Authors’ response to referee comment #1

RC - Referee’s comment

AR - Authors’ response

RM - Revised manuscript

RC *The authors present a feasibility study for using SNMR to detect an englacial channel at the Rhonegletscher. The study focuses on the challenges encountered in a low signal, high noise environment and how to still image an englacial channel. The approach uses a grid search of different glacial water content models to try and fit the acquired data. The results are compared against a Ground penetrating radar (GPR) survey and show consistency between the two methods.*

AR We thank the referee for taking the time to provide us with such detailed feedback on our manuscript. In the section below, we address each comment individually and highlight the changes in the revised manuscript (underlined and blue).

Main comments

Comment #1a

RC *My comments relate mostly to the noise estimation and how the grid search is performed. It is mentioned that the average noise level is 70 nV for most pulse moments (P.12 L.268), but inspecting Figure 7, the largest error bar found here is ~18nV wide.*

AR It is correct that the average noise level amounts to 70 nV, while the error bars in Fig. 7 (or Fig. 6a) show significantly lower values. The two values are related to each other but they represent different types of uncertainties:

- Average noise level (see P.11, L. 249 – 254): The average noise level corresponds to the average data uncertainty. We calculate it by averaging all standard deviations $\sigma_D(q, t_i)$ that we compute for each time sample, pulse moment, and the real and imaginary parts of the time series.
- Error bars in Fig. 7 (see Section 3.2.1 and 4.1, specifically P.9, L.191 – 200, P. 12, L. 263 – 268): The error bars in Fig. 7a correspond to the standard deviation of the initial value $e_0(q)$. The initial value is derived from the four parameters $m(q)$ describing the mono-exponential decay. They are estimated with a least-squares approach. To assess the posterior uncertainties of the four model parameters, we compute the covariance matrix at the maximum-likelihood point $\tilde{C}_m \approx \left(G^T C_D^{-1} G\right)^{-1}$. G is the linearised forward operator and C_D corresponds to the data covariance, containing all data variances $\sigma_D(q, t_i)^2$. Ultimately, the standard deviation of the initial value (i.e. error bars in Fig. 7a) is retrieved from Eq. 4 in the manuscript, which

takes into account the extrapolation of the amplitude to earlier times (see. Eq. (3) in the manuscript).

We understand that the sentence on P. 12, L. 267, *"We observe amplitudes between 0 and 110 nV corresponding to roughly the order of magnitude of the average noise level (70 nV) for most pulse moments."*, might be misleading. In this sentence, we refer to the S/N, which we define as the ratio between the initial value and the average noise level.

RM In the revised version of the manuscript, we state the definition of the error bar in Figs. 6 and 7 more clearly in the corresponding caption. In addition, we highlight the difference between the average noise level and the error bars in Figs. 6 and 7:

Figure 6. Estimation of the parameters from the mono-exponential fit (cf. Eqs. 2 and 3) with corresponding standard deviations (cf. Eq. 4 and the definition of the covariance matrix in Section 3.2.1) as a function of pulse moment. The coincident-loop data and the separate-loop data are shown in blue and orange, respectively. (a) Initial value e_0, (b) relaxation time T_2^*, (c) frequency offset δf, and (d) phase ϕ.

Figure 7. Comparison of the measurements (dots with error bars, cf. Fig. 6a) and the synthetic sounding curves based on the minimum RMS-misfit models (lines) for the coincident-loop (blue) and the separate-loop configuration (orange). The different line types correspond to the three different models presented in Figure 4. (a) Comparison of the synthetic and measured sounding curve based on the coincident-loop configuration. (b) Comparison of the synthetic and measured sounding curve based on the coincident- and separate-loop configuration (joint data).

Sec. 4.1, L. 268:

Figure 6a depicts the estimated initial values $e_0(q_i)$ as a function of the pulse moments q_i (sounding curve). We observe amplitudes between 0 and 110 nV corresponding to roughly the order of magnitude of the average noise level after processing σ_D (70 nV) for most pulse moments. Note the difference in the magnitude between the average noise level σ_D and the estimated standard deviation of the initial values σ_{e_0} represented by the error bars in Fig. 6a. The two values are related to each other, but they represent different types of uncertainties (see Sec. 4.1 and 3.2.1 for their definition).

Comment #1b

RC *In this figure (7), we see the forwarded data from three model scenarios having difficulties fitting the observed data within error bars. From one pulse moment to the next, the signal amplitude doubles and then drops by 35%, a difference way larger than the assigned error bars. Is the difficulty in fitting this data a product of the* *simplified model scenario, or could it be a product of underestimating the uncertainty affecting the initial values?*

AR This is an excellent question and we see three possible explanations for the discrepancy between the fit and the data in Fig. 7:

Data:

- Misestimation of the initial values:
  - Systematic misestimation as a result of processing (currently discussed in Sec. 5.4.1): RNC possibly distorts the signal up to 27 nV for the highest pulse moment (see L. 403). We expect a non-linear relationship between the amplitude of distortion and the pulse moment (see Fig. A5 and the fact that the phase of the transfer function does not have to be constant).
  - Misestimation as a result of assuming a mono-exponential decay (currently discussed in Sec. 5.4.2, L.417 – 437): We fit the complex envelopes (Fig. 3) with mono-exponential decays (Eq. 2), assuming that all spins contributing to the signal exhibit similar relaxation times. This assumption might be inaccurate because the presence of impurities could lead to a multi-exponential decay.
- Misestimation of the standard deviation of the initial values (currently mentioned on P.9, L.191-200): When computing the standard deviations, we linearise the forward operator and assume normally distributed model parameters. Both assumptions might lead to a misestimation of the standard deviation.

Fit:

- Overly simplified water models (currently discussed in Sec. 5.4.2, L. 418 – 444): The one-dimensionality of the water models considered in this study is a significant simplification, since other studies like the GPR survey presented in Figs. 1b and 9 suggest a three-dimensional structure.

RM In the revised version of the manuscript, we adjusted and slightly rearranged the last paragraph of Sec. 4.2.1 (L. 302 - 306). We point to the relevant sections discussing the possible explanations mentioned above:

If the synthetic data $e_0^{syn}(q_i)$ fit all of the observations $e_0(q_i)$ within their observational uncertainty $\sigma_{e_0(q_i)}$, we expect $\chi^{RMS} \approx 1$. In our case, none of the models reach this value, suggesting a slight under-fitting. For instance, even the best model fails to replicate the amplitudes at lower pulse moments for the separate-loop data (Fig. 7b). This under-fitting is likely an expression of our simplified forward problem (cf. discussion in Sec. 5.4.2), a misestimation of the initial values $e_0(q_i)$ (cf. discussion in Sec. 5.4) or the initial values' uncertainties $\sigma_{e_0(q_i)}$ (limitations mentioned in Sec. 3.2.1).

Comment #2

RC *Even with Equation 4 (P.9 L200), it is still unclear to me how the mean noise of 70nV becomes maximum 17nV in uncertainty on the model parameter e0. Please clarify.*

AR See response to comment #1a for the mathematical definition. We hope this sufficiently clarifies your question.

RM See response to comment #1a for how we clarified this in the text.

Comment #3

RC *In Figure 8b, the misfit for the models with a varying aquifer depth is shown. But unlike 8a, it seems it has not yet reached the lowest misfit, i.e., maybe an aquifer depth of 62m would be a better fit. Were the ranges chosen on previously acquired data (GPR)? If not, perhaps increasing the range here could reveal a similar parabola shape, like the one in Figure 8a.*

AR Yes, we chose the maximum depth based on the previously acquired GPR data, suggesting an average ice thickness of about 60 m in the survey area. Given the complexity of the aquifer geometry, e.g. non-horizontal ice surface and bedrock boundary, this is considered a robust estimate and thus did not investigate aquifer depths below that level.

RM In the revised version of the manuscript, we state more explicitly that we chose the maximum depth based on previously acquired GPR data (Sec. 3.2.2, L. 217-219):

…The one-layer model consists of a 60 m thick, uniform ice column with a homogeneous liquid-water content (LWC) x_{ice} (1 parameter). We chose the maximum depth based on the previously acquired GPR data, suggesting an average ice thickness of about 60 m in the survey area. …

Comment #4

RC *These results of aquifer depth are later discussed (P.18 L. 359-361) as broadly consistent with the GPR profile which finds a channel at 40m. But the lowest misfit for the SNMR was with a channel at 59m depth.*

AR It is true that the lowest misfit is found for an aquifer at 59 m depth and that GPR measurements indicate a more shallow conduit. Since our one-dimensional water model is a simplification of reality and cannot represent a three-dimensional conduit, however, we are not surprised by this deviation from the GPR data.

RM In the revised version of the manuscript, we explicitly state that the minimum RMS-misfit model has a more shallow channel than indicated by GPR data (Sec. 5.2., L.360). We also extended the end of the section with a short discussion about the possible origin of this discrepancy (L.365):

… From the GPR data, the average depth of the channel is around 40 m below the transmitter loop (Fig. 9). This is somewhat shallower than the minimum RMS-misfit model (59 m), but broadly consistent with the parameter distributions obtained from our SNMR investigations, which indicate a channel depth between 41 and 59 meters (Fig. 8b).

In addition to the channel, the GPR signals also reveal weak bedrock reflections and various features that we interpret as being part of the glacier's drainage system (including a surface water streams and possibly, a water-filled fracture; cf. Fig. 9). The spatial distribution of these partially englacial features indicates that our one-dimensional water models (cf. Fig. 4) might be an oversimplification as all of them have variable, three-dimensional shapes.

The simplified forward model could be the driver for the discrepancy between the aquifer depth of the minimum RMS-misfit model and the GPR findings. Additional factors may play a role too, such as signal distortions due to RNC, resulting in an overestimation of the aquifer depth. In Section 5.4, we further discuss the various limitations and their potential impact on the estimated model parameters.

Comment #5

RC *The RNC possibly distorting the signal up to 27nV is quite concerning since it is >25% of the maximum initial value seen (Figure 7). This is addressed in the conclusion, but only after stating that the RNC was the most crucial step in increasing S/N. Perhaps a more combined conclusion on RNC could highlight the usefulness and the issues with this approach.*

AR We generally agree with the comment that it is important to jointly highlight the usefulness and issues of RNC, which we tried to implement with a different introduction in Sec. 5.4.2. However, to maintain the current structure of the discussion section, we decided to still focus on the limitations of RNC in Sec. 5.4.1.

RM In the revised version of the manuscript, we now mention a possible distortion due to RNC already in Section 4.1; and start Section 5.4.1 by highlighting both the usefulness and the issues of RNC before providing more details.

Sec. 4.1., L. 247 - 249

After processing the data according to the scheme in Figure 2, the signal-to-noise ratio of the time series increased significantly. While the application of DS and HNC slightly improved the S/N, the application of RNC was essential to reduce the noise level by an order of magnitude (Fig. A1). We note that noise cancellation with RNC has limitations due to possible distortions of the signal and discuss these in Sec. 5.4.1.

Sec. 5.4.1.

While RNC is the most crucial step in our noise-cancellation sequence, its usefulness is limited by its potential to distort the SNMR signal. In the following section, we attempt to estimate the effect of this distortion. For optimal noise cancellation, one wants to maximise the correlation between the time series of the remote reference loops and the receiver loop while detecting the SNMR signal exclusively in the receiver loop.

Comment #6

RC *Additionally, since a noise record has been recorded, would it be possible to use RNC on the noise only data and examine if the transfer functions are different? If they are different, it might be a sign of signal being distorted.*

AR In principle, yes: If the noise was stable, we could have used a global transfer function. However, the noise conditions in this survey were not stable, and the transfer functions changed significantly between individual measurements. We thus used local transfer functions, i.e. transfer functions that are computed for each recording. Against these unstable noise conditions, a more detailed analysis of the transfer functions was not considered useful.

RM In the revised version of the manuscript, we add a sentence on the use of local transfer functions (Sec. 3.1.1., L. 142):

3) Remote Reference Noise Cancellation (RNC) targets the noise of unknown characteristics, which is dominating our data. We deployed two remote reference loops to record the time series simultaneously with the two receiver loops (Fig. 1b). For this analysis, we only use the data from the loop further away to perform RNC, thereby reducing the amount of SNMR-signal contamination in the remote reference loop (see discussion in Section 5.4.1). To perform the cancellation and since the noise conditions were not stable, we used so-called local transfer functions, i.e. functions that are computed for each recording (Müller-Petke and Costabel, 2014).

Comment #7

RC *When assuming 100% water it vastly reduces the aquifer thicknesses found fitting data within the threshold. But is the instrument capable of resolving a <1m thick layer at 40m to 60m depth? Perhaps add some discussion on whether this is feasible given the selected pulse moments and loop dimensions.*

AR We cannot resolve a 1 m layer at 40 – 60 m only using our SNMR data set. The data points in Fig. 8 should be seen as plausibility estimates for the respective model realisation. By using additional constraints based on assumptions or data from a different method, it is possible to further constrain the (water-model) parameter ranges. For example, in Fig. 8, we implicitly assume only positive water contents and set a maximum depth of 60 m (obtained from GPR data), which already constrains the ranges of the water-model parameters. In Fig. A4, we go one step further by assuming a minimum water content of 60% in the aquifer, which drastically reduces the estimated aquifer thickness to 1 m or thinner. Therefore, by adding further information (assuming a minimum water content of 60 %, because we expect a conduit mostly filled with water), the range of possible aquifer thickness was reduced to 1 m or less.

RM In the revised version of the manuscript, we added a sentence to Sec. 5.1 on the resolution of thin layers (L. 349):

…By doing so, the range of aquifer thicknesses decreases drastically, allowing for values between 0.2 and 1.0 meters. The ranges for the other parameters remain very similar to the ones in Figure 8. In conclusion, based on the information in Fig. 8 alone, we cannot resolve thin layers (≤ 1 m). However, by introducing additional information based on assumptions (e.g. minimum water content) or data from a different method (GPR in our case), it is possible to further constrain the range of the parameters in the water model – such as the aquifer thickness.

Comment #8

RC *A question about the englacial channel. I assume the water flowing within this channel, if so, how quickly? It might reduce the signal amplitude and should be discussed if appropriate.*

AR It is correct that the water is generally flowing within subglacial channels, typically with velocities not exceeding 1m/sec (Werder et al., 2010). Flow typically has an impact on the estimates of relaxation time and water content in lab- or borehole-

NMR measurements, where the water molecules move through a heterogeneous magnetic (either due to an artificial magnetic field gradient in lab or logging measurements or a pore space gradient) field during the measurement (Callaghan, 1991). To be relevant, both the gradient and the flow need to be high. A high flow in a homogeneous field does not impact the measurement. Thus, we assume this effect to be negligible in our measurement, because of the presumably homogeneous Earth's magnetic field, and a small displacement of the molecules during the recording time (since the recording time is 1 sec and assuming the maximal flow velocity of 1m/s mentioned above, the expected maximal displacement is in the order of 1 meter). However, one would need to further investigate this effect to fully understand its impact in our case.

We decided to not include this into the discussion as we assume this to be a minor effect and very likely not relevant .

RM No changes are made in the revised version of the manuscript.

Minor comments

Minor comment #1

RC *P.5 L.108: The 16th q was not completed. Could this have helped constrain the aquifer depth in Figure 8 by increasing the depth of investigation?*

AR In general, yes, since the higher the pulse moment, the higher the sensitivity at deeper layers. However, due to time constraints, we were not able to repeat the incomplete measurement at the 16th q.

RM No changes are made in the revised version of the manuscript.

Minor comment #2

RC *P.12 L.261: The peaks at -20Hz are not seen in noise only spectrum in Figure 5b. Are these harmonics or related to transmitting at high pulse moments? And what harmonics do you expect at this frequency?*

AR The peaks at around -20 Hz are present in both spectra but less visible in the noise-only one. One reason for the difference is the visualisation is that the scale of Fig. 5b is slightly smaller than in Fig. 5a. In addition, this Figure is a 3D plot that was manually tilted. The tilting angle of the two figures might differ slightly, resulting in a different height of the peaks. Another reason could be processing: The presence of the SNMR signal in the data in Fig. 5a might have had an impact on the processing, i.e. the processing result might slightly differ for traces with and without SNMR signal.

We removed higher harmonics of ~16.6 Hz and ~50 Hz. The origin of the peaks at around -20 Hz is currently unknown. We suspect that the appearance at higher pulse moments is a temporal effect (i.e. source started emitting noise later in the day when the recordings at higher pulse moments occurred) and has no causal relationship with the pulse moment.

RM We added a sentence in Sec. 4.1, L. 261:

The peaks at around 20 Hz indicate the presence of some residual higher harmonics that could not be removed with our processing routine. We suspect that the appearance of those peaks at higher pulse moments is a temporal effect (i.e. source started emitting noise later in the day when the recordings at higher pulse moments occurred) and has no causal relationship with the pulse moment.

Minor comment #3

RC *Figure 6a: Is it expected that the separate and coincident coil shows very different initial values? Is the water content lower here or is it mainly a product of less excitation?*

AR Coincident- and separate-loop measurements have a different spatial sensitivity below the loops. Consequently, the so-called sounding curves show quite different shapes. Separated loops generally show smaller initial values e_0 readings at small pulse moments and larger initial values at higher pulse moments, when larger quantities of excited spins are excited below the receiver loop. The curves can have largely different shapes, depending on the size and direction of the loop offset (Hertrich et. al, 2005).

RM We added a sentence on this effect in Sec. 4.2.1, L. 301:

…Based on these observations, we only consider three-layer models from now on. Note that the sounding curves of the separate- and coincident-loop data differ substantially (cf. Fig. 7b), which is a result of the difference in spatial sensitivity of the two configurations (Hertrich et al., 2005).

Minor comment #4

RC *Figure 8: Layout of figure is a bit confusing having the upper panel be (a),(b),(e), and the lower panel being (c),(d),(f). Perhaps consider three rows with a,b and c,d and lastly e,f..*

AR We chose a more compact layout on purpose, since the figure would need a whole page otherwise. To avoid the confusion, we added a vertical line between the second and third column which apparently was not visible enough.

RM In the revised version of the manuscript, we make the vertical line in Fig. 8 more visible.

Minor comment #5

RC *Figure 9: Consider marking the maximum observed dimension of the englacial channel according to Church et al., 2021, if feasible.*

AR The maximum observed dimension of the englacial channel according to Church et. al., 2021, can be estimated from Fig. 1a. We judge that including this information also in Fig. 9 would not directly contribute to the key message of our paper and could confuse readers. Thus, we decided to leave Fig. 9 as it is.

RM No changes are made in the revised version of the manuscript.

The following suggestions are directly implemented in the revised version of the manuscript.

P. 11 L.242: Indicate the abbreviation, i.e., "both the coincident(coi)- and separate(sep)-loop data…"

P20. L.426: a space missing between ",accumulate"

P.21 L. 464: A year is missing on the Ogier et al. reference.

References

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