S1 - Analytical solutions of AGWP and AGTP metrics for valid CO_2, CH_4 and N_2O.

$$AGWP_{CO2}(H) = A_{CO2}\left\{\alpha_0 H + \sum_j^3 \alpha_j \tau_j \left(1 - e^{-\frac{H}{\tau_j}}\right)\right\} \quad \text{(S1.1)}$$

$$AGTP_{CO2}(TH) = A_{CO2}\sum_{j=1}^{2}\left\{\alpha_0 c_j \left(1 - \exp(\frac{-TH}{d_j})\right) + \sum_{i=1}^{3}\frac{\alpha_i \tau_i c_j}{\tau_i - d_j}\left(\exp(\frac{-H}{\tau_i}) - \exp(\frac{-TH}{d_j})\right)\right\} \quad \text{(S1.2)}$$

For non-CO_2 gas i :

$$AGWP_i = A_i \tau_i (1 - e^{-\frac{H}{\tau_i}}) \quad \text{(S1.3)}$$

$$AGTP_i(TH) = A_i \sum_{j=1}^{2}\frac{\tau c_j}{\tau_i - d_j}\left(\exp(\frac{-TH}{\tau_i}) - \exp(\frac{-TH}{d_j})\right) \quad \text{(S1.4)}$$

S2 - Table S2: Detailed climate parameters values used for climate metrics

Variable	Definition	Unit	Value	Uncertainty	Uncertainty distribution type	Source
H	Time horizon	Years	1–1000	-	-	(Joos et al., 2013)
$AGWP_{CO2}$						
RE_{CO2}	Radiative efficiency in CO_2 mixing ratio	$W.m^{-2}.ppb^{-1}$	1.33×10^{-5}	0.16×10^{-5}	Normal 1.645σ	(Forster et al., 2021)
A_{CO2}	Radiative forcing scaling factor	$W.m^{-2}.kg^{-1}$	1.71×10^{-15}	0.21×10^{-15}	Normal 1.645σ	-
α_{0-3}	Coefficient for fraction of atmospheric CO_2 associated with a nominal timescale	Unitless	$\alpha_0 = 1- \alpha_1 - \alpha_2 - \alpha_3$; $\alpha_1 = 0.2240$; $\alpha_2 = 0.2824$; $\alpha_3 = 0.2763$	-	-	(Joos et al., 2013)
τ_{1-3}	IRF_{CO2} nominal timescale	Years	$\tau_1 = 394.4$; $\tau_2 = 36.54$; $\tau_3 = 4.304$			
$AGWP_{CH4}$						
RE_{CH4}	Radiative efficiency in CH_4 mixing ratio without indirect effects	$W.m^{-2}.ppb^{-1}$	3.89×10^{-4}	0.96×10^{-4}	Normal 1.645σ	(Forster et al., 2021)
τ_{CH4}^{OH}	Total chemical lifetime of methane	Years	9.7	1.1	Normal 1σ	(Szopa, 2021)
$\tau_{atm, CH4}$	Total atmospheric lifetime	Years	9.1	0.9	Normal 1σ	(Szopa, 2021)
f_{CH4}	Feedback factor	Unitless	1.30	0.07	Normal 1σ	(Thornhill et al., 2021)
τ_{CH4}	Perturbation lifetime	Years	11.8	1.8	Normal 1.645σ	(Forster et al., 2021)
f1	Tropospheric ozone indirect effect scaling	Unitless	0.36	0.18	Normal 1.645σ	(Forster et al., 2021)
f2	Stratospheric water vapour indirect effect scaling	Unitless	0.103	0.103	Normal 1.645σ	(Forster et al., 2021)
A_{CH4}	Radiative forcing scaling factor with indirect effect	$W.m^{-2}.kg^{-1}$	2.00×10^{-13}	0.49×10^{-13}	Normal 1.645σ	(Forster et al., 2021)
Y	Fractional molar yield of CO_2 from CH_4 oxidation	Unitless	0.75	[0.5 - 1]	Uniform	(Forster et al., 2021)
$AGWP_{N2O}$						
RE_{N2O}	Radiative efficiency in N_20 mixing ratio without indirect effects	$W.m^{-2}.ppb^{-1}$	3.19×10^{-3}	1.25×10^{-3}	Normal 1.645σ	(Forster et al., 2021)
A_{N2O}	Radiative forcing scaling factor with indirect effect	$W.m^{-2} .kg^{-1}$	3.6×10^{-13}	1.4×10^{-13}	Normal 1.645σ	(Forster et al., 2021)
$\tau_{atm, N2O}$	Mean atmospheric lifetime	Years	116	9	Normal 1.645σ	(Canadell, 2021)
f_{N20}	Feedback factor	Years	0.94	0.01	Normal 1.645σ	(Prather et al., 2015)
τ_{N2O}	Perturbation lifetime	Years	109	10	Normal 1.645σ	Canadell et al. (2021)
-	Methane lifetime effect in ppb(CH_4) per ppb(N_2O)	-	-1.7	1.7	Normal 1.645σ	(Forster et al., 2021)
RE_{N20}^{O3}	Upper stratospheric ozone depletion	$W.m^{-2}.ppb^{-1}$	5.5×10^{-4}	0.4×10^{-4}	Normal 1.645σ	(Forster et al., 2021)

AGTP						
λ	Net feedback parameter	$W.m^{-2}.K^{-1}$	1.16	0.40	Normal 1σ	(Forster et al., 2021)
$\Delta F_{2\times co2}$	Effective radiative forcing of CO_2 doubling	$W.m^{-2}$	3.93	0.47	Normal 1σ	(Forster et al., 2021)
ECS	Equilibrium climate sensitivity from constrained ensemble	$K.(W.m^{-2})^{-1}$	0.76	0.28	Normal 1σ	(Forster et al., 2021)
c_1	ECS fractional contribution of the fast mode	-	c_1 = 0.586		Normal 1σ	(Forster et al., 2021)
c_2	ECS fractional contribution of the slow mode	-	c_2 = 1 - c_1	-	Normal 1σ	(Forster et al., 2021)
d_1	Fast relaxation time	Years	d_1 = 3.4	-	Normal 1σ	(Forster et al., 2021)
d_2	slow relaxation time	Years	d_2 = 285		Normal 1σ	
CCf						
γ	IRF_{CCf} intensity term	$kgCO_2\ yr^{-1}\ °C^{-1}$	11.06 x 10^{12}	-	-	(Gasser et al., 2017)
$\beta_{1\text{-}3}$	Coefficient for fraction of IRF_{CCf} associated with a nominal timescale	Unitless	β_1=0.6368 ; β_2=0.3322 ; β_3=0.0310	-	-	(Gasser et al., 2017)
$\kappa_{1\text{-}3}$	IRF_{CCf} nominal timescale	Years	κ_1=2.376 ; κ_2=30.14 ; κ_3=490.1	-	-	(Gasser et al., 2017)

S3 - Analytical resolution of the convolution between IRF_{CH4} and $AGxx_{CO2}$ describing to methane oxidation

$$IRF_{CH4} * AGWP_{CO2}(H) = Y\frac{M_{CO2}}{M_{CH4}}\frac{1}{\tau_{\mathrm{CH4}}^{\mathrm{OH}}}\int_0^H e^{-\frac{t}{\tau_{CH4}^{OH}}}\int_0^{H-t} A_{CO2} IRF_{CO2}(x)\,dxdt \tag{S3.1}$$

$$= Y\frac{M_{CO2}}{M_{CH4}}\frac{1}{\tau_{\mathrm{CH4}}^{\mathrm{OH}}}A_{CO2}\left[\alpha_0\tau_{CH4}^{OH}\left(H+\tau_{CH4}^{OH}\left(e^{-\frac{H}{\tau_{CH4}^{OH}}}-1\right)\right)\right.$$
$$\left.-\sum_i\left\{\frac{\alpha_i{\tau_i}^2\tau_{CH4}^{OH}}{\tau_i-\tau_{CH4}^{OH}}\left(e^{-\frac{H}{\tau_i}}-e^{-\frac{H}{\tau_{CH4}^{OH}}}\right)+\alpha_i\tau_i\tau_{CH4}^{OH}\left(1-e^{-\frac{H}{\tau_{CH4}}}\right)\right\}\right]$$

$$IRF_{CH4} * AGTP_{CO2}(H) = Y\frac{M_{CO2}}{M_{CH4}}\frac{1}{\tau_{\mathrm{CH4}}^{\mathrm{OH}}}\int_0^H e^{-\frac{t}{\tau_{CH4}}}\int_0^{H-t} A_{CO2} IRF_{CO2}(x) IRF_T(t-x)\,dxdt \tag{S3.2}$$

$$= Y\frac{M_{CO2}}{M_{CH4}}\frac{1}{\tau_{\mathrm{CH4}}^{\mathrm{OH}}}A_{CO2}\times ECS$$
$$\times\sum_{j=1}^{2}\left\{\frac{\alpha_0 c_j\tau_{\mathrm{CH4}}^{\mathrm{OH}}d_j}{\tau_{\mathrm{CH4}}^{\mathrm{OH}}-d_j}\left(e^{-\frac{H}{\tau_{\mathrm{CH4}}^{\mathrm{OH}}}}-e^{-\frac{H}{d_j}}\right)+\alpha_0 c_j\tau_{\mathrm{CH4}}^{\mathrm{OH}}\left(1-e^{-\frac{H}{\tau_{\mathrm{CH4}}^{\mathrm{OH}}}}\right)\right.$$
$$\left.+\sum_{i=1}^{3}\frac{\alpha_i{\tau_i}^2 c_j\tau_{\mathrm{CH4}}^{\mathrm{OH}}}{(d_j-\tau_i)(\tau_i-\tau_{CH4}^{OH})}\left(e^{-\frac{H}{\tau_i}}-e^{-\frac{H}{\tau_{\mathrm{CH4}}^{\mathrm{OH}}}}\right)+\frac{\alpha_i\tau_i c_j d_j\tau_{\mathrm{CH4}}^{\mathrm{OH}}}{(d_j-\tau_i)(\tau_{CH4}^{OH}-d_j)}\left(e^{-\frac{H}{\tau_{\mathrm{CH4}}^{\mathrm{OH}}}}-e^{-\frac{H}{d_j}}\right)\right\}$$

S4 - Table S4: Fitting parameters of model responses in CO_2 (*IRF_{CO2}*) for a 100 GtC emission pulse added to a constant background of 389 ppm from (Joos et al., 2013, Supplementary Information, Table S1) considered to compute uncertainty range

model	*nryears*	a_0	a_1	a_2	a_3	τ_1	τ_2	τ_3
~~NCAR CSM1.4~~	~~289~~	~~2.935E-07~~	~~3.665E-01~~	~~3.542E-01~~	~~2.793E-01~~	~~1.691E+03~~	~~2.836E+01~~	~~5.316E+00~~
~~HadGEM2-ES~~	~~101~~	~~4.340E-01~~	~~1.973E-01~~	~~1.889E-01~~	~~1.798E-01~~	~~2.307E+01~~	~~2.307E+01~~	~~3.922E+00~~
~~MPI-ESM~~	~~101~~	~~1.252E-07~~	~~5.864E-01~~	~~1.826E-01~~	~~2.310E-01~~	~~1.781E+02~~	~~9.039E+00~~	~~8.989E+00~~
Bern3D-LPJ	1000	6.345E-10	5.150E-01	2.631E-01	2.219E-01	1.955E+03	4.583E+01	3.872E+00
Bern3D-LPJ (ensemble)	585	2.796E-01	2.382E-01	2.382E-01	2.440E-01	2.762E+02	3.845E+01	4.928E+00
Bern2.5D-LPJ	1000	2.362E-01	9.866E-02	3.850E-01	2.801E-01	2.321E+02	5.850E+01	2.587E+00
CLIMBER2-LPJ	1000	2.318E-01	2.756E-01	4.900E-01	2.576E-03	2.726E+02	6.692E+00	6.692E+00
DCESS	1000	2.159E-01	2.912E-01	2.410E-01	2.518E-01	3.799E+02	3.631E+01	3.398E+00
GENIE (ensemble)	1000	2.145E-01	2.490E-01	1.924E-01	3.441E-01	2.701E+02	3.932E+01	4.305E+00
LOVECLIM	1000	8.539E-08	3.606E-01	4.503E-01	1.891E-01	1.596E+03	2.171E+01	2.281E+00
MESMO	1000	2.848E-01	2.938E-01	2.382E-01	1.831E-01	4.543E+02	2.500E+01	2.014E+00
UVic2.9	1000	3.186E-01	1.748E-01	1.921E-01	3.145E-01	3.046E+02	2.656E+01	3.800E+00
ACC2	985	1.779E-01	1.654E-01	3.796E-01	2.772E-01	3.862E+02	3.689E+01	3.723E+00
Bern-SAR	1000	1.994E-01	1.762E-01	3.452E-01	2.792E-01	3.331E+02	3.969E+01	4.110E+00
MAGICC6 (ensemble)	604	2.051E-01	2.533E-01	3.318E-01	2.098E-01	5.961E+02	2.197E+01	2.995E+00
TOTEM2	984	7.177E-06	2.032E-01	6.995E-01	9.738E-02	8.577E+04	1.118E+02	1.583E-02
multi-model mean	1000	2.173E-01	2.240E-01	2.824E-01	2.763E-01	3.944E+02	3.654E+01	4.304E+00

In this study, all models that compute IRF CO_2 at least up to 500 years (long-term time horizon in climate metrics) have their fit extended up to 1000 years for the uncertainty range. In other words, fits from NCAR CSM1.4, HadGEM2-ES and MPI-ESM are not considered here.

S5: IRF_T to match MAGICC's behaviour used in AR6

https://gitlab.com/magicc/ar6-wg1-plots-and-processing/-/blob/main/notebooks/calculate-temperature-irf-for-ch6/110_calculate_temperature_irf_from_2xCO2.ipynb?plain=0&blame=1

See out [33]