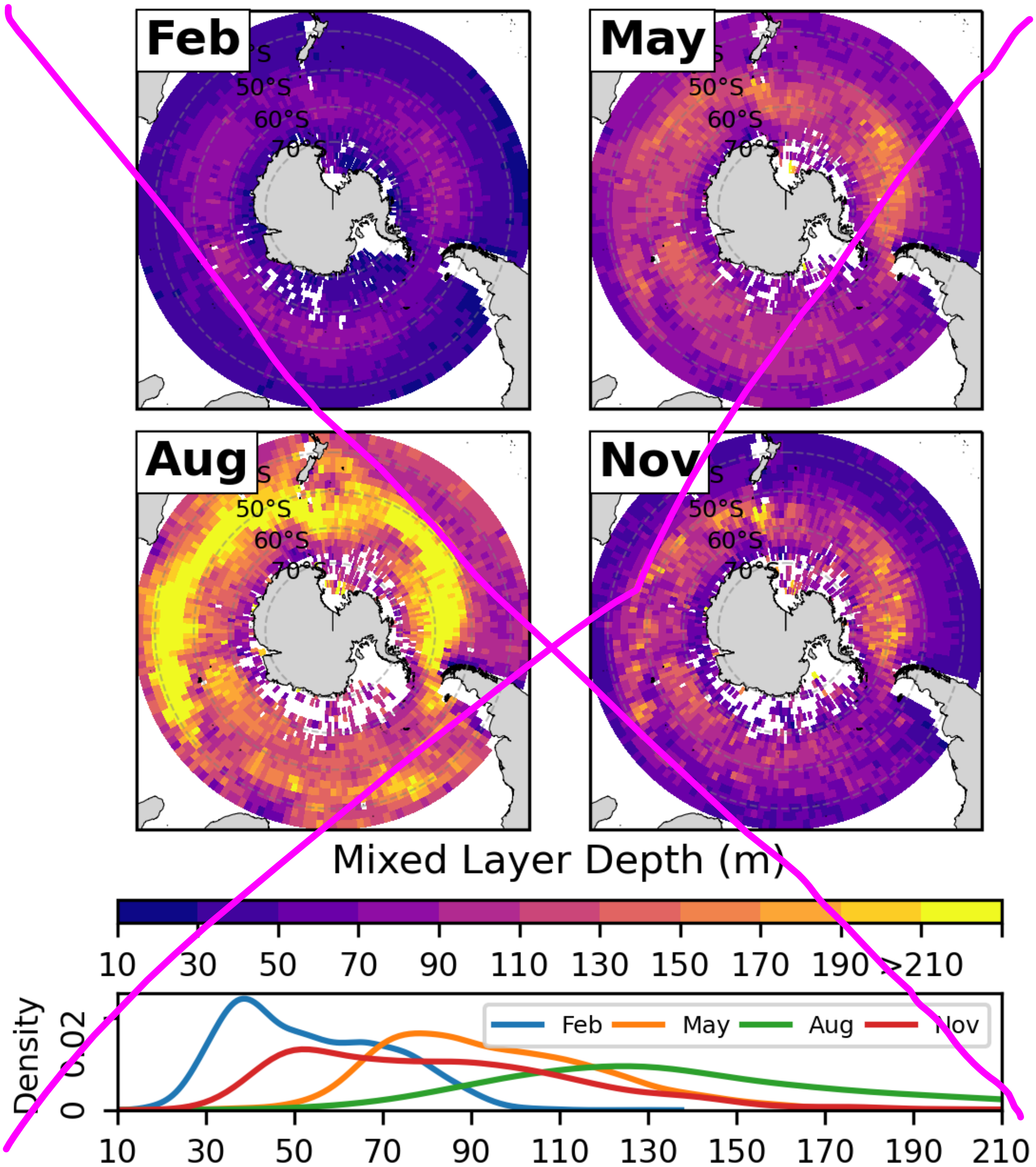


Figure 6. Upper panels: energy-based MLD in the Southern Ocean in a typical climatological month for each austral season: February (summer), May (autumn), August (winter), and November (spring). Lower panel: histograms (expressed in densities) of the corresponding MLD during each typical month.

The previous analyses showed that the energy-based methodology provides realistic estimates of the MLD, consistent with the density stratification intensity in all world regions, demonstrating the good performance of the methodology and its global applicability during all seasons. The energy-based methodology determines a mixed layer homogeneous in buoyancy energy, which does not always coincide with a homogeneous density or temperature layer. Figures 7 and 8 show a monthly climatology of potential density differences and conservative temperature differences from the reference depth (10 m) to the energy-based MLD. These figures also show histograms (expressed in densities) of such density and temperature differences considering all the months; thus, they represent the conjoint distribution in space and time of the density and temperature differences.

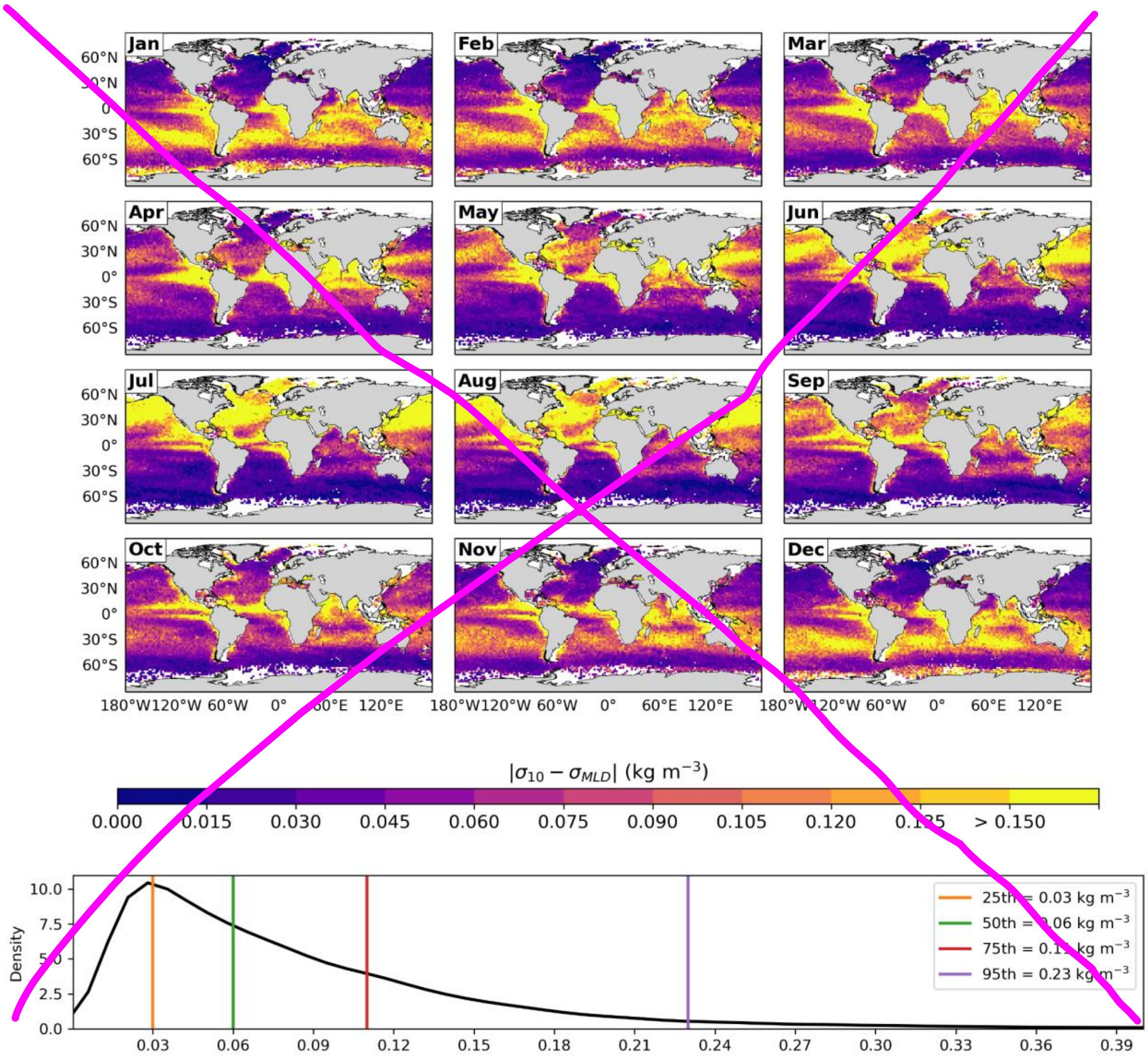


Figure 7. Upper panel: global monthly climatology of absolute differences in potential density from the reference depth of 10 m to the energy-based MLD. The maximum density difference is 1.43 kgm-3. Lower panel: the histogram (expressed in densities) of the conjoint distribution in space and time of the density differences. The values of the histogram at various percentiles are also shown.

In general, the potential density differences (Fig. 7) have a behavior opposite to that of the MLD (Fig. 5), mainly due to the density stratification characteristics. The stronger the density stratification, the smaller the MLD and the larger the density differences; the weaker the density stratification, the larger the MLD and the smaller the density differences. The density differences are highly heterogeneous in space, larger in the northern hemisphere than in the southern hemisphere. The tropical oceans have large differences with a strong seasonal variation. As for the MLD, semiannual cycles can be discerned in the region of barrier layer formation, approximately located in [15°S, 15°N] x [150°E, 150°W], and in the northern Indian Ocean. The mid-to-high latitudes oceans exhibit strong seasonal behavior, with small density differences during wintertime and large during summertime. The Southern Ocean (65°S-40°S) exhibits small density differences of around 0.03 kgm-3 almost throughout the year despite having a strong MLD seasonal variation; the density differences slightly increase during winter. The corresponding histogram shows that 75% of the world ocean has density differences of up to 0.11 kgm-3 throughout the year. The energy-based methodology determines a homogeneous mixed layer in buoyancy energy and a quasi-homogeneous density layer for most of the world ocean.

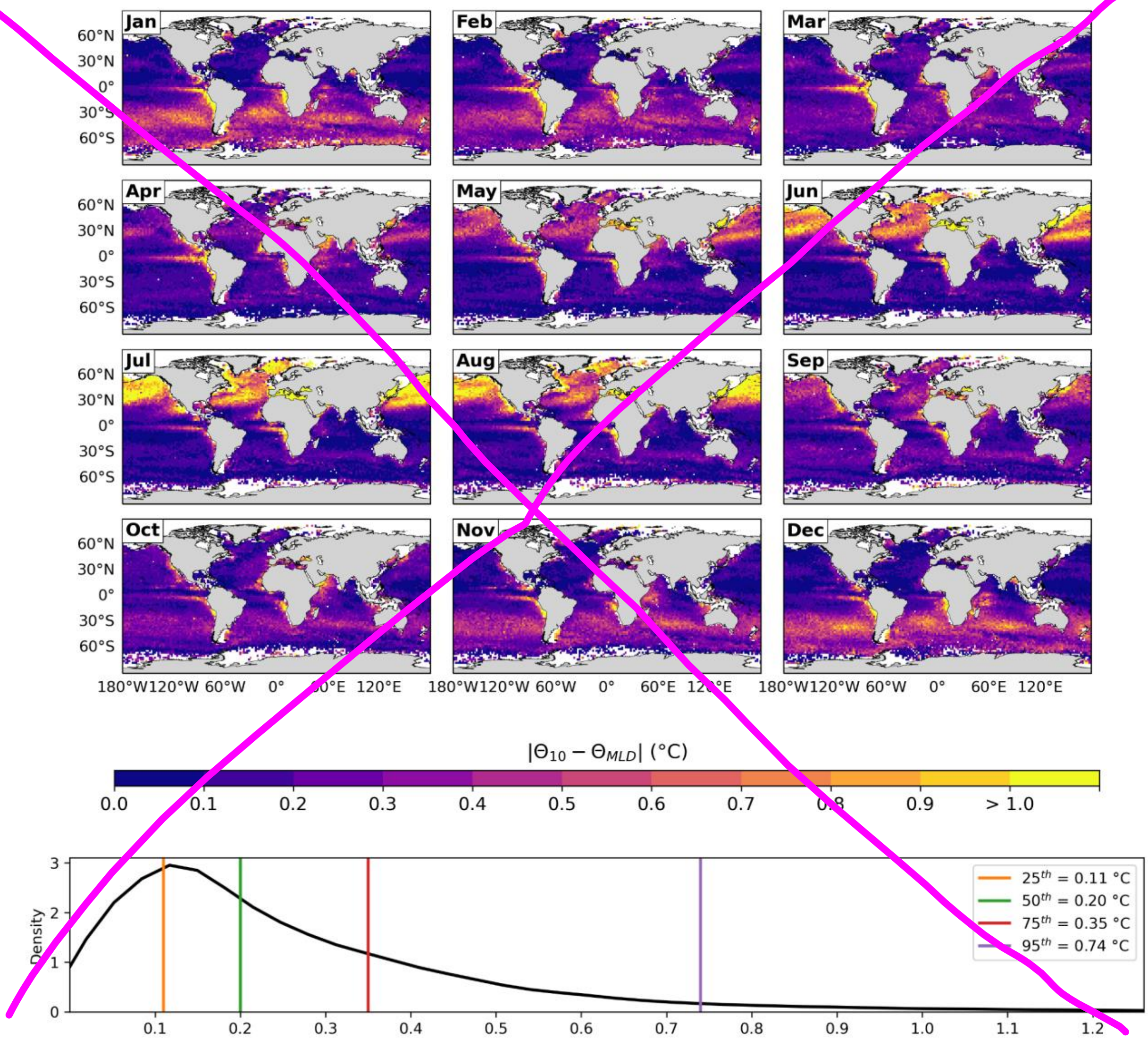


Figure 8. Figure 7. Upper panel: global monthly climatology of absolute differences in conservative temperature from the reference depth of 10 m to the energy-based MLD. The maximum temperature difference is 6.33°C. Lower panel: the histogram (expressed in densities) of the conjoint distribution in space and time of the temperature differences. The values of the histogram at various percentiles are also shown.

As expected, the monthly climatology of conservative temperature differences (Fig. 8) has characteristics that are opposite to those of the MLD climatology (Fig. 5). The stronger the temperature stratification, the smaller the MLD and the larger the temperature differences; the weaker the temperature stratification, the larger the MLD and the smaller the temperature differences. The temperature differences are highly heterogeneous in space. The tropical oceans have the smallest differences, with a strong seasonal variation in the eastern Pacific Ocean and a semiannual cycle in the northern Indian Ocean. The mid-to-high latitudes oceans exhibit strong seasonal behavior, with small temperature differences during wintertime and large during summertime. The temperature differences are larger in the northern hemisphere than in the southern hemisphere. The Southern Ocean (65-40°S) exhibits seasonal temperature differences from around 0.1°C during winter to more than 0.4°C during summer, consistent with its strong MLD seasonal variation. The corresponding histogram shows that 95% of the world ocean has temperature differences of up to 0.74°C throughout the year. Again, the homogeneous mixed layer in buoyancy energy approximately coincides with a quasi-homogeneous temperature layer for most of the world ocean.

3.1. Energy-based global monthly MLD climatology

The global monthly MLD climatology calculated with EBM, considering $\overline{\Delta\rho^{\theta}}$ =0.0150 kg m^{-3}, is shown in Fig. 3. In agreement with the expected physical behavior, the MLD exhibits clear seasonality, being shallow during summer and deep during winter, and having a high heterogeneity in space. Figure 3 also shows the corresponding cumulative density function (CDF) of the MLD for the world ocean, considering all the months; thus, it represents the conjoint distribution in space and time of the MLD. Throughout the year, 50% of the world ocean has MLDs up to 44 m, while only 1% reaches MLDs over 269 m. The probability density function (PDF) used to compute the CDF was obtained through kernel density estimation (Rosenblatt, 1956; Parzen, 1962) using the gaussian-kde function from Python's SciPy library.

The following describes the MLD's spatio-temporal variability shown in Fig. 3. The tropical oceans have relatively shallow mixed layers throughout the year, with moderate seasonal changes; the MLD varies in a range of a few tens of meters. Semiannual cycles can be discerned in the region of barrier layer formation, approximately located in [15°S, 15°N] x [150°E, 150°W], and in the northern Indian Ocean, mainly in the Arabian Sea. In contrast to the tropical oceans, the regions from midlatitudes to high latitudes have deeper mixed layers with strong seasonal changes, with the MLD ranging from several tens of meters during summer and early fall to several hundred meters during winter and early spring. The seasonal changes are smaller in the North Pacific than in the North Atlantic and Southern Oceans. The MLD values in the North Pacific are asymmetric during wintertime; they are larger in the northwest than in the northeast. Concerning the seasonal behavior of the MLD in the Southern Ocean, the mixed layer is shallow in the continental shelves during each season, deepens in the Antarctic Circumpolar Current region, and becomes shallower towards the north of the Antarctic Circumpolar Current. The largest MLD values across the Southern Ocean are not located between the same latitudes; they are located between 60°S-50°S in the Pacific, between 50°S-40°S in the Indian Ocean, and between 60°S-50°S in the Atlantic Ocean. The largest MLD values occur during wintertime in deep and intermediate water formation regions and polar seas in the North Atlantic (south of Iceland and the Labrador, Greenland, Iceland, and Norway Seas) and in the South Pacific and South Indian Oceans between 65°S and 45°S. The MLD can reach values of up to 945 m in the Labrador Sea, 1074 m in the Greenland, Iceland and Norwegian seas region, and 614 m in the South Pacific and South Indian Oceans.

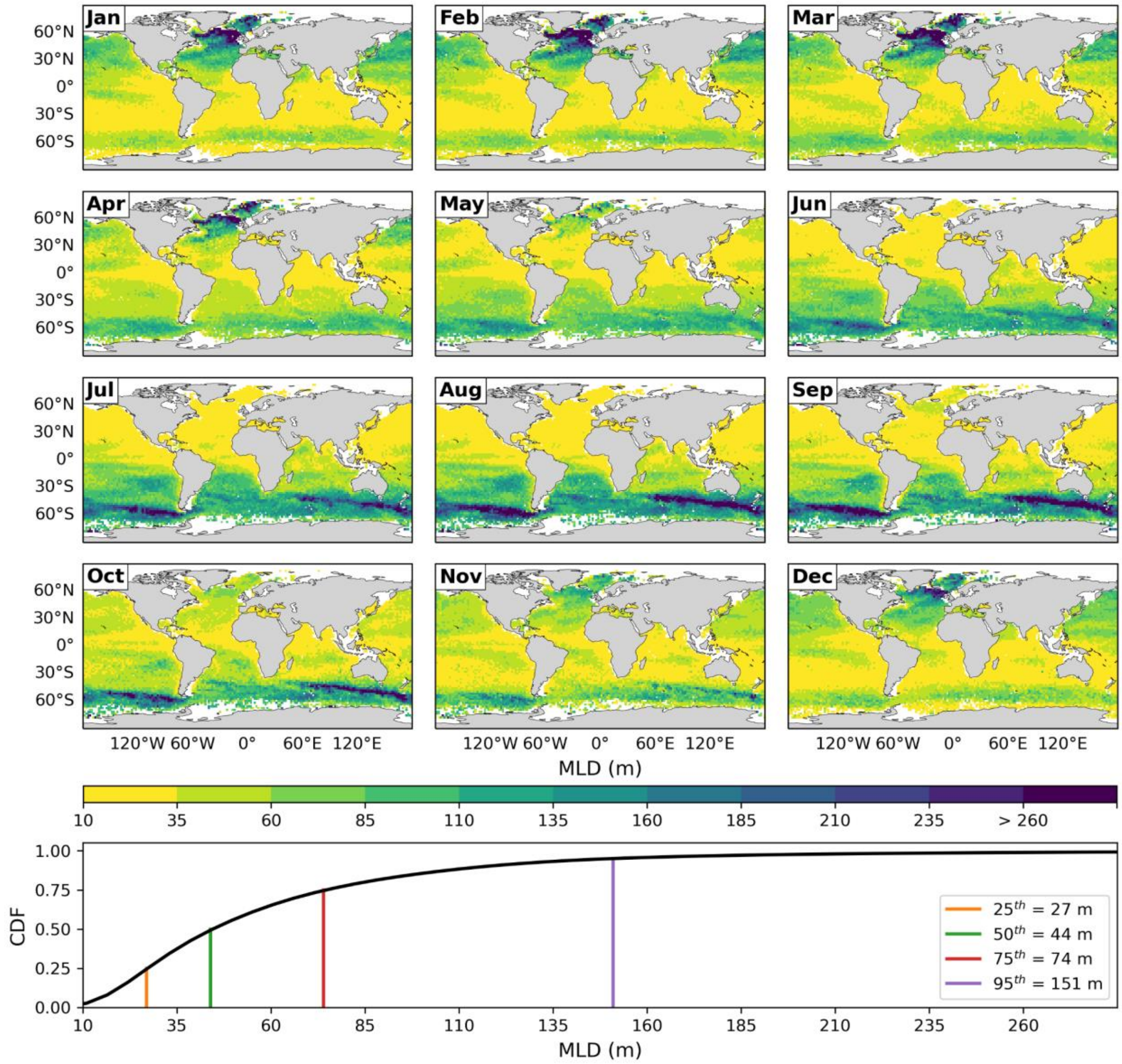


Figure 3. Upper panel: global monthly MLD climatology calculated with EBM, considering $\Delta\overline{\rho^{\theta}}$ =0.0150 kg m^{-3}. Lower panel: the cumulative density function (CDF) of the conjoint distribution in space and time of MLD, with various percentiles shown.

The global monthly climatology of the WB threshold characterizing the MLD, considering $\Delta\overline{\rho^{\theta}}$ =0.0150 kg m^{-3}, is shown in Fig. 4. This figure also shows the CDF of WB at the MLD for the world ocean, considering all the months. The spatial and temporal variability of the WB threshold is very similar to that of the associated MLD (Fig. 3), but with small variations in the WB magnitude through time. Equation (8) and Fig. 2 establish that the MLD derived from a unique, non-zero $\Delta\overline{\rho^{\theta}}$ threshold will not result in a unique, non-zero WB threshold. Interestingly, results for this $\Delta\overline{\rho^{\theta}}$ threshold showed that for most of the world ocean, the WB thresholds seem small enough to characterize an energetically homogeneous ocean layer, which would be consistent with our energy definition of the mixed layer. 75% of the world ocean has WB thresholds not exceeding 9.5 J m^{-3} year-round, and up to 95% has WB thresholds below 20.8 J m^{-3}; the largest WB thresholds only occur in high latitudes during wintertime.

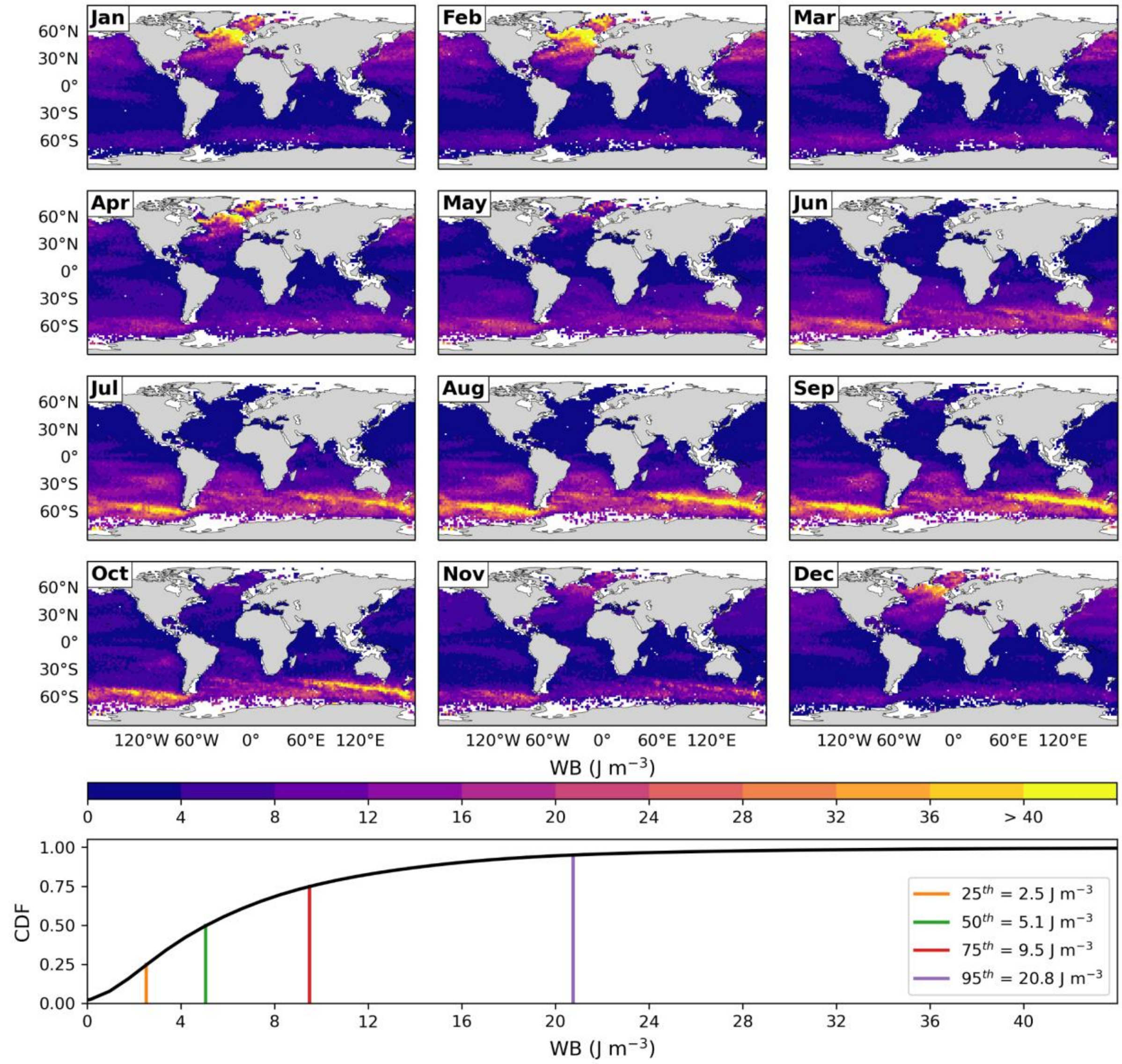


Figure 4. Upper panel: global monthly climatology of the WB threshold characterizing the EBM-MLD, i.e., WB(z=MLD), considering $\Delta\overline{\rho^{\theta}}$ =0.0150 kg m^{-3}. Lower panel: the cumulative density function (CDF) of the conjoint distribution in space and time of WB(z=MLD), with various percentiles shown.

To explore if the mixed layer is energetically homogeneous, we computed a quality index for WB (QI_{WB}) following Lorbacher et al. (2006), who defined it assuming a near-surface layer with quasi-homogeneous properties in which the standard deviation of the property along its vertical mean is close to zero. Following Lorbacher et al. (2006), QI_{WB} can evaluate the degree of homogeneity in WB from the 10 m depth to the MLD and, consequently, the EBM performance in determining the MLD according to the following criteria: QI_{WB}>0.8 indicates a well-homogeneous layer in WB, 0.5< QI_{WB}<0.8 indicates increased uncertainty in the existence of a quasi-homogeneous layer in WB, and QI_{WB}<0.5 indicates that there is no a quasi-homogeneous layer in WB (a common result for profiles where WB changes gradually with depth). Figure 5 shows the global monthly climatology of QI_{WB} for the EBM-MLD, considering $\Delta\overline{\rho^{\theta}}$ =0.0150 kg m^{-3}, along with its corresponding CDF. EBM performs very well in almost all the world ocean year-round: 96.72% of the world ocean has QI_{WB} 0.8, whereas only in 0.03% of the world ocean EBM has QI_{WB}<0.5. During the transition from wintertime to springtime in the North Atlantic and Southern Oceans, QI_{WB}~0.7, indicating a reduced but still good

EBM performance; the reasons for this behavior are beyond the scope of this study but will be investigated in the future. The most relevant aim of this analysis is to evaluate the general performance of EBM.

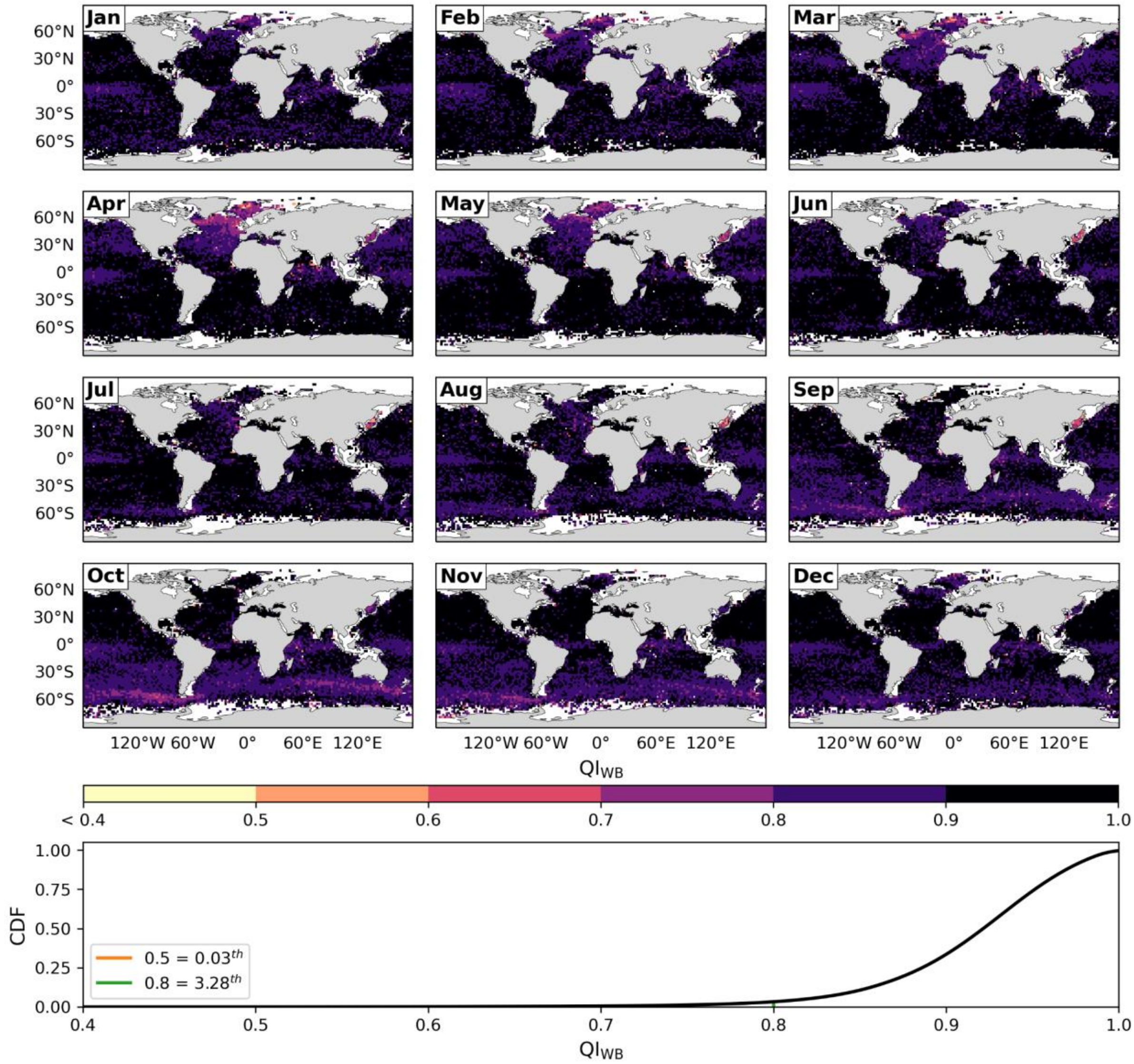


Figure 5. Upper panel: global monthly climatology of QI_{WB} for the EBM-MLD, considering $\Delta\overline{\rho^{\theta}}$ =0.0150 kg m^{-3}. Lower panel: the cumulative density function (CDF) of the conjoint distribution in space and time of QI_{WB}, with two QI_{WB} values and their corresponding percentiles shown.

The previous analyses showed that EBM provides MLD estimates consistent with the space-time variability in stratification across the world ocean throughout the year, which agrees with the expected physical behavior. The resulting MLD delimitates a well-mixed layer in energetic terms under different ocean conditions in highly and slightly stratified regions, suggesting it can represent a good standard in determining the MLD with global applicability during all seasons. However, it remains to investigate to what extent the layer quasi-homogeneous in energy is homogeneous in density and temperature. Figures 6 and 7 show a monthly climatology of the absolute differences in potential density and conservative temperature from the reference depth of 10 m to the EBM-MLD, respectively; their corresponding CDFs are also shown.

In constructing the mixed layer definition, the density variations along the mixed layer throughout the year were established. The maximum differences in potential density from the reference depth of 10 m to the EBM-MLD are limited by approximately $2\overline{\Delta\rho^{\theta}}$, and the minimum ones by values little larger than $\overline{\Delta\rho^{\theta}}$. For the EBM-MLD climatology, considering $\overline{\Delta\rho^{\theta}}$ =0.0150 kg m^{-3}, most of the world ocean should have potential density differences in the interval (0.0150, 0.0300) kg m^{-3} throughout the year (Fig. 6). Density differences smaller than 0.0150 kg m^{-3} can occur in regions with very shallow mixed layers where the density profiles are noisy, with values oscillating around the density value at the reference depth. The spatial variability of the density differences is not expected to have particular characteristics, such as showing high heterogeneity or seasonal behavior. EBM was constructed to have density variations along the mixed layer that are very restricted globally year-round (see the CDF in Fig. 6). Note that this result does not contradict the expected physical behavior relating the density stratification and the MLD, according to which the stronger the density stratification, the smaller the MLD and vice versa.

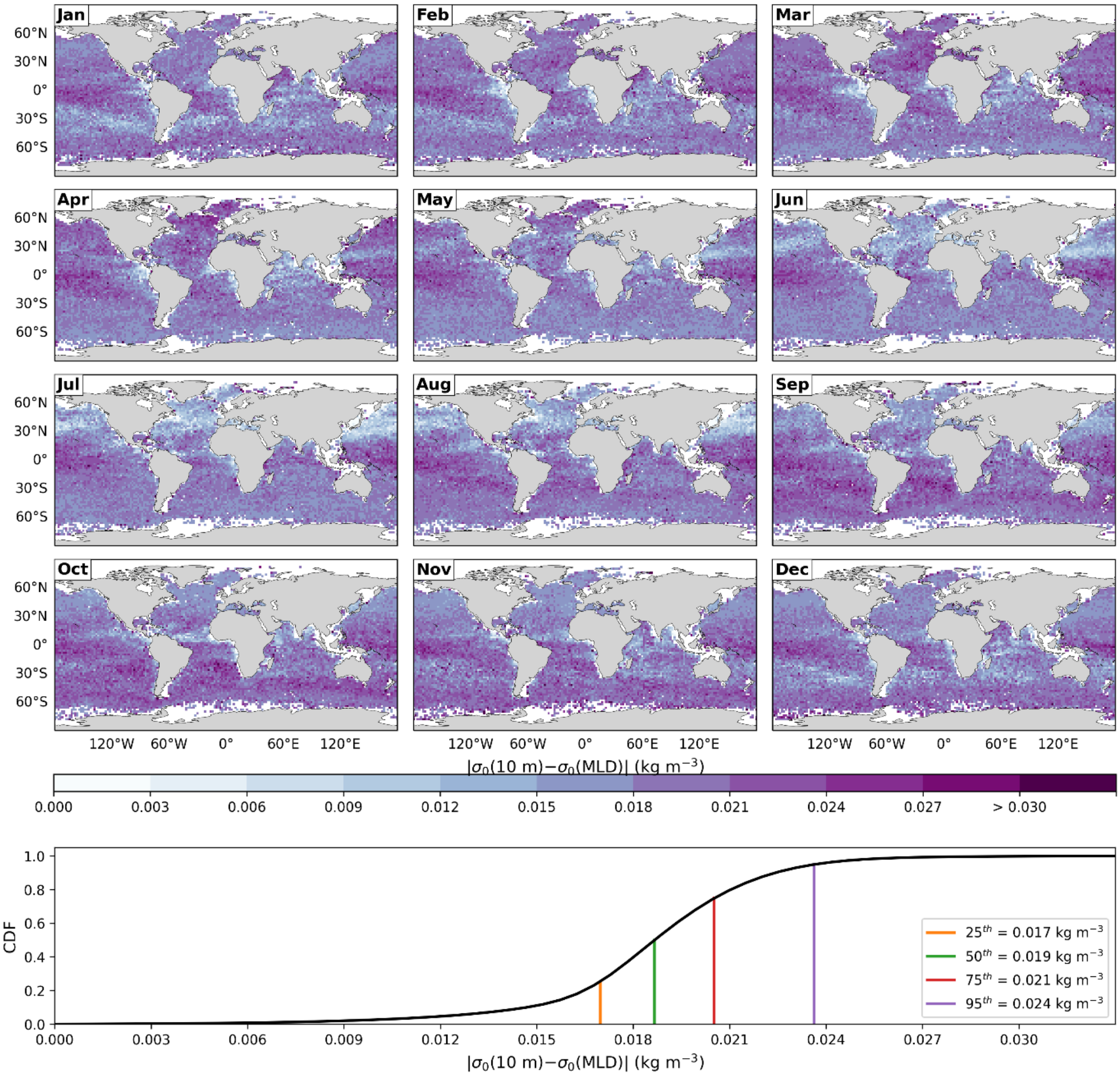


Figure 6. Upper panel: global monthly climatology of the absolute differences in potential density from the reference depth of 10 m to the EBM-MLD, i.e., |σ_0(10 m)-σ_0(MLD)|. The maximum density

difference is 0.102 kg m^{-3}. Lower panel: the cumulative density function (CDF) of the conjoint distribution in space and time of $|\sigma_0(10\text{ m})-\sigma_0(\text{MLD})|$, with various percentiles shown.

The differences in conservative temperature from the reference depth of 10 m to the MLD shown in Fig. 7 are heterogeneous in space and change over time with a type of seasonal variation. The temperature differences are generally large for large MLDs and vice versa; however, the temperature differences do not have the same structure or seasonal variation as those of the MLD. The corresponding CDF shows that 95% of the global ocean has temperature differences of less than 0.2°C throughout the year. The most relevant contribution of this figure is the homogeneity in temperature rather than a detailed analysis of the spatiotemporal variability of these differences. In summary, EBM determines a quasi-homogeneous mixed layer in WB, density, and temperature for the global ocean throughout the year, in agreement with our definition and those of de Boyer Montégut et al. (2004).

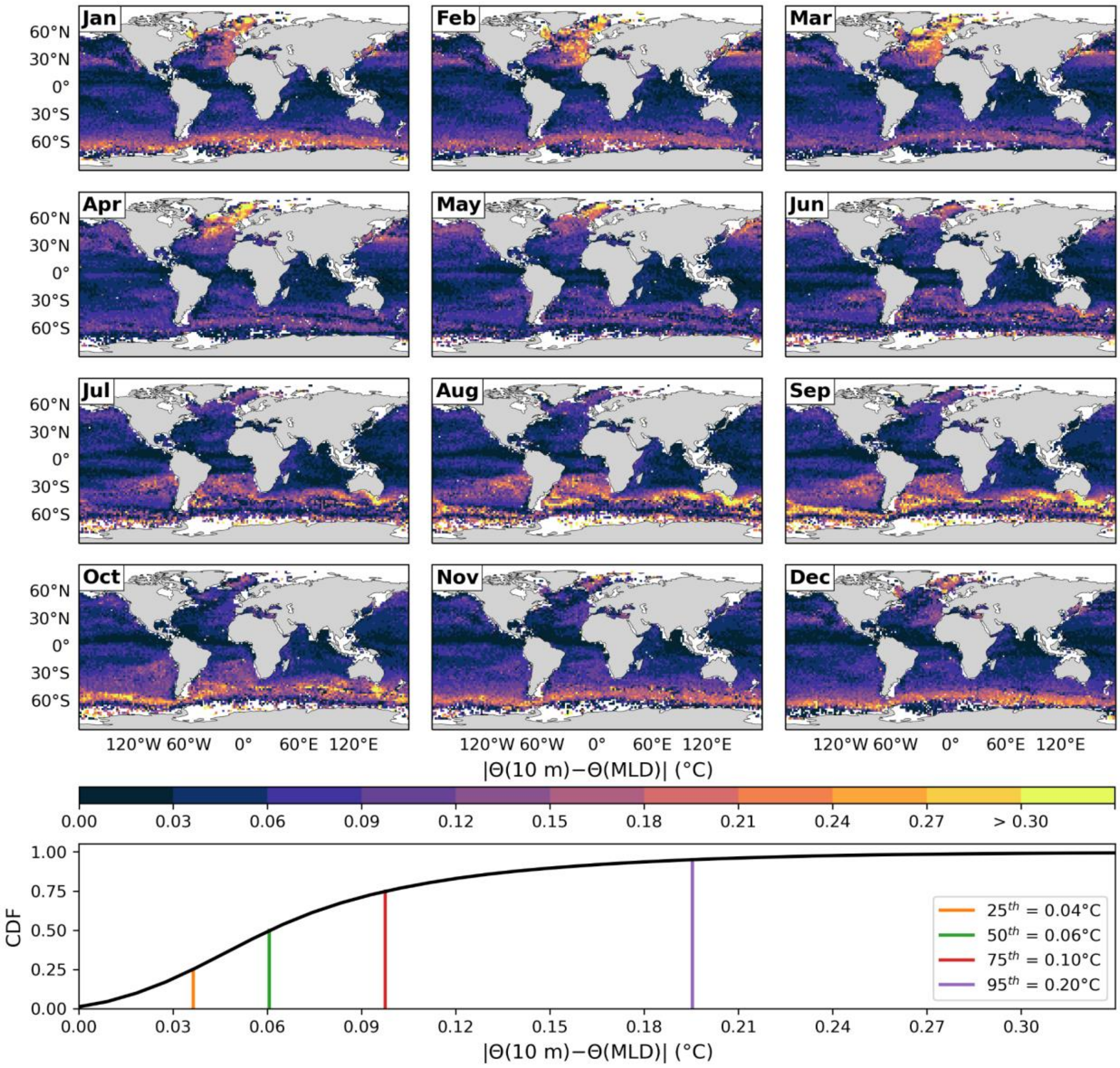


Figure 7. Upper panel: global monthly climatology of the absolute differences in conservative temperature from the reference depth of 10~m to the EBM-MLD, i.e., $|\Theta(10\text{ m})-\Theta(\text{MLD})|$. The maximum temperature difference is 1.82°C. Lower panel: the cumulative density function (CDF) of the conjoint distribution in space and time of $|\Theta(10\text{ m})-\Theta(\text{MLD})|$, with various percentiles shown.

Finally, we analyzed the global monthly MLD climatology calculated with EBM and its corresponding WB threshold climatology, considering $\Delta\overline{\rho^{\theta}}$ =0.0625 kg m^{-3} (Figs. S2 and S3 in the Supplement). Compared to the case with $\Delta\overline{\rho^{\theta}}$ =0.0150 kg m^{-3}, the MLD magnitude is larger, and its seasonal changes are somewhat blurred. The associated WB thresholds are large enough to be representative of layers with small WB values, with 50% of the world ocean having WB thresholds exceeding 30 J m^{-3} year-round. Therefore, it is concluded that the MLD obtained with $\Delta\overline{\rho^{\theta}}$ =0.0625 kg m^{-3} does not produce a quasi-homogeneous layer in WB and is inconsistent with our mixed layer definition. A corollary of this result is that MLD methodologies based on density thresholds of about 0.125 kg m^{-3} along the mixed layer produce overestimated MLDs and are inadequate to define a well-mixed layer in energetic terms.

3.2. MLD methodologies intercomparison

The mixed layer definition depends on the parameter being addressed, which has resulted in numerous MLD methodologies whose estimates do not completely agree with each other. Figures S4, S6, S8, and S10 in the Supplement show the global monthly MLD climatologies calculated with B04T, B04D, HT09, and R23, respectively. From those climatologies and the corresponding EBM climatology (Fig. 3), we evaluated the conjoint uncertainty in the MLD estimation via a percent error,

$$\text{MLD uncertainty} = \frac{0.5 \times \Delta\text{MLD}}{\overline{\text{MLD}}} \times 100\%, \tag{9}$$

where ΔMLD is the range in the MLD estimated by the five methodologies, and $\overline{\text{MLD}}$ is the corresponding mean MLD; the smaller the MLD range, the smaller the MLD uncertainty, and vice versa.

The global monthly climatology of the MLD uncertainty is shown in Figure 8; it exceeds 19% for half of the world ocean throughout the year. All the methodologies approximately coincide in the MLD calculation for profiles near the ideal, that is, for hydrographic profiles with a clear homogeneous upper section and sharp density and temperature gradients below it, without temperature inversions. For profiles with increasing density and decreasing temperature from the near surface, all the methodologies give very shallow MLD values, which do not necessarily coincide. The most significant disparity in the MLD values is obtained for profiles with a quasi-homogeneous upper section and smooth gradients below it. In some cases, some common methodologies fail to provide an MLD value. Large MLD uncertainties are common during winter and spring when mixing is more active, eroding sharp density and temperature gradients in winter and creating near-surface restratification in spring. Large uncertainties are ubiquitous across the ocean; however, they are mainly located in regions where salinity significantly influences density, such as polar seas, intermediate and deep-water formation regions, and barrier and compensated layers; they are also predominant along the equator.

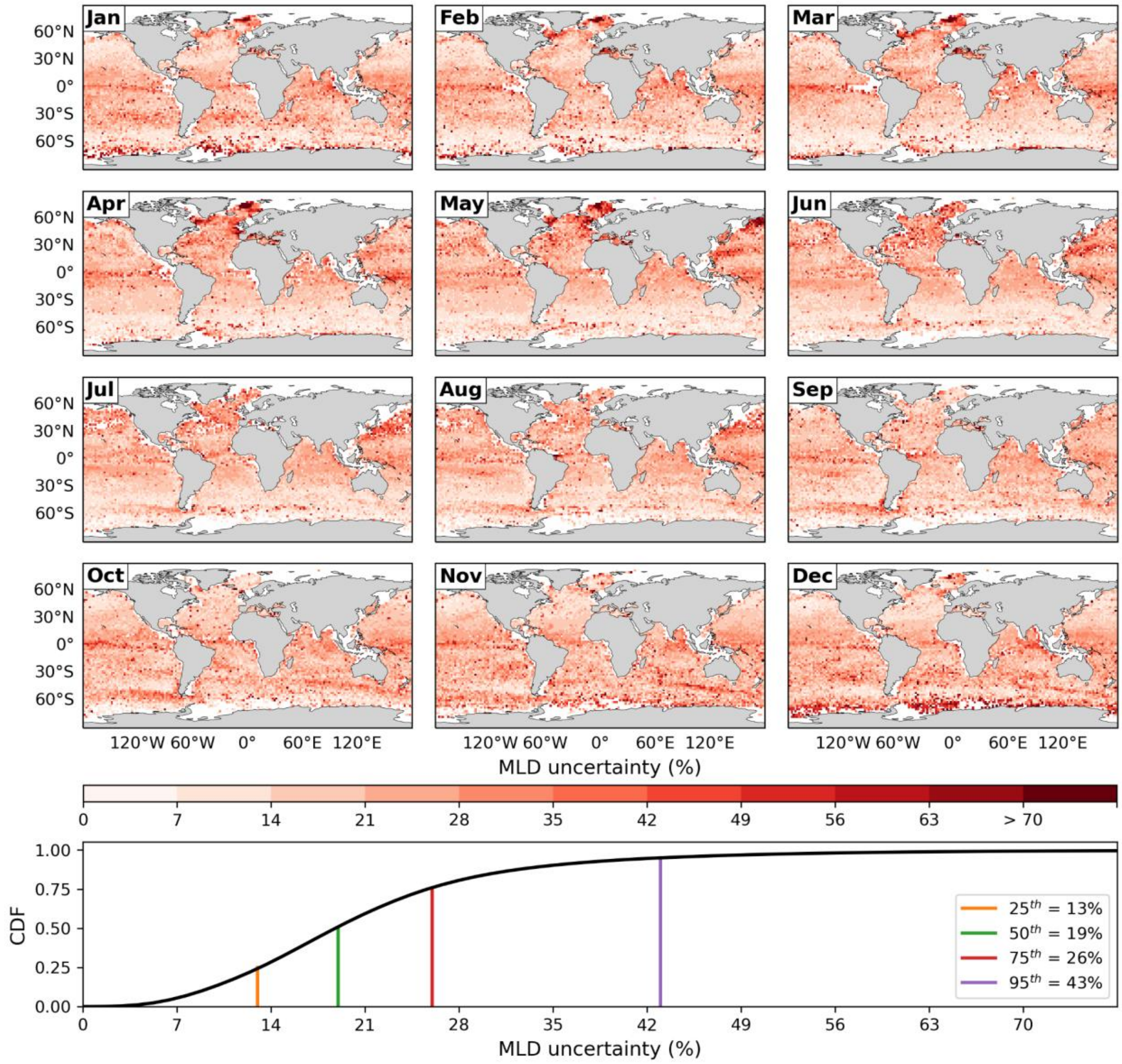


Figure 8. Upper panel: global monthly climatology of the conjoint uncertainty in the MLD estimation, considering the methodologies B04T, B04D, HT09, R23, and EBM. Lower panel: the cumulative density function (CDF) of the conjoint distribution in space and time of the MLD uncertainty, with various percentiles shown.

The above analysis underscores the subjective nature of the mixed layer definition and the resulting lack of consistency among the methodologies in determining the MLD. It is almost impossible to ensure that the true value of the MLD in any location and time is known; the different MLD methodologies distinctly evaluate the mixing conditions across the ocean. Consequently, the accuracy of any methodology can not be determined, that is, the closeness of any MLD estimation to the true value. When comparing MLD methodologies, we can only evaluate their precision: the closeness between the different MLD estimates. The reliability of EBM in determining the MLD was evaluated, to a certain extent, using the quality index QI_{WB}. To complete the EBM's reliability evaluation, we evaluated the EBM's precision by comparing its MLD estimates with those of other methodologies (i.e., B04T, B04D, HT09, and R23). Qualitatively, the structure of the spatiotemporal variability of the MLD is consistent among all the methodologies (see Figs. 3, S4, S6, S8, and S10). In this regard, EBM is precise compared to the others and provides a realistic description of the MLD, consistent with the seasonal variation of ocean conditions across the ocean.

To quantitatively analyze the EBM's precision, we considered the global PDF and CDF of the MLD for each season, obtained with each methodology (Fig. 9). A clear seasonal behavior of the MLD, with the largest MLDs during wintertime and the smallest ones during summertime, is consistent across all methodologies. However, the MLD magnitude differs among methodologies, with the smallest MLDs obtained with EBM and the largest ones obtained with B04T (Fig. 9). The global PDF of the MLD for each season was analyzed using some precision measures: median, variance, and skewness. Table 1 shows the value of each precision measure for each season and methodology. To evaluate EBM, we calculated the average value and range for each precision measure considering the four common methodologies; then, we calculated the corresponding relative difference of EBM according to the following formula,

$$\text{relative difference of EBM} = \frac{\text{EBM measure - average value of common methodologies}}{\text{average value of common methodologies}} \times 100\%, \qquad (10)$$

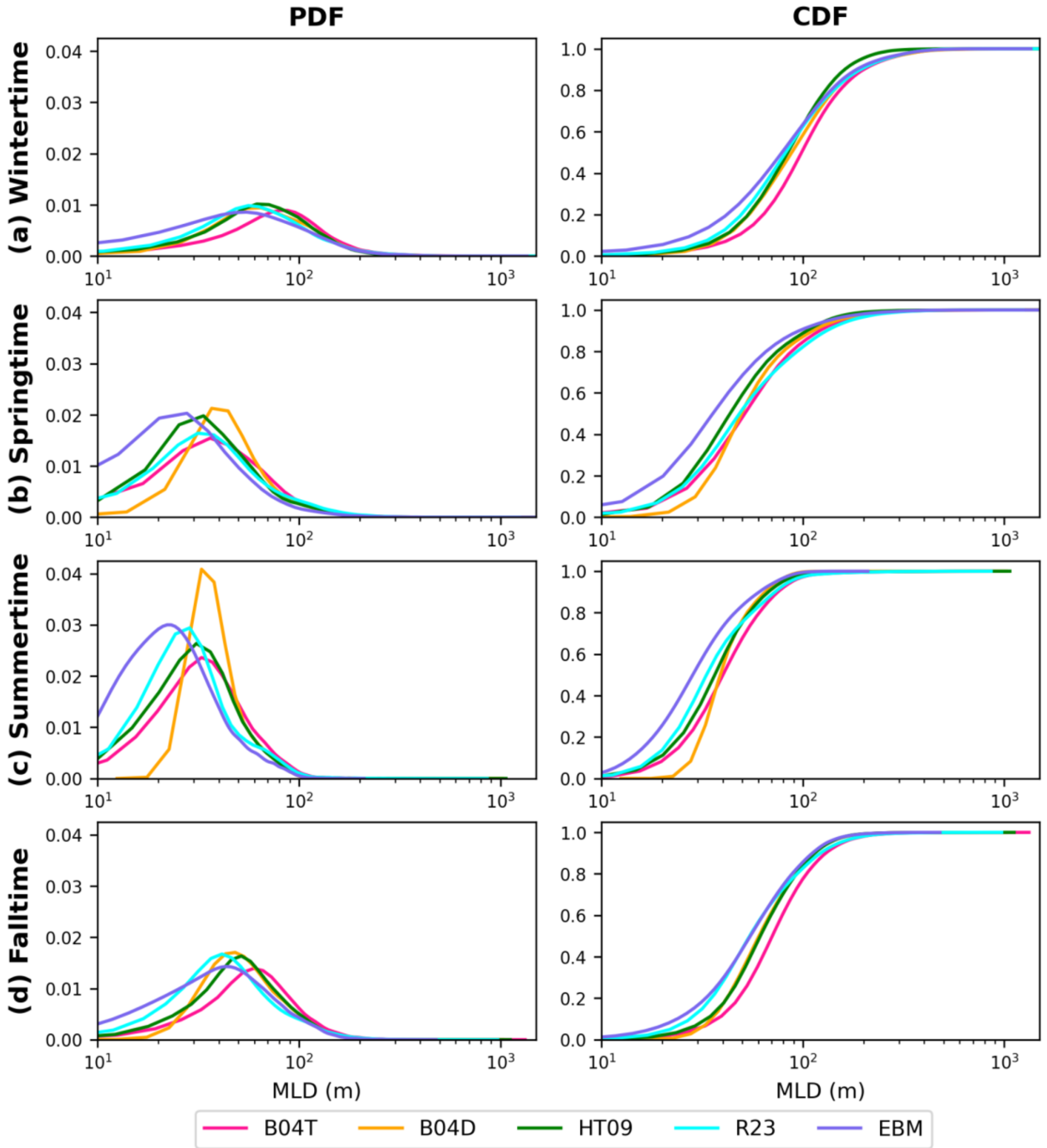


Figure 9. The global probability density function (PDF) and cumulative density function (CDF) of the MLD for (a) wintertime, (b) springtime, (c) summertime, and (d) falltime, obtained with each methodology. The PDF and CDF represent the conjoint distribution in space and time of the MLD.

Table 1. Precision measures (median, variance, and skewness) of the global PDF of the MLD for each season and methodology. The average value and range for each precision measure, calculated considering the four common methodologies (B04T, B04D, HT09, and R23), are also shown. The relative difference of EBM relative to the different average values of common methodologies is also shown.

Season	Statistic	B04T	B04D	HT09	R23	Average values of common methodologies	EBM (relative difference)
Wintertime	Median (m)	98	88	84	82	Average = 88 Range = 16	78 (-11%)
	Variance (m^2)	5932	6807	3059	5695	Average = 5373 Range = 3748	5888 (10%)
	Skewness	3.13	4.34	3.69	2.55	Average = 3.43 Range = 1.79	2.57 (-25%)
Springtime	Median (m)	51	48	43	47	Average = 47 Range = 8	35 (-27%)
	Variance (m^2)	3204	2981	1899	3403	Average = 2872 Range = 1504	2384 (-17%)
	Skewness	6.7	7.86	6.25	3.77	Average = 6.14 Range = 4.1	4.93 (-20%)
Summertime	Median (m)	40	39	36	33	Average = 37 Range = 7	28 (-23%)
	Variance (m^2)	895	302	810	902	Average = 727 Range = 600	360 (-50%)
	Skewness	6.51	11.96	8.83	6.1	Average = 8.35 Range = 5.86	1.44 (-83%)
Falltime	Median (m)	70	60	61	54	Average = 61 Range = 17	55 (-11%)
	Variance (m^2)	1966	1356	1389	2135	Average = 1711 Range = 779	1400 (-18%)
	Skewness	3.53	3.73	3.73	3.09	Average = 3.52 Range = 0.65	1.76 (-50%)

The global PDF of the MLD for each season varies among methodologies (Table 1). However, the most relevant contribution of this table is the evaluation of the EBM's precision, measured via its relative difference. The common methodologies provide an ensemble of the possible extent of the precision measures, in which the smallest and largest values account for the uncertainty in calculating it (the range). In this ensemble approach, EBM can be considered precise if its values are inside that interval. According to the relative difference, EBM underestimates the median of the MLD throughout the year, from 11% during winter and fall to 27% during spring; EBM is not precise in calculating the MLD median. Regarding the MLD variance, EBM is precise in calculating it. Finally, EBM underestimates the MLD skewness throughout the year but provides precise values during winter and spring.

The MLD methodologies are qualitatively consistent in the spatiotemporal variability of the MLD. Quantitatively, EBM is precise in a statistical ensemble sense; regarding the statistical distribution of the MLD, the methodologies agree in their variance but differ in their median and skewness. The above raises the question of whether it is possible to determine the best MLD methodology. All the methodologies perform well under the oceanographic conditions for which they were built according to the parameter being addressed; Tang et al. (2025) evaluated 12 MLD methodologies and found that each has unique merits and limitations that depend on the analyzed ocean conditions. The determination of the best MLD methodology thus depends on the criterion used to rank the methodologies. However, we can go a step further in addressing this question by evaluating the energy-consistency of the MLD methodologies. If the behavior of an MLD methodology deviates from the energy definition of the mixed layer, it is not energy-consistent and can not be considered physically realistic. According to the aforementioned definition, the mixed layer is the energetically homogeneous layer characterized by zero or small WB values; moreover, according to Eq. (8), it is acceptable to have shallow mixed layers associated with small WB values and deep mixed layers associated with large WB values.

To evaluate the energy-consistency of the MLD methodologies, we considered the global monthly MLD climatology obtained with each methodology and calculated the WB value at the MLD (Figs. 4 and S5, S7, S9, and S11 in the Supplement); in this way, we were able to analyze their energy-consistency on a global scale throughout the year. The methodologies B04T, HT09, and R23 are not energy-consistent because the spatiotemporal variability of the WB at the MLD is not consistent with that of the MLD through space and time. According to our mixed-layer definition, they are not expected to calculate the MLD accurately based on energy considerations. Figure 10 exemplifies the WB value at the MLD on global meridional transects in the Pacific and Atlantic Oceans during winter and summertime.

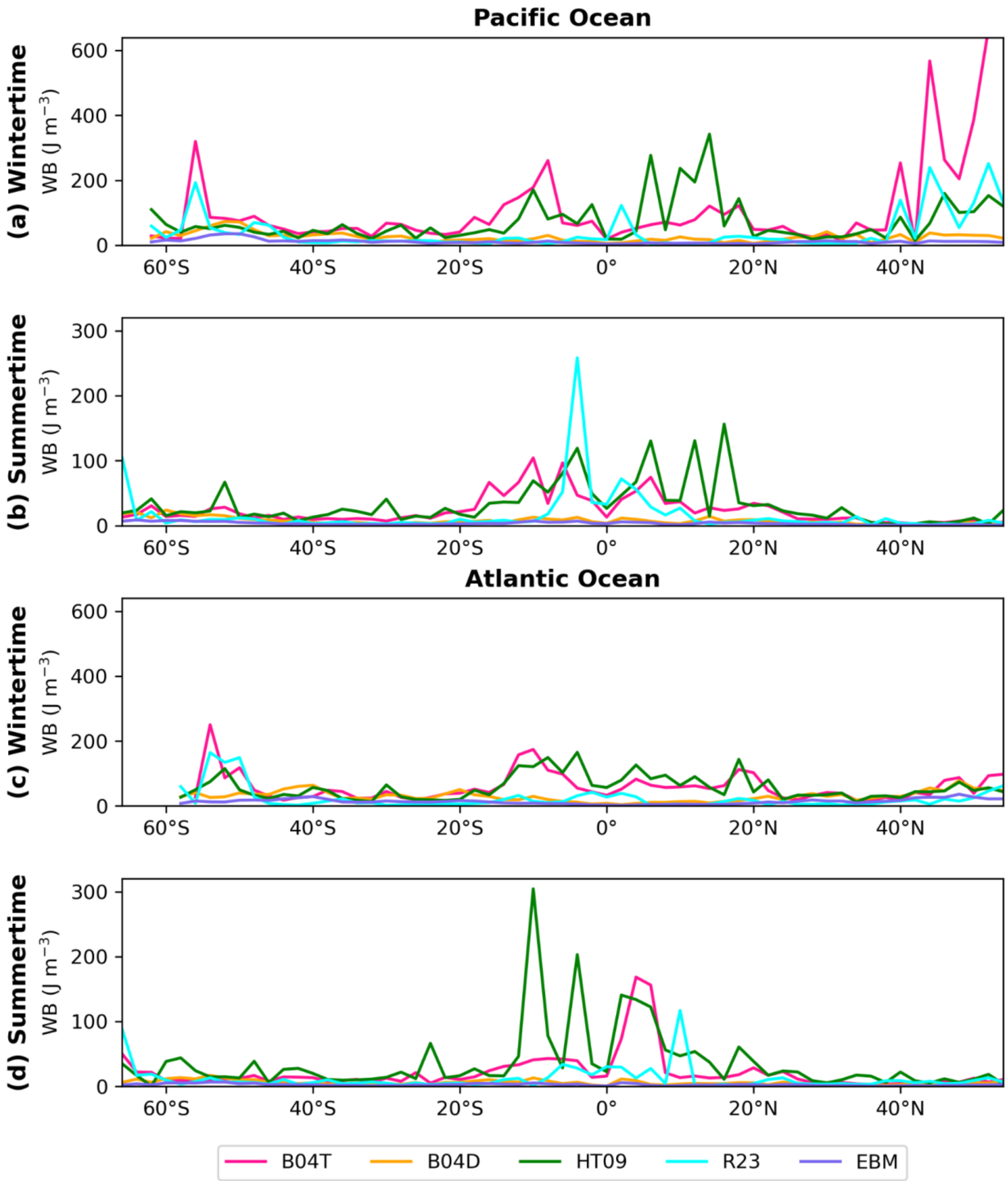


Figure 10. The WB value at the MLD for each methodology on global meridional transects in the Pacific (along 150°W) and Atlantic (along 30°W) Oceans during wintertime (February in the northern hemisphere and August in the southern hemisphere) and summertime (August in the northern hemisphere and February in the southern hemisphere). The global monthly MLD climatology obtained with each methodology was used for the calculation.

B04D has a behavior that is very close to being energy-consistent (Fig. 10). It has the highest concordance with EBM, as both are built from the density; however, B04D has WB values larger than EBM's. During winter, B04D tends to overestimate the WB threshold (and consequently, the MLD) in latitudes higher than 40°S and 40°N; during summer, it largely coincides with EBM. Of all the methodologies, B04T and HT09 are the ones that are furthest from being energy-consistent. The largest discrepancies between B04T and HT09 with EBM occur during winter throughout almost all latitudes, especially in low and high latitudes; during summer, the discrepancies are concentrated in low latitudes (between 20°S-20°N); in latitudes south of 20°S and north of 20°N, the methodologies are close to each other. R23 has a behavior close to being energy-consistent, although it persistently exhibits large WB values in low latitudes near the equatorial zone every month. During winter, R23 is close to EBM between 40°S-40°N (where the methodology has its best fit); beyond those latitudes, the WB values are larger than EBM's. During summer, R23 is very close to EBM throughout almost all latitudes, except near the equatorial zone, where its WB values are larger than EBM's.

The above analysis showed that B04D and EBM are energy-consistent and can be considered physically realistic, although WB in B04D is almost twice that of EBM in some regions and months, making it difficult to reconcile the large WB values of B04D with our mixed layer definition. By being physically derived and based on energy, EBM could be superior to B04D in estimating the MLD; EBM could represent an improved or well-founded version of the threshold density criterion proposed by de Boyer Montégut et al. (2004) to define the MLD because EBM considers the density vertically integrated. Nonetheless, despite its qualities, EBM has some downsides; the EBM-MLD intrinsically depends on the $\Delta\overline{\rho^{\theta}}$ threshold, which may negatively influence its performance in analyzing highly stratified or vertically compensated layers. It is important to note that the common methodologies can also struggle or even fail when analyzing profiles that strongly differ from the ideal ones (an upper homogeneous layer above a strong pycnocline or thermocline). For highly stratified layers, different $\Delta\overline{\rho^{\theta}}$ thresholds could lead to very different MLDs; however, the requirement of having small WB values could lessen this limitation and restrict the variation in the MLD values. For vertically compensated layers, like B04D, EBM may also overestimate the MLD; nonetheless, using WB, we can measure the degree of inhomogeneity of the water column associated with the compensated layer and investigate whether it is intense enough to suppress mixing. While EBM may not provide a better or more meaningful MLD estimate than other methodologies, it does measure the water column inhomogeneity in terms of energy, a unique feature that other methodologies lack. Furthermore, EBM provides realistic MLD estimates and performs without failure in complex profiles, demonstrating its robustness under different ocean conditions.

Similar to other MLD methodologies, EBM is sensitive to the choice of the reference depth, mainly in regions with very thin mixed layers and during winter and early spring when mixing is more active, eroding sharp density and temperature gradients in winter and creating near-surface restratification in spring. The above is not a limitation for EBM, as it is based on the analysis of WB with depth; EBM can still be used to find the MLD and its associated WB. For those regions and during those periods, the reference depth can be adapted to be consistent with the local dynamics; then, the procedure described in the Methods (section 2) can be applied. We also explored the influence of the vertical resolution of the density profiles on the MLD calculation. We found that as long as the vertical characteristics of the density are correctly resolved and sampled, the estimated MLD will be accurate to the order of the vertical resolution. This adaptability provides flexibility in applying the methodology in specific regions and under different ocean conditions.

3.3. What WB values can define the MLD globally throughout the year?

Our results showed that the mixed layer obtained with EBM is quasi-homogeneous in WB on a global scale. To a certain degree, the WB threshold is region—and season—independent (Fig. 4), which raises the possibility that a few or even a unique WB threshold can characterize the MLD globally year-round.

To what extent is the WB threshold region—and season—independent? To explore this question, we present a very preliminary result delving into what WB values can define the MLD globally during all seasons, a question posed by Treguier et al. (2023). The above question was addressed by analyzing three global meridional transects in the Pacific, Atlantic, and Indian Oceans during August and February (not shown). For each transect, the EBM-MLD was calculated along with the differences in depth between the MLD and three WB values (Fig. 11). It was found that a unique WB value does not consistently locate the MLD. For latitudes north of 20°S, the 5 J m^{-3} WB isoline well locates the MLD; for latitudes south of 20°S, the 12.5 or 20 J m^{-3} WB isolines are more appropriate. The average difference in depth between the MLD and the three WB isolines along each transect is shown in Table 2; for this calculation, we used the absolute values of the differences in depth.

Given the small differences in depth of the 12.5 and 20 J m^{-3} WB isolines, some WB equipotential in that interval could be a good choice to delineate an energetically well-mixed upper ocean layer, thereby defining the MLD. The above suggests that a unique WB equipotential could define the MLD throughout the three ocean transects, a remarkable finding that would indicate that the mixed layer is close to being energy-consistent across space and time. Whether this result can be extended and applied to a global scale during all seasons is an endeavor beyond the scope of this study that deserves further research.

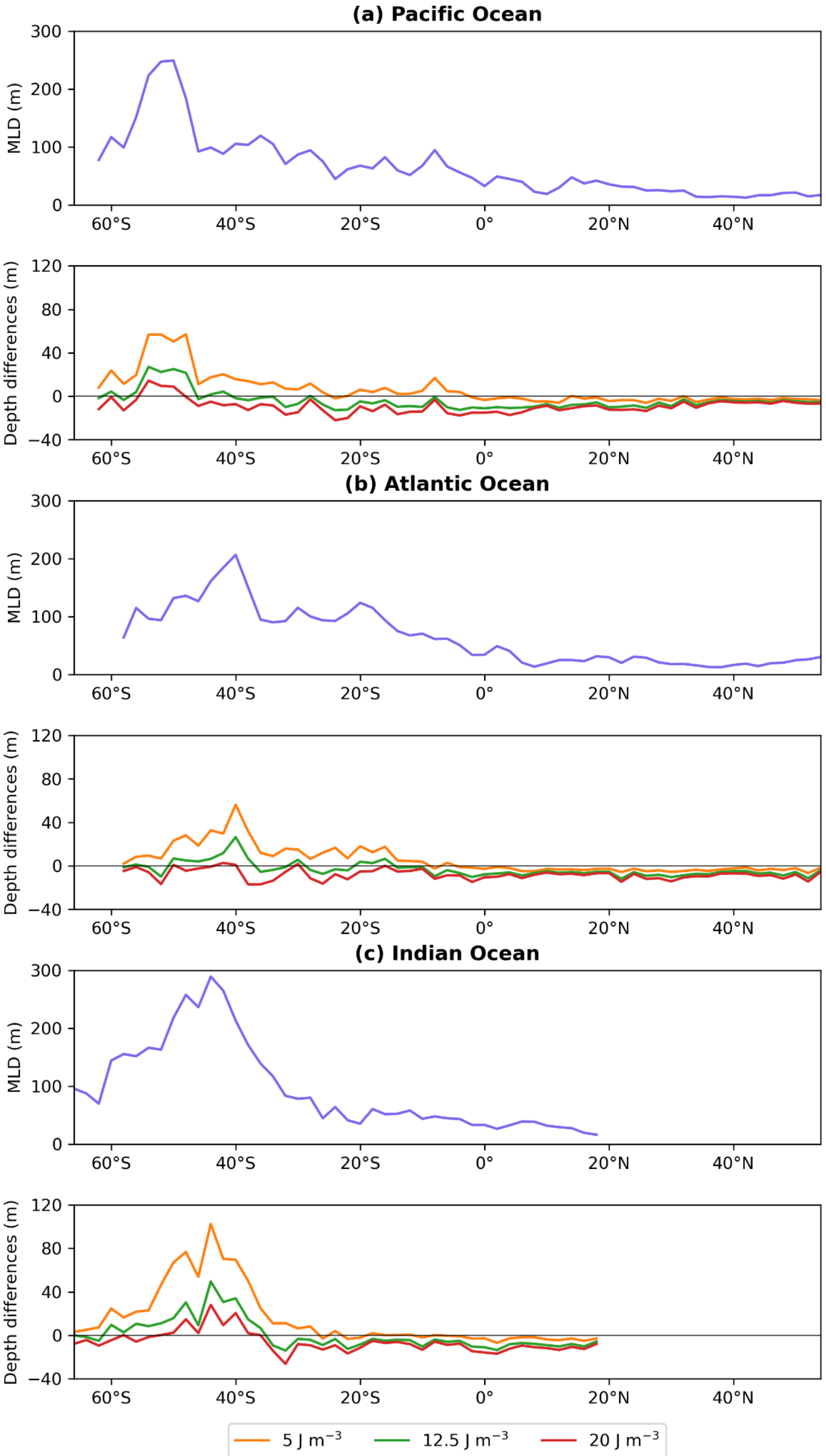


Figure 11. The EBM-MLD (purple line) along three global meridional transects in the Pacific (150°W), Atlantic (30°W), and Indian (90°E) Oceans during August. The differences in depth between the MLD and three WB equipotentials are also shown for each transect.

Table 2. The average difference in depth between the EBM-MLD and the three WB isolines along each transect shown in Fig. 11. For this calculation, the absolute values of the differences in depth were used.

Ocean	5.0 Jm^{-3}	12.5 Jm^{-3}	20.0 Jm^{-3}
Pacific	9.3 m	7.0 m	9.6 m
Atlantic	8.4 m	6.2 m	8.2 m
Indian	17.2 m	10.1 m	9.5 m
Average value	11.6 m	7.7 m	9.1 m

4. Discussion

~~In this study, we developed a new methodology constructed from physical principles and energy considerations for calculating the MLD. Our study advances the development of energy-based methodologies to define the MLD, adding to that of Reichl et al. (2022). The most important characteristic of our energy-based methodology is that it provides realistic estimates of the MLD in all world regions under different ocean conditions using a consistent criterion in space and time, that is, a criterion that does not vary in space or time depending on the hydrography of the study region. The procedure for calculating the MLD using the energy-based MLD methodology is as follows. From a vertical profile of surface-referenced potential density, calculate the WB required to displace a water parcel from all depths to 10 m and find the depth of the 20 Jm^{-3} WB isoline, which defines the MLD. The energy-based methodology performs well during periods and in regions where the common methodologies show low consistency in the MLD calculation (in the so-called challenging regions). The methodology is, therefore, accurate, robust, and of global applicability. It can be the base methodology for performing MLD model intercomparison studies, as in the OMIP and CMIP projects (Griffies et al., 2016; Treguier et al., 2023). It is also easy to implement numerically.~~

~~The energy-based methodology identifies the upper section of the ocean, well-mixed in energetic terms, that can be considered in contact with the atmosphere and thus be referred to as the mixed layer. The methodology uses the work done by the buoyancy force and the depth of its structural change to define the MLD. From the physical principles of the ocean boundary layer mixing, Reichl et al. (2022) demonstrated that diagnosing the MLD from density stratification establishes a connection between the turbulent boundary layer and the mixed layer. They demonstrated that the mixed layer can be defined through the potential energy of the water column and advocated using an energy threshold to define the MLD. There is a correspondence between our methodology and that of Reichl et al. (2022), suggesting that our energy-based methodology is consistent with the turbulence approach of the mixed layer formation (D'Asaro, 2014; Sutherland et al., 2014; Franks, 2015; Sallee et al., 2021). The mixed layer is thus determined by energy processes instead of density, temperature, or salinity thresholds, which vary in space and time according to the hydrography of the study region (Griffies et al., 2016; Treguier et al., 2023).~~

~~One of our most significant contributions is the finding that the 20 Jm-3 WB isoline delimitates an upper section of the ocean that is well-mixed in energetic terms, defining the MLD globally during all months. This finding contrasts with Reichl et al. (2022), who did not provide specific energy values to define the MLD across the world ocean during all seasons. They found that a spatially and temporally variable energy threshold should be used to reproduce, to some extent, MLDs similar to those obtained with the~~

HT09 methodology. However, since the common methodologies are non-energy-based, trying to match the performance of energy-based methodologies to that of non-energy-based ones may not be meaningful. To what extent are the common methodologies energy-consistent in space and time? To address this question, we considered the global MLD monthly climatology obtained with each methodology and calculated the WB value at the corresponding MLD (see Figs. S3-S6 in the Supplement). As an example, Fig. 9 shows the WB value at the MLD on a global meridional transect along 170°W during wintertime and summertime.

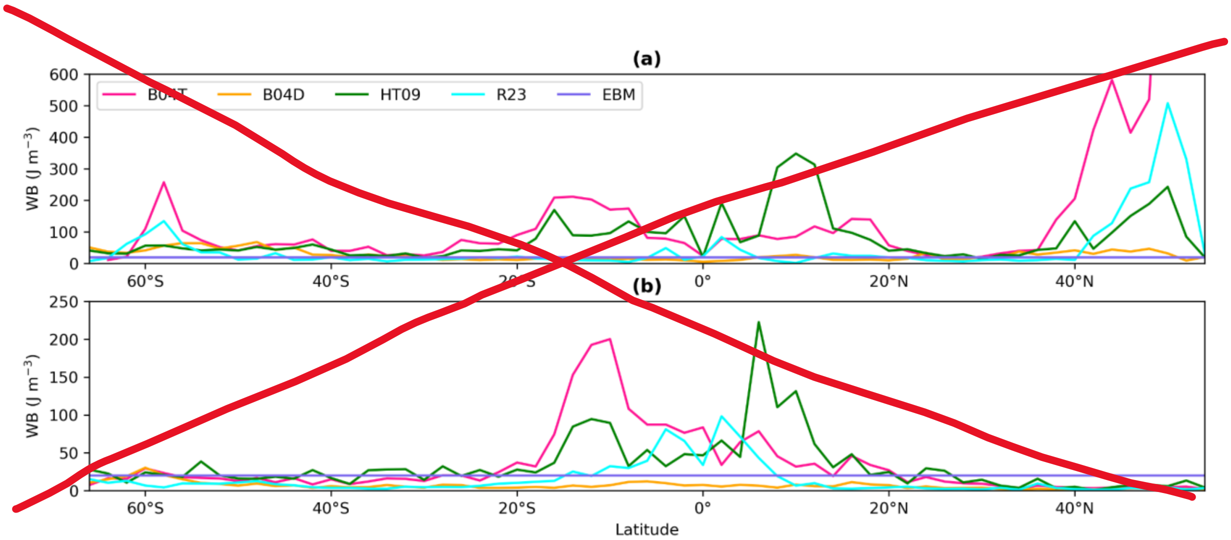


Figure 9. The WB value at the MLD for each methodology on a global meridional transect along 170°W during (a) wintertime (August in the southern hemisphere and February in the northern hemisphere) and (b) summertime (February in the southern hemisphere and August in the northern hemisphere). The global MLD monthly climatology obtained with each methodology was used for the calculation.

The 20 Jm-3 WB isoline of the EBM methodology can be used to analyze the energy consistency of the common MLD methodologies shown in Fig. 9. The common methodologies are not energy-consistent: no unique WB value exists at the MLD. The WB variations are region- and season-dependent and differ among the methodologies. B04T has the largest WB values, followed by HT09; the largest WB values occur during winter in the polar and tropical regions and summer in the tropical regions. B04T and HT09 have WB values larger than 20 Jm-3 throughout the transect during winter; during summer, they have WB values close to 20 Jm-3 south of 20°S, WB values close to 20 Jm-3 between 20°S-20°N, and WB values smaller 20 Jm-3 north of 20°N. R23 has WB values close to 20 Jm-3 between 55°S-40°N (where the methodology has its best fit) during winter; beyond that, the WB values are larger than 20 Jm-3. During summer, R23 has WB values smaller than 20 Jm-3 along most of the transect, except near the equator, where it has WB values larger than 20 Jm-3. B04D shows the greatest concordance with EBM, as both are built from the density. During winter, B04D has WB values close to 20 Jm-3 between 40°S-40°N, which increase beyond those latitudes; during summer, it has WB values smaller than 20 Jm-3 along most of the transect.

If a MLD methodology is not energy-consistent, the homogeneity of the mixed layer in energetic terms is not equal throughout space and time; large WB values are associated with high-energy stratified layers and vice versa. The above suggests that the methodologies in which the WB at the MLD varies a lot in space and time are not expected to calculate the MLD accurately, as we showed. The common MLD methodologies analyzed in this study are non-energy-based and consequently not energy-consistent.

Common and energy-based methodologies use a threshold to define the MLD. However, the nature of their thresholds is substantially different: common thresholds only consider the difference in values of some hydrographic variable between two depths, while energy thresholds consider the cumulative effects of those differences along the vertical. As shown in this study and by Reichl et al. (2022), mixing resistance depends on the differences in density and the physical distance between two depths. While the buoyancy force is directly proportional to the difference in density between two depths, the associated work is not. That means that the difference in density between two given depths cannot be used as a proxy for the energy required to homogenize the ocean's upper layer. Similarly, density-derived measures of the local stability or homogeneity of the water column, such as the buoyancy frequency, also cannot be used as proxies for the energy required to homogenize the water column if they do not consider the cumulative effect of the buoyancy force along a water column section. Therefore, the density threshold, density gradient, and buoyancy frequency criteria may not be sufficient to calculate the MLD, as done in previous research (e.g., Lukas and Lindstrom, 1991; Large et al., 1997; de Boyer Montégut et al., 2004; Lorbacher et al., 2006; Dong et al., 2008; Holte and Talley, 2009; Chu and Fan, 2011; Carvalho et al., 2017).

The definition of the mixed layer as the ocean's surface layer whose properties (density, temperature, salinity, and other tracers) are relatively homogeneous in the vertical is challenging to achieve when considering constant increases in density or constant decreases in temperature from the corresponding values of these variables at the reference depth. Since the coefficients of the equation of state of seawater vary with pressure, temperature, and salinity, a given density change does not correspond to a unique temperature change, and vice versa. In order to determine a homogeneous mixed layer from density or temperature thresholds, the thresholds should vary according to the hydrography of the study region; moreover, the implementation of spatially variable thresholds in a set of models and observations would be complex and daunting (Griffies et al., 2016; Treguier et al., 2023). Our results showed that constant density or temperature thresholds do not correspond to constant increases in WB across the world ocean. The above supports the definition of the mixed layer as the ocean's upper layer quasi-homogeneous in buoyancy energy, even if that leads to spatially variable increments in density and decrements in temperature. According to Levitus (1982) and Kara et al. (2000), variations of up to 0.125 kgm-3 in density and up to 0.8°C in temperature can be considered typical in a well-mixed layer. Although our methodology does not seek to determine mixed layers homogeneous in density or temperature, the energy-based mixed layer is not far from such quasi-homogeneity: 75% of the world ocean has density differences of up to 0.11 kgm-3, and 95% of the world ocean has temperature differences of up to 0.74°C throughout the year.

The energy-based methodology is sensitive to the choice of the reference depth, mainly in regions with very thin mixed layers and during winter and early spring when mixing is more active, eroding sharp density and temperature gradients in winter and creating near-surface restratification in spring (the so-called challenging regions). The above is not a limitation of our methodology since it is based on the structural change in WB; it still can be used to find the exact WB isoline that defines the MLD. For those regions and during those periods, the reference depth has to be adapted to be consistent with the local dynamics; then, the depth of the structural change in WB can be found using its vertical gradient or a specific ad-hoc method and thus locate the MLD. Furthermore, we explored the influence of the vertical resolution of the density profiles on the MLD calculation. We found that as long as the vertical characteristics of the density are correctly resolved and sampled, the estimated MLD will be accurate to the order of the vertical resolution. This adaptability provides flexibility in applying the methodology in specific regions and under different ocean conditions.

This study analyzed the MLD on long spatial and temporal scales: spatial scales larger than mesoscale and timescales larger than diurnal cycles. Active mixing and high-frequency MLD variability, mainly driven by synoptic atmospheric forcing, ocean eddies, and fronts (Brainerd and Gregg, 1995; Whitt et al., 2019), were not addressed. To explore the above processes, the surface turbulent boundary layer is a more relevant measure (Reichl et al., 2022). In computing monthly MLD values from daily values, the sub-monthly variability was omitted, potentially underestimating the MLD compared to the corresponding daily MLD values, as shown by Toyoda et al. (2017). A thorough analysis of regional differences between the monthly and daily MLD values is out of the scope of this study and is proposed for future research. The MLD analysis in the Southern Ocean revealed additional differences between the energy-based and non-energy-based methodologies regarding the skewness of the MLD distribution and its persistence. Further analyses of those differences are beyond the scope of this study and are also proposed for future research. Finally, future research proposes quantifying the runtime to estimate the MLD in on-the-fly or offline computations, considering different MLD methodologies.

In this study, we developed a new methodology for calculating the MLD based on physical principles and energy considerations, advancing the development of energy-based methodologies such as that of Reichl et al. (2022). The energy-based methodology EBM identifies the upper section of the ocean, well-mixed in energetic terms, in which water parcels can move with little or no work, which can be considered in contact with the atmosphere and thus be referred to as the mixed layer. EBM uses the work done by the buoyancy force and considerations about the density structure of the water column to define the MLD. The most important characteristic of EBM is that it provides realistic MLD estimates in all world regions and performs without failure in complex profiles, demonstrating its robustness under different ocean conditions. We showed a connection between WB and the turbulent kinetic energy budget, suggesting that EBM is consistent with the turbulence approach of the mixed layer formation (D'Asaro, 2014; Sutherland et al., 2014; Franks, 2014; Sallée et al., 2021). Similar to Reichl et al. (2022), we showed that diagnosing the MLD from density stratification establishes a connection between the turbulent boundary layer and the mixed layer. The mixed layer is thus determined by energy processes instead of density, temperature, or salinity thresholds, which vary in space and time according to the oceanographic conditions of the study region (Griffies et al., 2016; Treguier et al., 2023). EBM can be the base methodology for performing MLD model intercomparison studies, as in the OMIP and CMIP projects (Griffies et al., 2016; Treguier et al., 2023). The numerical implementation of EBM only requires the potential density profile referred to 0 dbar, which is easily obtained from simple survey ocean data or numerical data. The script to compute the MLD is very short, and its formulae are not complex. In that regard, EBM is easy to implement numerically.

Common and energy-based methodologies use a threshold to define the MLD. However, the nature of their thresholds is substantially different: common thresholds only consider the difference in values of some oceanic variable between two depths, while energy thresholds consider the cumulative effects of those differences along the vertical. As shown in this study and by Reichl et al. (2022), mixing resistance depends on the differences in density and the physical distance between two depths. EBM is based on a threshold in WB and is more than a common threshold. While the buoyancy force is directly proportional to the difference in density between two depths, the associated work is not. That means that the difference in density between two given depths cannot be used as a proxy for the energy required to homogenize the ocean's upper layer. Similarly, density-derived measures of the local stability or homogeneity of the water column, such as the buoyancy frequency, also cannot be used as proxies for the energy required to homogenize the water column if they do not consider the cumulative effect of the buoyancy force along a water column section. Therefore, the density threshold, density gradient, and buoyancy frequency criteria may not be sufficient to calculate the MLD, as done in previous research (Lukas and Lindstrom, 1991; Large et al., 1997; de Boyer Montégut et al., 2004; Lorbacher et al., 2006; Dong et al., 2008; Holte and Talley, 2009; Chu and Fan, 2011; Carvalho et al., 2017).

The definition of the mixed layer as the ocean's surface layer whose properties (density, temperature, salinity, and other tracers) are relatively homogeneous in the vertical is challenging to achieve when considering constant increases in density or constant decreases in temperature from the corresponding values of these variables at the reference depth. Since the coefficients of the equation of state of seawater vary with pressure, temperature, and salinity, a given density change does not correspond to a unique temperature change, and vice versa. To determine a homogeneous mixed layer from density or temperature thresholds, the thresholds should vary according to the oceanographic conditions of the study region; moreover, the implementation of spatially variable thresholds in a set of models and observations would be complex and daunting (Griffies et al., 2016; Treguier et al., 2023). The above supports the definition of the mixed layer as the ocean's upper layer, quasi-homogeneous in buoyancy energy, even if that leads to spatially variable increments in density and decrements in temperature. According to Levitus (1982) and Kara et al. (2000), variations of up to 0.125 kg m^{-3} in density and up to 0.8°C in temperature can be considered typical in a well-mixed layer. Although our methodology does not seek to determine mixed layers homogeneous in density or temperature, the energy-based mixed layer is very close to such quasi-homogeneity: almost 100% of the world ocean has density differences of less than 0.03 kg m^{-3}, and 95% of the world ocean has temperature differences of less than 0.2°C throughout the year.

EBM has several interesting qualities; however, it has some downsides and room for improvement. This study analyzed the MLD on long spatial and temporal scales: spatial scales larger than mesoscale and timescales larger than diurnal cycles. Active mixing and high-frequency MLD variability, mainly driven by synoptic atmospheric forcing, ocean eddies, and fronts (Brainerd and Gregg, 1995; Whitt et al., 2019), were not addressed. The surface turbulent boundary layer can be a more relevant measure to explore the above processes. In computing monthly MLD values from daily values, the sub-monthly variability was omitted, potentially underestimating the MLD compared to the corresponding daily MLD values, as shown by Toyoda et al. (2017). A thorough analysis of regional differences between the monthly and daily MLD values is out of the scope of this study and is proposed for future research. Additionally, due to limitations in the spatial coverage of Argo data, this study could not explore the MLD in coastal zones, and the robustness of the findings in the subpolar oceans may be limited; for future research, we propose incorporating additional observational datasets covering the regions not extensively mapped by Argo to expand the scope and robustness of this study. Also, it would be instructive to extend the intercomparison of MLD methodologies by incorporating additional methodologies such as those analyzed by Tang et al. (2025), who found that the linear fitting method of Chu and Fan (2010) resulted in the most robust one in calculating the MLD.

Recent research has highlighted temperature inversions as a significant limitation of various MLD methodologies, not only those based on temperature, which restricts their application in regions where temperature inversions are common (Tang et al., 2025). However, density-based methodologies, such as WB, could have an advantage over temperature-based ones because they adequately incorporate the effects of temperature on mixing conditions into density via the equation of state of seawater. Consequently, WB could adequately account for the effects of temperature on mixing and in the MLD calculation. Although the mixed layer has been commonly described in terms of physical variables (temperature or density), ecological and chemical variables (chlorophyll and oxygen) are also very relevant in evaluating mixing conditions along the vertical (Sutherland et al., 2014; Tang et al., 2025).

The performance of EBM in calculating the MLD and the associated vertical distribution of different ecological and chemical variables is proposed for future research.

Because the turbulence and its associated energy levels are spatially and temporally variable on a global scale, the water column's stratification and vertical homogenization are not spatially uniform through the seasons. Globally, the mixed layer is not associated with a unique density threshold, e.g., see Table 1 of Kara et al. (2000), de Boyer Montégut et al. (2004), and Peralta-Ferriz and Woodgate (2015); the compensated layers exemplify that a unique density threshold is inappropriate for the world ocean (de Boyer Montégut et al., 2004). Therefore, the $\Delta\overline{\rho}^{\theta}$ threshold is not expected to be globally uniform year-round, and WB's associated spatial distribution across time is still an open question. A very preliminary analysis of the energy levels at the mixed layer base suggested that a unique WB equipotential in the interval 12.5-20 J m^{-3} could define the MLD globally year-round. This finding contrasts with Reichl et al. (2022), who did not provide specific energy values to define the MLD across the world ocean during all seasons. They found that a spatially and temporally variable energy threshold should be used to reproduce, to some extent, MLDs similar to those obtained with HT09. However, since HT09 is non-energy-based, trying to match the performance of energy-based methodologies to that of non-energy-based ones may not be meaningful. Testing the hypothesis that a few or even a unique WB threshold can characterize the MLD globally year-round could determine if the mixed layer is energetically consistent across space and time. Such a study would require long-term data and a regionalization of the WB thresholds on a global scale. Exploring this hypothesis is an endeavor that deserves further research since it could enhance the way we understand the mixed layer and the different ocean-atmosphere phenomena in which the MLD is relevant.

The EBM-MLD depends on the choice of the WB threshold, which we set based on a $\Delta\overline{\rho}^{\theta}$ threshold. A significant improvement for EBM would be constructing a criterion to unequivocally determine the WB threshold characterizing a well-mixed layer independently of density. A mathematical problem of this nature would lead to trying to find the solution of only one equation with two unknowns (Eq. 8), an ill-posed problem. Solving this problem is not trivial because we must have an extra condition or equation to have uniqueness and turn the problem into a well-posed one. In the absence of an additional equation to determine the unique solution for WB, the choice of the value of the remaining variable would be subjective or, at least, based on experience (like in the common threshold MLD methodologies). However, from the energy definition of the mixed layer, we can explore some geometric methods to determine the layer quasi-homogeneous in energy with small WB values without specifying an associated density variation. Methods like the quality index of Lorbacher et al. (2006) or the maximum angle method of Chu and Fan (2011) could be helpful; however, they assume a structural change in the variable of interest and are not suitable for strongly stratified or very smooth density profiles in which a structural change is difficult to find. For these profiles, a specific WB threshold characterizing a quasi-homogeneous layer in energy is needed. Additional research concerning the physical properties of WB and the values that accurately determine the vertical extension of the mixed layer is required and proposed for future research.

5. Conclusions

Recent research has proposed energy-based methodologies as the best option to calculate the MLD, as they can provide accurate estimates while maintaining the calculations without the unnecessary complexities of the turbulent mixing theory. We contribute to the development of energy-based methodologies to define the MLD. Based on energy considerations, our proposed MLD methodology is globally applicable and produces realistic estimates of the MLD. ~~We found that a unique energy equipotential can define the MLD across the world ocean throughout the year.~~ The mixed layer, determined by energy processes, is ~~also~~ quasi-homogeneous in energy, density and temperature in most of the global ocean ~~during most of~~ throughout the year. A practical contribution of our work is an observation-based global MLD climatology, useful for seasonal to climate time scale studies from regional to large spatial scales. This climatology can also be used as a reference to validate Oceanic General Circulation Model solutions and perform MLD model intercomparison studies. Currently, we are working on investigating the potential of this new MLD methodology to better interpret various dynamic (e.g., vertical exchanges within the ocean and between the ocean and the atmosphere), thermodynamic (e.g., upper ocean heat content), and ecological (e.g., chlorophyll-a content and phytoplankton dynamics) processes at regional and global scales.

Supplement

The Supplement was rewritten to provide a detailed comparison of our MLD methodology with the common ones; we included global monthly climatologies of the MLD and the WB at the MLD.